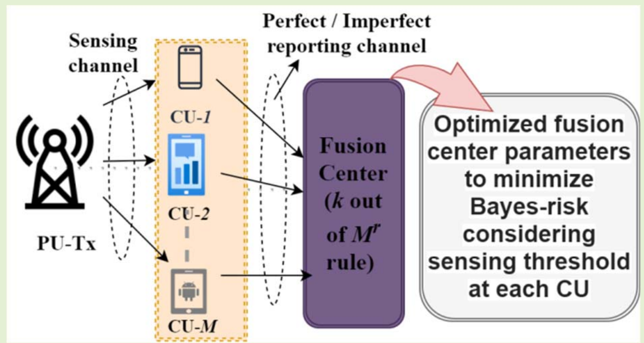
Optimization of Fusion Center Parameters With Threshold Selection in Multiple Antenna and Censoring-Based Cognitive Radio Network

Alok Kumar, Shweta Pandit, Prabhat Thakur, and Ghanshyam Singh

***Abstract*—Cognitive radio technology is a potential contender to fulfil the demand of spectrum/bandwidth for a large number of connected devices of the next-generation internet of things (IoT) network. Spectrum sensing is the crucial step of cognitive radio, and its performance is affected by the selection of sensing threshold and cooperation among multiple cognitive users (CUs). The accuracy of spectrum sensing results is a major concern in cognitive radio networks (CRN). Therefore, in this paper, we have minimized the Bayes risk which deals with the spectrum sensing error. In general, the *k*-out-of-*M* fusion rule is employed at fusion center (FC) in the cooperative CRN and optimal *k* and *M* with selection of spectrum sensing threshold results in minimum Bayes risk. Further, we have derived the expression for optimal value of CUs (*k* and M^r) in *k*-out-of-M^r rule in the cooperative spectrum sensing (CSS) at all signal-to-noise ratio (SNR) while employing different threshold selection approaches to minimize the Bayes risk at FC. The considered scenario employs multiple antennas at each CU where each CU report to the FC over the perfect/imperfect reporting channel with non-censoring ($M^r = M$) and censoring ($M^r = M^c$) approaches. Further, we have also validated the proposed results with recently reported literature and have shown that the existing expressions are the special case of the proposed generalized expressions.**



***Index Terms*—Cognitive radio, cooperative spectrum sensing, threshold selection, low SNR, imperfect reporting channel, Bayes risk.**

I. Introduction

CURRENTLY the researchers and industrialists are eagerly awaiting the Industry 4.0 (I4.0) for industrial growth [1], [2]. The revolution in I4.0, is helpful for human life in various direction such as: Internet of Things (IoT), Internet of Vehicle (IoV), Internet of Video Thing (IoVT), and Industrial Internet of Things (IIoT) [3] etc. The IoT plays an important role to connect a larger number of diverse objects together. To support larger number of connected devices in the future IoT, more operating frequency bands are required however fulfilling this requirement is challenging due to spectrum scarcity [4]. Cognitive radio technology has been employed to overcome the spectrum scarcity problem by permitting unlicensed/cognitive user (CU) to access the licensed frequency spectrum such that it does not affect the communication of licensed/primary user (PU) [5]. In the cognitive radio network (CRN), spectrum sensing (SS) is the key step for the CU to identify the status of PU on the licensed channel. Though, the sensing decision of single CU is not reliable in poor channel condition (i.e. in multipath fading and shadowing) and at low SNR consequently, the cooperative spectrum sensing (CSS) is employed to improve the reliability of sensing decision of the CRN [6], [7]. In CSS, each CU send the individual binary sensing result to fusion center (FC), where different fusion rules are employed to take final global decision about the presence or absence of PU on the licensed channel. In the reported literature [8]–[10], most popular fusion rule is k out of M in which sensing decision of FC comes in favor of active licensed channel when at least k CUs decision out of total M CUs received at FC is in the favor of active licensed channel. However, the major challenges of spectrum sensing in cognitive radio network (CRN) are the selection of sensing threshold at each CU and optimized value of k or/and M at FC to minimize or maximize the different objective

Manuscript received November 18, 2021; accepted January 6, 2022. Date of publication January 11, 2022; date of current version February 28, 2022. This work was supported by the Centre for Smart Information and Communication Systems, Department of Electrical and Electronic Engineering Science, University of Johannesburg, Johannesburg, South Africa. The associate editor coordinating the review of this article and approving it for publication was Dr. Dongyan Wei. *(Corresponding author: Ghanshyam Singh.)*

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Digital Object Identifier 10.1109/JSEN.2022.3142197

TABLE I
SUMMARY OF VARIOUS REPORTED LITERATURES TO COMPUTE THE OPTIMIZED VALUE OF k OR/AND M

Ref	Optimal value of	Minimizing parameters	Maximizing parameters	SNR	Threshold	No of Antennas at each CU	Censoring
[8]	k	Total error rate	--	High	Predefined	Single	×
[9]	k, M	--	Throughput	High	Predefined	Single	×
[10]	M	Bayes Risk	---	High	Predefined	Single	×
[11]	k, M	Bayes Risk	---	High	Predefined	Single	×
[12]	k	--	Energy Efficiency	High	Predefined	Single	×
[13]	M	Total error rate	--	Low	Predefined	Multi antenna	×
[14]	k	--	Energy Efficiency	High	Predefined	Single	×
[15]	k	Total error rate		Low	Predefined	Single	×
[16]	k, M	False Decision	Energy Efficiency	--	-----	Single	×
[17]	k	Total error rate	---	--	Predefined	Multi antenna	×
[18]	k	Bayes Risk	--	High	Predefined	Single	×
[19]	k	-------	Throughput	--	-----	Single	×
[20]	k	Total error rate	--	Low	CFAR, MEP	Multi antenna	✓
Proposed	k, M^r	Bayes Risk, Total error rate	--	Low, High	CFAR, MEP	Multi antenna	✓

functions (e.g., Bayes risk, total error rate, throughput, energy efficiency etc.) In this context, various researchers [8]–[10] achieved the optimized value of k or M in k out of M rule in different channel conditions while assuming predefined fixed value of sensing threshold, summary of which is presented in Table I. However, in most of the researcher's analysis, they have reported the optimal value of k or M at high SNR, for single antenna and for predefined fixed value of sensing threshold.

