JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

T-3, EXAMINATION- 2023

Ph.D.: Mathematics

COURSE CODE (CREDITS): 13P1WMA232 (3) MAX. MARKS: 35

COURSE NAME: MATHEMATICAL ANALYSIS

COURSE INSTRUCTORS: MDS MAX. TIME: 120 Minutes.

***Note: Note: Note: (a)** All questions are compulsory.*
(b)Marks are indicated against each question in square brackets.
(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

Quest.(1) Let $\mathcal{S}$ denote the set of all (bounded or unbounded) sequences and the metric d defined by

$$d(x,y)=\sum_{j=1}^{\infty}\frac{1}{2^j}\left(\frac{|\xi_j-\eta_j|}{1+|\xi_j-\eta_j|}\right),$$

where $x=(\xi_j)_{j=1}^{\infty}$ and $y=(\eta_j)_{j=1}^{\infty}$. Show that $(\mathcal{S},d)$ is a metric space [5]

Quest.(2) Prove that if M is a nonempty subset of a metric space (X,d), and $\bar{M}$ its closure, then- $x\in\bar{M}$, if and only if there is a sequence (x_n) in M, such that $x_n\to x$, as $n\to\infty$. [5]

Quest.(3) Show that the space $l^p=\{x=(\xi_j)_{j=1}^{\infty}:\sum_{j=1}^{\infty}|\xi_j|^p<+\infty\}$, *where* $(p\geq 1)$, is a

(a) Normed linear space, with norm $\|x\|=\left(\sum_{j=1}^{\infty}|\xi_j|^p\right)^{\frac{1}{p}}$

(b) Banach space with induced metric $d(x,y)=\|x-y\|=\left(\sum_{j=1}^{\infty}|\xi_j-\eta_j|^p\right)^{\frac{1}{p}}$ [3+4]

Quest.(4) Evaluate $\oint_C\frac{z^3+3}{z\,(z-i)^2}\,dz$, where C is the contour shown in figure(a) [4]

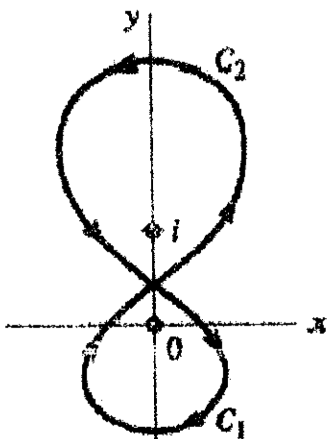

Figure (a)

Quest.(5) Evaluate the integral , by using residue theorem [5]

$$\oint_C \frac{z^3+3}{z\,(z-i)^2}\,dz,$$

where $C: |z| = 2$.

Quest.(6) Expand $f(z) = \frac{1}{(z-1)^2(z-3)}$, in a Laurant series valid for $0 < |z-1| < 2$. [3]

Quest.(7) (a) Consider $f(t) = 3t - 5$ and $g(t) = t^2$ in the polynomial space $P(t)$, with inner product

$$\langle f, g\rangle = \int_0^1 f(t)g(t)dt$$

Find (i) $\langle f, g\rangle$ and (ii) $\|f\|$

(b) Define Hilbert space with an example and also check whether the space $C[a,b]$ (space of all real valued functions, defined and continuous in the closed interval [a,b]) is a Hilbert space or not ? Justify your answer. [3+3]