JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -1 EXAMINATION- MARCH-2023 (Ph.D. Maths, I Sem)

COURSE CODE(CREDITS): 17P1WMA111(3) MAX. MARKS: 15

COURSE NAME: DIFFERENTIAL GEOMETRY

COURSE INSTRUCTOR: P K Pandey MAX. TIME: 1 Hour

***Note:** All questions are compulsory. Marks are indicated against each question in square brackets.*

1. Suppose $V = y^2U_1 - xU_3$, and let $f = xy$, $g = z^3$. Compute $fV[g] - V[V[f]]$. [CO1] [2 M]

2. For a unit speed curve $\beta \rightarrow I: R^3$ with curvature $k > 0$ and torsion τ, state and prove the Frenet formulae. [CO1] [3 M]

3. Compute the directional derivative $\boldsymbol{v}_p[f]$ for the function $f = \sqrt{3}\, x \sin yz$, with $p = (1, 3, 0)$ and $\boldsymbol{v} = (1,\ 2, -1)$. [CO1] [2M]

4. Find the coordinate functions of the curve $\beta = \alpha(h)$, where α is the curve defined by $\alpha(t) = \left(1 + \cos t, \sin t, 2\sin\frac{t}{2}\right), \forall t \in \mathbb{R}$, and $h(s) = \cos^{-1} s$ for $0 < s < 1$. [CO1] [2M]

5. If r, θ, z are the cylindrical coordinate functions of $\mathbb{R}^3$, and $x = r\cos\theta, y = r\sin\theta, z = z$. Compute the volume element $dx \wedge dy \wedge dz$ in terms of r, θ, z and their differentials. [CO1] [2M]

6. Show that the tangent vectors:

$$e_1 = \frac{(1, 2, 1)}{\sqrt{6}}, \quad e_2 = \frac{(-2, 0, 2)}{\sqrt{8}}, \quad e_3 = \frac{(1, -1, 1)}{\sqrt{3}}$$

Constitute a frame. Express $\mathbf{v} = (6, 1, -1)$ as a linear combination of these vectors. [CO2] [2M]

7. For curve $\boldsymbol{\alpha}(t) = \left(2t, t^3, \frac{t^3}{3}\right)$, find the arc length function $s = s(t)$ based at $t = 0$.[CO2][2M]

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