Alam *et al.* in [8] have achieved the optimal value of k to minimize the total error rate while considering a single antenna and imperfect reporting channel at high SNR. However, the researchers have computed the optimized values of k and M to maximize the throughput in [9] and optimized values of M to minimize the Bayes risk in [10] while employing the predefined fixed value of the spectrum sensing threshold at each CU in single antenna under the imperfect reporting channel in the CSS scenario. Moreover, Singh *et al.* in [13] have obtained the optimal value of M required for cooperation to minimize the total error rate at low SNR and imperfect reporting channel and multi-antenna scenario while employing fixed value of threshold. Further, at low SNR, they also shown reduced total error rate with multiple antennas employed by CUs with respect to single antenna in CRN. In [17], Singh *et al.* have achieved the optimal value of k to minimize the total error rate in multi-hop CRN. Moreover, the spectrum sensing performance of CU is improved in CSS however, the energy consumption also increases due to more number of CUs reporting to the FC in CRN, which is crucial issue for deficient battery powered CUs [14]. In this context, Althunibat and Granelli in [14] have computed the optimal value of k to maximize the energy efficiency. To achieve the optimal k value they have proposed an algorithm which reduces the energy consumption without affecting the detection accuracy. Moreover, Hu *et al.* in [12] computed the optimal value of k to maximize the energy efficiency at high SNR and for fixed value of threshold. Furthermore, Althunibat *et al.* in [16] computed the optimal value of k and M to maximize the energy efficiency and detection accuracy. Subsequently, Chauhan *et al.* in [21] have maximized the energy efficiency by proposing a technique called long/short term memory network to perform the spectrum prediction result in less energy consumption in the spectrum sensing process. However, in [22], Alhamad *et al.* have proposed a scheme for reporting channels to maximize the spectrum sensing performance at FC while employing k out of M rule. Further, Tan and Jing in [23] found out the closed-form expression of spectrum sensing threshold to maximize the sensing performance while employing k out of M rule under CSS in the heterogeneous CRN. However, Banavathu and Khan in [18] minimized the Bayes risk and computed the optimal value of k in the heterogeneous environment over imperfect reporting channels at high SNR. Further, Lin *et al.* in [24] considered the imperfect reporting channels and multi-antenna scenario and showed that decision making of CU is maximized when beam-forming reception is used under k out of M in CSS. Moreover, Liu *et al.* in [19] have achieved the value of k in k-out-of-M rule to maximize the throughput of an energy-harvesting cognitive radio network. Moreover, Bala and Ahuja in [25] have proposed a new frame structure to achieve higher throughput and energy efficiency in the non-cooperative and single antenna scenarios of CRN. Moreover, to reduce the energy consumption, censoring approach is exploited by several researchers [26], [27]. In the censoring approach, the spectrum sensing decision of limited CUs nodes are sent to the FC whose sensing decision is reliable [28]. In the context of censoring scenario, Kumar *et al.* in [20] have derived the expression for optimal value of k to minimize the total error rate at low SNR while employing CFAR and MEP threshold selection approach. Moreover, the work done by the various researchers to find the optimal value of k and M in different channel conditions are presented in Table I. Based on the above-mentioned reported literature presented in Table I, we have concluded that the researchers in [10]–[12] and [17]–[19] have optimized the value of k or M in order to minimize or maximize the different parameters (such as total error rate, Bayes risk, throughput and energy efficiency) by employing the predefined value of spectrum sensing threshold in single or multi-antenna based non-censoring scenario either at low or high SNR. Since these reported literature presented in [10]–[12] and [17]–[19] have considered fixed predefined sensing threshold either at low or high SNR which provides the limitation of computation of sensing threshold at all SNR (low as well high) in the practical wireless scenario. It is crucial to compute the sensing threshold instead of considering the predefined value of the fixed threshold at different SNR to achieve accurate spectrum sensing results. Moreover, the selection of spectrum sensing threshold at each CU affects the performance of k out of M rule in the cooperative CRN. Therefore, in this paper, we have been motivated to achieve the optimal value of k for the fixed value of M^r and the optimal value of M^r for a fixed value of k to minimize the Bayes risk in the more practical wireless scenario by considering the selection of the threshold. The novelty of the proposed model lies in the computation of optimal value of k and M^r at all SNR (low SNR as well as high SNR), single/multi-antenna system in the perfect/imperfect reporting channels with the censoring/non-censoring. In the proposed system, for the non-censoring scenario $M^r = M$ and in the censoring approach $M^r = M^c$. We have considered low and high SNR where low SNR is represented over the region with received SNR $\leq$ -12dB. In this article, we have computed the optimal value of both k, as well as M^r to minimize the Bayes risk at low and high SNR. This article is the extension of our reported work presented in [20] where we have only shown the optimal k to minimize the error probability. Moreover, the error probability is the special case of Bayes risk and results achieved in [20] on optimal k are the special cases of proposed approach presented in this article. In addition, the author's potential contributions in this article are as follows.

- We have employed a multiple antenna system with consideration of censoring scenario for Bayes risk computation in CSS cognitive radio network. The expressions for Bayes risk in the considered scenario are derived with CFAR and MEP threshold detection approaches in the imperfect reporting cognitive radio spectrum sensing system.
- For the given value, the optimization problem focuses to yield the optimal k value to minimize the Bayes risk under CFAR and MEP threshold selection approaches by considering the combined effect of the licensed channel's active/idle state probability, the number of antennas employed by each CU and imperfect reporting channel.
- In addition, we have illustrated that for a given value of k, the optimization problem emphasizes to yield the optimal value of which minimizes the Bayes risk under CFAR and MEP threshold selection approach in the considered scenario.
- The validation of achieved results of the proposed communication system has been presented with existing literature on optimal M [10] and optimal k [11], [20]. We have also illustrated that the results presented for optimal M in [10] and optimal k in [11], 20] are the special cases of the proposed approach.

This paper has been divided in the following Sections. In Section II, the proposed system model is introduced. Then, Section III discusses the analysis of the proposed spectrum sensing system model, and Section IV describes its simulation results. Finally, Section V concludes the presented work and recommends its future scope.

II. System Model

In the proposed system model, we have considered single PU transmitter (PU-Tx), M number of CU nodes employing L_a number of antennas, and one central unit called fusion center (FC) as demonstrated in Fig. 1. Each CU employs the energy detector spectrum sensing (EDSS) technique for spectrum sensing to find the status of PU channel. Nasser *et al.* [29] have presented that the selection of sensing technique depends on several factors such as PU-CU cooperation, computational complexity etc. and then concludes that the computational complexity of EDSS technique is less among other spectrum sensing techniques. Further, Atapattu *et al.* [30] have also proposed that the EDSS can detect the presence of PU at low SNR by the cooperation among CUs or/and employing large number of samples of received signal at CU terminal. Therefore, in the present analysis, we have employed EDSS technique in cooperative scenario to have better detection performance at low SNR. In addition to this, we have assumed the homogenous environment [31] and constant activity of PU [32] during the spectrum sensing process. The state of PU activity remains same during sensing duration means if PU is active on the licensed channel at the start of sensing duration it remains active or if PU is not present on the licensed channel at the start of sensing period of time, then it will not occupy the licensed channel during the sensing duration.

The CU senses the status of PU/licensed channel over the sensing channels and reports the spectrum sensing decision to the FC over reporting channels with non-censoring or censoring approach. Moreover, the reporting channels may be perfect or imperfect. The perfect reporting channels are those channels over which the received spectrum sensing results at FC is same as spectrum sensing result transmitted by the CUs. However, in case of the imperfect reporting channels, the received spectrum sensing results at FC is not same as the spectrum sensing results transmitted by the CUs. It may be inaccurate and is affected with the value of error probability in reporting channels (P_e^r). The perfect reporting channels could be considered as the special case of imperfect reporting channel when $P_e^r = 0$. In the non-censoring technique, all CUs (M) send their sensing results to the FC while in censoring scenario, M^c ($M^c \leq M$) number of CUs are reporting to the FC. The total number of reporting CUs to the FC are M^r and $M^r = M$ for the non-censoring while $M^r = M^c$ for the censoring approach. Moreover, FC apply k out of M^r rule on the received binary sensing results to take the global final decision about the active/presence or idle/absence state of PU on the licensed channel. Moreover, in Table II, we have presented the various notations and their interpretations used in the proposed system model.

III. Analysis of Proposed System Model

In this section, we have provided the method to compute the spectrum sensing threshold for false alarm and detection probability analysis at low as well as high SNR. Further, the computed value of false alarm and detection probability is used to analyze the effect of multiple antennas, perfect/imperfect reporting channel under the non-censoring/censoring scenario and to achieve the optimal value of k and M^r to minimize the Bayes risk. Initially, we have demonstrated the expressions employed for false alarm and detection probabilities at low as well as high SNR and computed the value of sensing threshold with CFAR and MEP threshold selection approaches. Afterwards, we have presented the analysis of

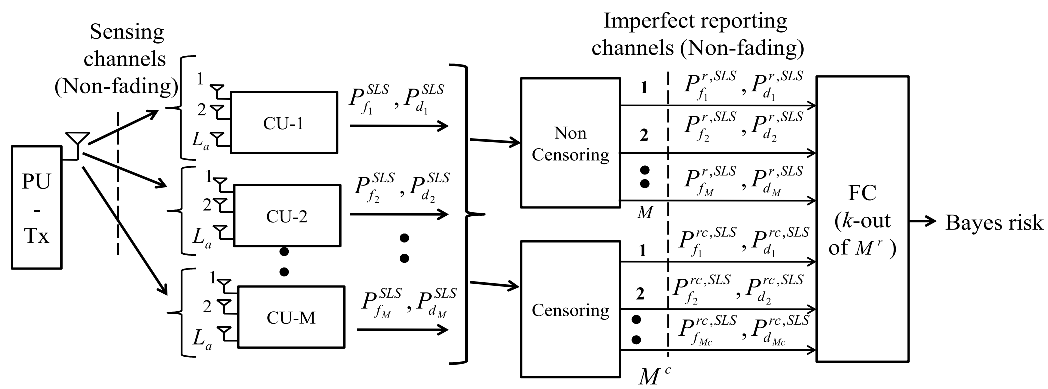


Fig. 1. Proposed system model.

TABLE II
LIST OF NOTATIONS USED IN THE PROPOSED SYSTEM MODEL

Variable	Interpretation
$P_{f_i}^{SLS}$	i^{th} CU false-alarm probability after employing SLS scheme
$P_{d_i}^{SLS}$	i^{th} CU detection probability after employing SLS scheme
$P_{f_i}^{r,SLS}$	i^{th} CU false alarm probability received at FC with non-censoring approach over imperfect reporting channel
$P_{d_i}^{r,SLS}$	i^{th} CU detection probability received at FC with non-censoring approach over imperfect reporting channel
$P_{f_i}^{rc,SLS}$	i^{th} CU false alarm probability received at FC with censoring approach over imperfect reporting channel
$P_{d_i}^{rc,SLS}$	i^{th} CU detection probability received at FC with censoring approach over imperfect reporting channel
L_a	Number of antennas employed at each CU
PU-TX	Primary user transmitter
Q_f	Global false-alarm probability at FC
Q_d	Global detection probability at FC

multiple antennas, imperfect reporting channel in censoring and non-censoring scenario under CSS. In addition, we have derived the expressions for optimal value of k for fixed value of M^r and optimal value of M^r for fixed value of k to minimize the Bayes risk under proposed system model.

A. Computation of False Alarm and Detection Probability

In this section, we have computed the false alarm (P_f) and detection (P_d) probabilities at high as well as low SNR with different threshold selection approaches (CFAR and MEP). The computation of false alarm (P_f) and detection (P_d) probabilities at each CU is given as: $P_f = P_r(T(r) \geq \lambda|H_0)$ and $P_d = P_r(T(r) \geq \lambda|H_1)$, where, $P_r(T(r))$ is the probability of test statistics of the received signal [20], H_0 and H_1 are the binary hypothesis of spectrum sensing decision comes in favor of idle and active licensed channel, respectively and λ is the sensing threshold at each CU. Moreover, these values (P_f and P_d) at high and low SNR under Gaussian channel can be computed as follows [10], [30]:

1) At High SNR:

$$P_f = \frac{\Gamma\left(u, \frac{\lambda}{2}\right)}{\Gamma(u)} \tag{1}$$

$$P_d = Q_u\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) \tag{2}$$

where, $\Gamma(.,.)$ is the upper incomplete Gamma function, $\Gamma(.)$ is the Gamma function and $Q_u(.,.)$ are the generalized Marcum Q-function of order $u-1$. where, u represents the time-bandwidth product, and γ is the received signal-to-noise ratio (SNR).

2) At Low SNR:

$$P_f = \frac{1}{2} Erfc\left(\frac{\lambda - N\sigma_n^2}{\sqrt{2N\sigma_n^4}}\right) \tag{3}$$

$$P_d = \frac{1}{2} Erfc\left(\frac{\lambda - N\sigma_n^2(1+\gamma)}{\sqrt{2N\sigma_n^4(1+\gamma)^2}}\right) \tag{4}$$

$$P_m = 1 - P_d \tag{5}$$

where, N is the number of samples of the received signal, σ_n^2 is the noise variance, P_m is the probability of missing the detection at each CU. We have selected the threshold at each CU with CFAR (λ_{CFAR}) and MEP (λ_{MEP}) approach which is given as [30], [33].

$$\lambda_{CFAR} = \left\{\sqrt{\frac{2}{N}} Erfc^{-1}\left(2\bar{P_f}\right) + 1\right\} N\sigma_n^2 \tag{6}$$

$$\lambda_{MEP} = \frac{N\sigma_n^2}{2}\left\{1 + \sqrt{1 + \frac{2(2+\gamma_p)\, ln(1+\gamma_p)}{N\gamma_p}}\right\} \times \left(\frac{1+\gamma_p}{1+\frac{\gamma_p}{2}}\right) \tag{7}$$

where, $\bar{P_f}$ is the target value of false alarm probability and γ_p is the received SNR at CU due to PU communication.

B. Analysis of Different Performance Affecting Parameters

In this section, we have analyzed the effect of different parameters (multi-antennas and reporting approach to FC over imperfect reporting channel) that affect the performance of the system.

1) Multiple Antennas: In CRN, when CUs employs multiple antennas then sensing results at each CU can be computed with either square law combining (SLC) or square law selection, (SLS) scheme. In the proposed system model, we have assumed that each CU employs SLS scheme because of its least complexity. In SLS scheme, each CU selects that antenna branch of the receiver for signal detection which has maximum SNR (γ_j) [28] i.e.

$$\gamma^{SLS} = \max_{j=1,2,\ldots L_a} \gamma_j \tag{8}$$

TABLE III
FALSE ALARM AND DETECTION PROBABILITIES RECEIVED AT FC UNDER IMPERFECT REPORTING CHANNEL WITH NON-CENSORING/CENSORING APPROACHES

Licensed channel state	Licensed channel state detected by CU	Licensed channel state received at FC	False alarm or detection probabilities received at FC under non-censoring approach	False alarm or detection probabilities received at FC under censoring approach
Active	Active	Active	$P_d^{SLS}(1-P_e^r)$	$P_d^{SLS}(1-P_e^r)$
Active	Idle	Active	$(1-P_d^{SLS})P_e^r$	0
Idle	Active	Active	$P_f^{SLS}(1-P_e^r)$	$P_f^{SLS}(1-P_e^r)$
Idle	Idle	Active	$(1-P_f^{SLS})P_e^r$	0

Therefore, the false alarm and detection probabilities by employing SLS scheme in multi-antenna scenario at each CU can be computed with the help of (9) and (10), respectively and are given as [20]:

$$P_f^{SLS} = 1 - \left(1 - P_f\right)^{L_a} \tag{9}$$

$$P_d^{SLS} = 1 - (1 - P_d)^{L_a} \tag{10}$$

where P_f is the probability of false alarm and P_d is the probability of detection at each CU employing single antenna while P_f^{SLS} and P_d^{SLS} are the false-alarm and detection probability at each CU after employing SLS scheme respectively in multi-antenna scenario.

2) Imperfect Reporting Channel Under Non-Censoring and Censoring Scenario: The spectrum sensing results of each CU is sent to the FC through imperfect reporting channels with error probability (P_e^r). There is total eight possible cases of false alarm and detection probabilities at FC while considering the licensed channel state, licensed channel state detected by CU, and licensed channel state received at FC. Among total eight possible states, we have considered only those states in which decisions on licensed channel state received at FC is active which are presented in Table III. With the help of Table III, we have computed the false alarm probability received at FC in the non-censoring and censoring scenario which is presented in (11) and (12), respectively.

$$P_f^{r,SLS} = (1 - P_f^{SLS})P_e^r + P_f^{SLS}\left(1 - P_e^r\right) \tag{11}$$

$$P_f^{rc,SLS} = P_f^{SLS}\left(1 - P_e^r\right) \tag{12}$$

Further, with the help of Table III, we compute the detection and miss detection probabilities received at FC in non-censoring and censoring scheme and presented as:

$$P_d^{r,SLS} = (1 - P_d^{SLS})P_e^r + P_d^{SLS}\left(1 - P_e^r\right) \tag{13}$$

$$P_d^{rc,SLS} = P_d^{SLS}\left(1 - P_e^r\right) \tag{14}$$

$$P_m^{r,SLS} = 1 - P_d^{r,SLS} \tag{15}$$

$$P_m^{rc,SLS} = 1 - P_d^{rc,SLS} \tag{16}$$

where, $P_f^{r,SLS}$ and $P_f^{rc,SLS}$ is the false alarm, $P_d^{r,SLS}$ and $P_d^{rc,SLS}$ is the detection and $P_m^{r,SLS}$ and $P_m^{rc,SLS}$ are the miss detection probabilities received at FC when CU employs SLS scheme over imperfect reporting channel in non-censoring and censoring scenario, respectively. Afterwards, FC employs k-out of M^r fusion rule to take the global decision about the presence or absence of PU on licensed channel in the non-censoring and censoring scenario. In the non-censoring approach, all M CUs report to the FC while, in censoring scenario M^c ($M^c \le M$) number of CUs report to the FC. Moreover, the value of M^c can be computed with the help of Table III and is expressed as:

$$M^c = \left\lceil \left(M\left\{P\left(H_0\right)P_f^{SLS} + P\left(H_1\right)P_d^{SLS}\right\}\right)\right\rceil \tag{17}$$

where, $\lceil . \rceil$ indicate the ceiling function, $P\left(H_0\right)$ and $P\left(H_1\right)$ are the probability of hypothesis H_0 and H_1 to be true. Therefore, global false alarm ($Q_{f'}^r$) and detection probability ($Q_{d'}^r$) at FC under non-censoring/censoring scenario are expressed as follows:

$$Q_F^r = \sum_{l=k}^{M^r} \binom{M^r}{l} \left(P_f^r\right)^l \left(1 - P_f^r\right)^{M^r - l} \tag{18}$$

$$Q_D^r = \sum_{l=k}^{M^r} \binom{M^r}{l} \left(P_d^r\right)^l \left(1 - P_d^r\right)^{M^r - l} \tag{19}$$

$$Q_M^r = 1 - Q_D^r \tag{20}$$

$$Q_e^r = P(H_0)Q_f^r + P(H_1)(1 - Q_d^r) \tag{21}$$

where, M^r is the number of CUs reporting to the FC, k *is* the number of CU nodes reporting to the FC in favor of active licensed channel, P_f^r, P_d^r are the received false alarm and detection probabilities at FC, respectively in the non-censoring/censoring scenario. Moreover, to find out the value of M^r, P_f^r, and P_d^r under the non censoring or censoring scenario, we have considered two cases which are as follows:

Case-1 (Non-Censoring Scenario): In the non-censoring scenario, the value of $M^r = M$ and the total false alarm (Q_F^r) and detection probability (Q_D^r) used in (18) and (19) can be computed with the help of (11) and (13), respectively while employing $P_f^r = P_f^{r,SLS}$ and $P_d^r = P_d^{r,SLS}$.

Case-2 (Censoring Scenario): In the censoring scenario, the value of $M^r = M^c$ and the total false alarm (Q_F^r) and detection probability (Q_D^r) used in (18) and (19) can be computed using (12) and (14), respectively while employing $P_f^r = P_f^{rc,SLS}$ and $P_d^r = P_d^{rc,SLS}$.

C. Optimization of Bayes Risk

In this section, we have derived the expression for Bayes risk for the proposed system model. The reason for selection of Bayes risk for the proposed CRN system model is that the accurate spectrum sensing results are required to efficiently utilize the spectrum therefore, we have emphasized to minimize the sensing error or total error rate. However, the spectrum sensing error is the special case of Bayes risk therefore, we have minimized the Bayes risk with the proposed system model and compared it with earlier reported literature considering the Bayes risk. Further, we have illustrated that the existing expressions of optimal value of k and M are the special case of our proposed generalized expressions. To find out the expression for Bayes risk, we have considered the cost associated with the actual state of licensed channel and state of licensed channel decided by the FC in Table IV.

where, C_{ij} is the cost associated with global decision at FC when actual state is H_j and decided state H_i. Further, with

TABLE IV
COST MATRIX ASSOCIATED WITH ACTUAL AND DECIDED STATE OF LICENSED CHANNEL

Actual State (H_j)	Decide State (H_i)	Cost (C_{ij})
0	0	C_{00}
0	1	C_{10}
1	0	C_{01}
1	1	C_{11}

the help of Table IV, we compute the Bayes risk of CRN and is expressed as [10]:

$$R = \sum_{i=0}^{1} \sum_{j=0}^{1} C_{ij}\, P\left(\frac{H_i}{H_j}\right) P\left(H_j\right) \tag{22}$$

After solving and rearranging (22), the Bayes risk (R) is expressed in (23). Further, the derivation of expression for R given in (23) is presented in Appendix A of this paper and final equation of Bayes risk is given below:

$$\begin{aligned} R = C_{00} P\left(H_0\right) + C_{11} P\left(H_1\right) + \left(C_{10} - C_{00}\right) P\left(H_0\right) Q_F^r \\ + \left(C_{01} - C_{11}\right) P\left(H_1\right) Q_M^r \end{aligned} \tag{23}$$

In (23), when $C_{00} = C_{11} = 0$ and $C_{01} = C_{10} = 1$, then Bayes risk (R) is same as total error rate computed in [20].

$$R = Q_e = P\left(H_0\right) Q_F^r + P\left(H_1\right) Q_M^r \tag{24}$$

where, Q_e is the total error rate. Further, we have derived the expression for optimal value of k and M to minimize the Bayes risk.

1) Optimal Value of k: The Bayes risk (R) is the function of k and M^r therefore, for fixed value of M^r we find the exact value of k at which the Bayes risk is minimum by differentiating the Bayes risk with respect to k ($\frac{\partial R}{\partial k} = 0$). Therefore, the value of k at which Bayes risk is minimum is given in (25) which is represented by K. Since, the number of reporting users should be an integer quantity, therefore we have to take the ceiling of K which is given in (26) as k_{opt}. The derivation of expression for k_{opt} given in (26) is presented in Appendix B of this manuscript.

$$K = \frac{log\left[\left(\frac{C_{01}-C_{11}}{C_{10}-C_{00}}\right)\left(\frac{P(H_1)}{P(H_0)}\right)\right]}{log\left(\frac{P_f^r}{1-P_m^r}\right) - log\left(\frac{1-P_f^r}{P_m^r}\right)} + \frac{M^r.log\left(\frac{P_m^r}{1-P_f^r}\right)}{log\left(\frac{P_f^r}{1-P_m^r}\right) - log\left(\frac{1-P_f^r}{P_m^r}\right)} \tag{25}$$

$$k_{opt} = \lceil K \rceil \tag{26}$$

Moreover, the expression achieved for k_{opt} in (26) is the generalized expression and the expression achieved by several researchers [11], [20] are the special case of generalized expression. In non-censoring scenario while employing single antenna i.e. ($L_a = 1$), the achieved value of k_{opt} in (26) is same as that given in [11]. Further, in the non-censoring multiple antenna scenario, equation (26) gives the same expression for finding the optimized value of k that minimizes the error probability as that presented in [20] while substituting $C_{00} = C_{11} = 0.0$, and $C_{01} = C_{10} = 1$.

Algorithm 1 Computation of Optimal Rule at FC (k_{opt} or M_{opt})

Input: Sensing threshold selection approach (STSA) = {CFAR, MEP}, Event sequence (ES) = {Censoring, Non-censoring}, Reporting channel (RC) = {Imperfect, Perfect}, γ, k, M
Output: k_{opt}, M_{opt}

Initialization: N, σ_n^2, M, $P(H_0)$, $P(H_1)$, $p \in (0, 1]$, k, M, $\gamma_{set} = [-20, 10]$, u, L_a, C_{00}, C_{01}, C_{10}, C_{11}
if $\gamma \in \gamma_{set}$
  if STSA == CFAR
    $\lambda \leftarrow \lambda_{CFAR}$
  else
    $\lambda \leftarrow \lambda_{MEP}$
  end if
if $\gamma > -12$dB
  compute P_f and P_d using (1) and (2) respectively
else
  compute P_f and P_d using (3) and (4) respectively
end if
  compute P_f^{SLS} and P_d^{SLS} using (9) and (10) respectively
  if RC == Imperfect
    $P_e^r \leftarrow p$
  else
    $P_e^r \leftarrow 0$
  end if
if ES == Censoring
    compute M^c using (17)
    find $P_f^{rc,SLS}$ from (12) and $P_d^{rc,SLS}$ using (14)
else
    find $P_f^{r,SLS}$ from (11) and $P_d^{r,SLS}$ using (13)
end if
  find Q_F^r from (18), and Q_M^r using (20)
  compute R from (23)
  compute k_{opt} from (26) and M_{opt} using (28)
  end if

---

2) Optimal Value of M^r: The Bayes risk is the function of k and M^r therefore, for fixed value of k, we find the optimal value of M^r, by minimizing the Bayes risk with respect to M ($\frac{\partial R}{\partial M^r} = 0$) and which is given in (27). Since, the total number of CUs should be an integer quantity, therefore we have to take the ceiling of m which is given in (28) as M_{opt}. The derivation of expression for M_{opt} given in (28) is presented in Appendix C.

$$m = (k-1) + \frac{log\left[\left(\frac{P(H_1)}{P(H_0)}\right)\left(\frac{C_{01}-C_{11}}{C_{10}-C_{00}}\right)\right]}{log\left(\frac{1-P_f^r}{P_m^r}\right)} + \frac{k.log\left(\frac{1-P_m^r}{P_f^r}\right)}{log\left(\frac{1-P_f^r}{P_m^r}\right)} \tag{27}$$

$$M_{opt} = \lceil m \rceil \tag{28}$$

where, m is that value of M^r at which Bayes risk is minimum, while $\lceil . \rceil$ indicates the ceiling function. Substituting, the values of P_f^r and P_m^r in non-censoring scenario with the help of (11) and (15) while employing single antenna scenario i.e. ($L_a = 1$), then the expression achieved in (28) for M_{opt} is same as the expression computed in [10]. Further, the flow chart showing the methodology for computation of optimal rule at FC (k_{opt} for fixed value of M^r and M_{opt} for

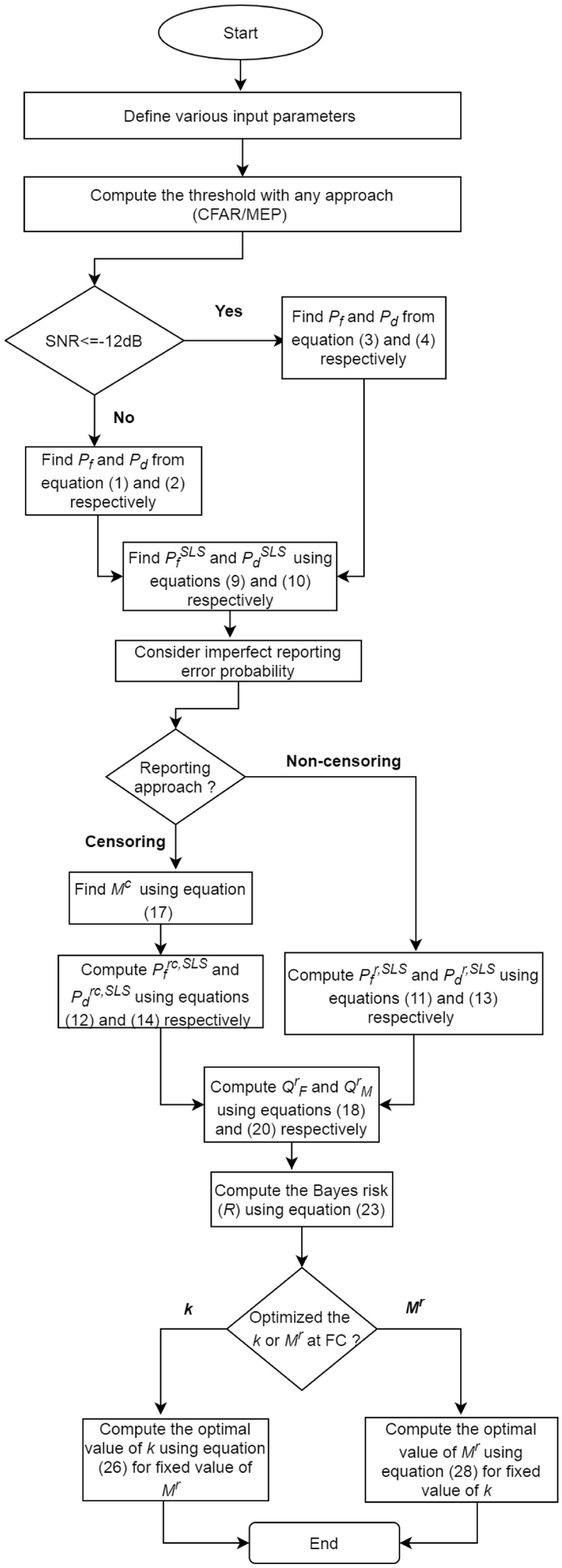


Fig. 2. Flow chart for finding the optimal k and M^r.

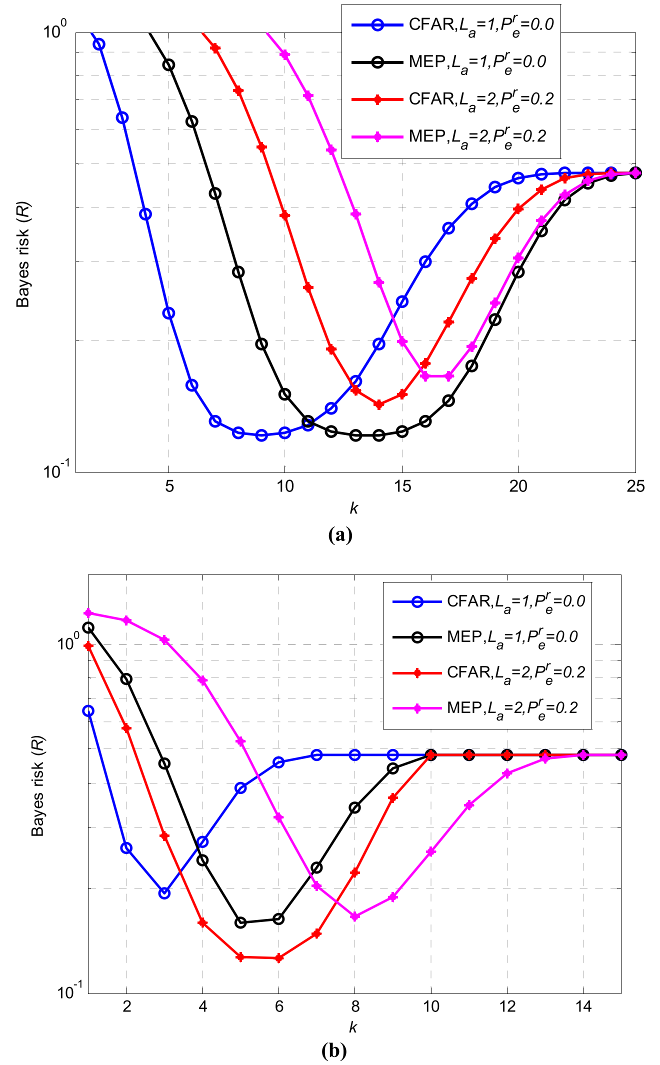
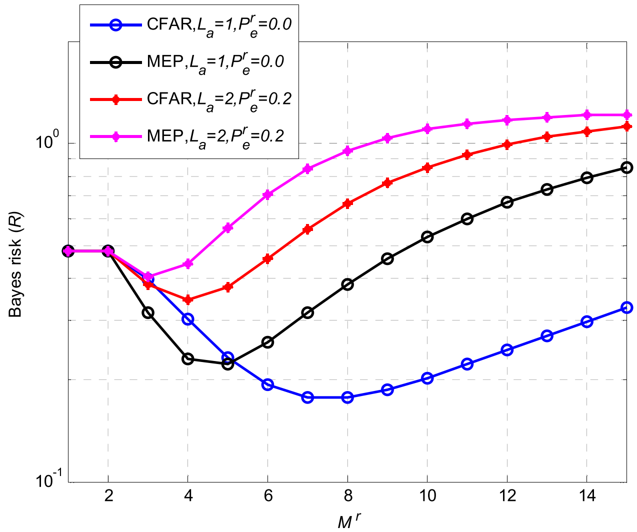


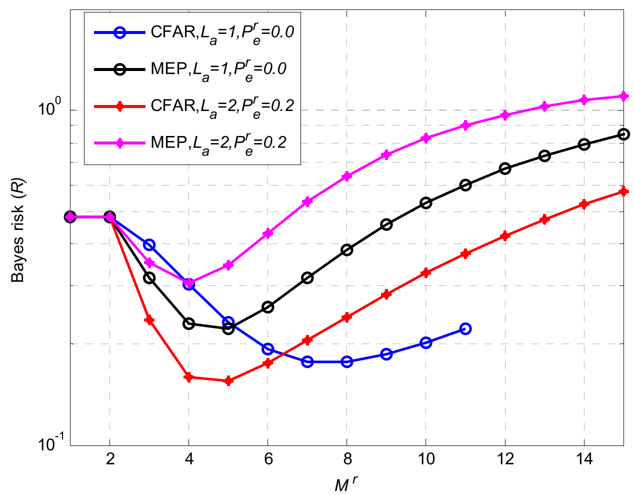
Fig. 3. Variation in Bayes risk with k while employing different threshold selection approaches for $M = 25$, $C_{00} = 0.1$, $C_{01} = 2$, $C_{10} = 1.5$, $C_{11} = 0.2$, $P(H_0) = 0.8$ $N = 25000$, $\sigma_n^2 = 1$, $\bar{P}_f = 0.1$, and $\gamma = -20$dB (a) non-censoring and (b) censoring scenarios.

fixed value of k) while employing CFAR or MEP threshold selection approaches at low and high SNR under censoring and non-censoring approaches is presented in Fig. 2 and in Algorithm-1.

3) Complexity Analysis of Proposed Algorithm: In algorithm 1, we have computed the k_{opt} and M_{opt} to minimize the Bayes risk under considered system model. However, the Bayes risk depends on the false alarm and detection probabilities (P_f and P_d) and threshold selection approaches (CFAR (λ_{CFAR}) and MEP (λ_{MEP})). Further, above mentioned parameters depend on the Erfc function which is clear from (3) to (7). Moreover, the Erfc function is related with Q function as $Q(t) = \frac{1}{2}Erfc(\frac{t}{\sqrt{2}})$. Further, $Q(t)$ included infinite terms and can be approximated as [34]: $Q(t) = \frac{e^{-\frac{t^2}{2}}\sum_{n=1}^{n_a}(-1)^{n+1}(C)^n}{D\sqrt{\pi}\left(\sqrt{2}\right)^{n+1}n!}$, where C and D are constants. Further, the complexity of Q function ($Q(t)$) is given as $O(n_a)$ due to its dependence on the term n_a. In addition, the value of n_a is selected as per the accuracy requirement. Moreover, in the proposed algorithm those steps which involve either P_f, P_d or threshold selection approaches have complexity $O(n_a)$ under considered system model. However, in algorithm 1 we have also employed k_{opt} or M_{opt} which consists of summation of k_{opt} or M_{opt} terms and therefore its complexity is $O(k_{opt})$ and $O(M_{opt})$, respectively while remaining steps have complexity $O(1)$. Hence, we have observed that the complexity of the

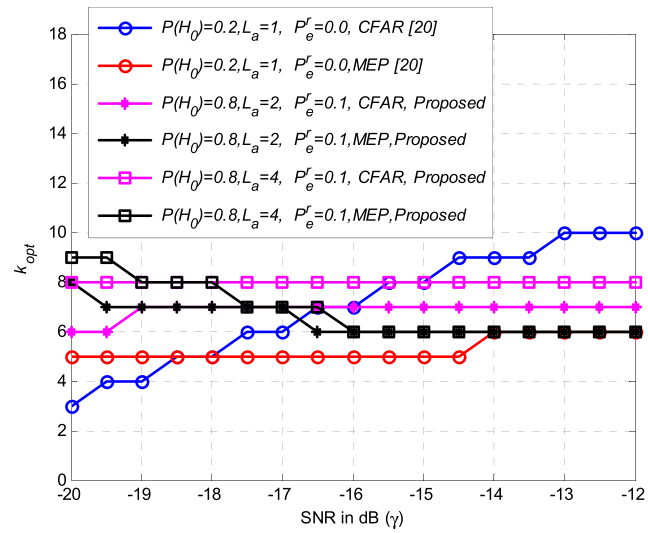


(a)

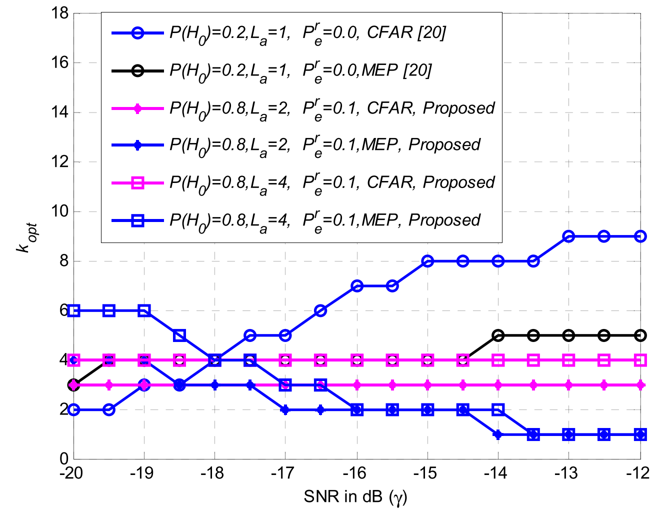


(b)

Fig. 4. Variation in Bayes risk with number of cognitive user reporting to FC(M^r) while employing different threshold selection approaches for $k = 3$, $C_{00} = 0.1$, $C_{01} = 2$, $C_{10} = 1.5$, $C_{11} = 0.2$, $P(H_0) = 0.8$, $N = 25000$, $\sigma_n^2 = 1$, $\bar{P}_f = 0.1$, and $\gamma = -20$dB (a) non-censoring and (b) censoring scenarios.



(a)



(b)

Fig. 5. Variation in the optimal k value with SNR to minimize the Bayes risk for CFAR and MEP threshold selection approach in different channel scenario when $C_{00} = 0.1$, $C_{11} = 0.2$, $C_{01} = 2$, $C_{10} = 1.5$ (a) non-censoring and (b) censoring scenarios.

proposed algorithm 1 is given as $\text{Max}(O(k_{opt}),\ O(n_a))$ for computation of k_{opt} and $\text{Max}(O(M_{opt}),\ O(n_a))$ for computation of M_{opt} while employing any threshold selection approach under proposed system model.

IV. Simulation Result

In this section of the paper, we have demonstrated the simulated results for the proposed system model with the help of MATLAB 2013. For the simulation, we have assumed that the SNR is known at CU and having in the range of -20dB to 10 dB. First, we have shown the variation in Bayes risk with k and number of CUs reporting to FC (M^r) in Fig. 3 and Fig. 4, respectively while employing different combinations of the threshold selection approaches (CFAR and MEP), number of antennas employed at each CUs (L_a) and reporting error probability (P_e^r) in the non-censoring and censoring scenarios. Further, it is observed that the Bayes risk initially decreases and then increases before becoming constant as k or number of CUs reporting to FC (M^r)increases in Fig. 3 and Fig. 4, respectively, which reveals that the Bayes risk is convex function and the optimal value of k and M^r exist at which Bayes risk is minimum.

In the non-censoring scenario, the value of $M^r = M$ and in the censoring scenario, the value of $M^r = M^c$ as described in the analysis Section IIIB. Therefore, in the analysis, we have achieved the optimal value of k and M^r at different SNRs at which the Bayes risk is minimized for the proposed system model. Fig.5(a) and Fig.5(b) illustrated the variation in optimal value of k with SNR to minimize the Bayes risk while considering the combined effect of licensed channel's active/idle state probability $(P(H_1)/P(H_0))$, number of antennas employed by each CU (L_a) and error in reporting channel (P_e^r) while employing different threshold selection approaches in non-censoring and censoring scenario, respectively.

Further, in Fig. 5(a) and Fig. 5(b), we have shown that results achieved in [20] is the special case of the proposed

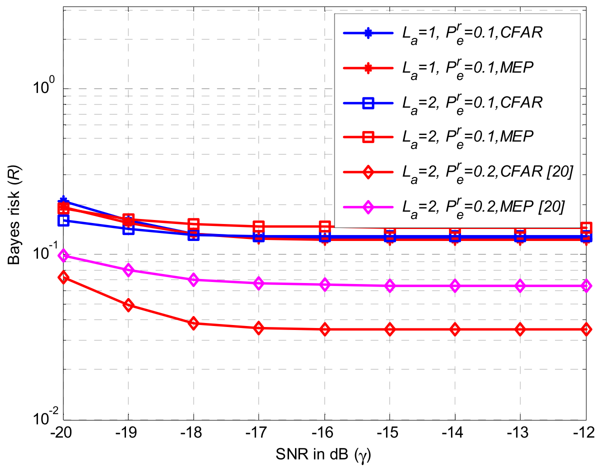


Fig. 6. Variation in Bayes risk with SNR while employing different threshold selection approaches in non-censoring scenario for optimal value of $k_{opt}C_{00} = 0.1$, $C_{11} = 0.2$, $C_{01} = 2$, $C_{10} = 1.5$, and $M = 10$.

approach when $C_{00} = C_{11} = 0.0$, $C_{01} = C_{10} = 1$. From Fig. 5, we have observed that for fixed value of γ, $P(H_0)$ and P_e^r, the k_{opt} increases with increase in L_a due to increment in both P_f and P_d while employing any threshold selection approach (CFAR/MEP).

Further in Fig. 6, we have illustrated the variation in Bayes risk with SNR for optimal value of k at $M = 10$ while employing different threshold selection approaches in the non-censoring scenario under different channel conditions. From Fig. 6, it is clear that Bayes risk (R) decreases with increase in SNR due to increment in both P_f and P_d. From Fig.6, it is also observed that at very low value of SNR, the Bayes risk decreases with increase in L_a due to decrement in Q_F^r and increment in Q_M^r while employing any threshold selection approach under imperfect reporting channel. Further in Fig. 6, we have shown that when $C_{00} = C_{11} = 0.0$, $C_{01} = C_{10} = 1$, the results achieved in [20] is the special case of our proposed approach.

Further, in Fig. 7(a) and Fig. 7(b), we have illustrated the combined effect of multiple antennas used by each CU, active/idle state probability of licensed channel, and probability of error in reporting channel (P_e^r) on finding the optimal value of M^r (M_{opt}) for minimizing the Bayes risk at low SNR while employing various threshold selection approaches, i.e., CFAR and MEP, respectively in the non- censoring scenario. From Fig. 7, we have concluded that for the fixed value of SNR, the required value of M_{opt} decreases with increase in L_a while employing CFAR or MEP threshold selection approach due to increment in both P_f and P_d under perfect reporting channel ($P_e^r = 0$). Further, the required value of M_{opt} increases with increment in P_e^r (0.0 to 0.1) for fixed value of SNR, $P(H_0)$, and L_a while employing any threshold selection approach (CFAR or MEP) due to increment in P_f and decrement in P_d.

In Fig. 8, we have illustrated the variation in M_{opt} with SNR while considering the combined effect of multiple antennas and error in reporting channel (P_e^r) with CFAR and MEP threshold approach in censoring scenario. From Fig. 8, we have concluded that for the fixed value of SNR and P_e^r, M_{opt} is less for higher values of L_a due to increment in both $P_f^{rc,SLS}$ and $P_d^{rc,SLS}$ received at FC while employing CFAR or MEP threshold selection approach. However, at high SNR both $P_f^{rc,SLS}$ and $P_d^{rc,SLS}$ values become constant for CFAR threshold selection approach with variation in number of antennas resulting M_{opt} nearly constant at high SNR. Further, we have observed that at fixed SNR and L_a, the M_{opt} increases with increase in P_e^r due to decrement in both $P_f^{rc,SLS}$ and $P_d^{rc,SLS}$ received at FC while employing CFAR or MEP threshold selection approaches. In addition, from Fig. 8, we have observed that for fixed value of P_e^r and L_a, the M_{opt} decreases and become constant with increase in SNR due to constant $P_f^{rc,SLS}$ and $P_d^{rc,SLS}$ increases while employing CFAR threshold selection. Further, for fixed value of P_e^r and L_a, the M_{opt} increases with increase in SNR due to decrement in $P_f^{rc,SLS}$ and increment in $P_d^{rc,SLS}$ received at FC. Moreover, in Fig. 9, we have illustrated the variation in Bayes risk with SNR for M_{opt} at fixed value of k while employing different threshold approaches in different channel conditions under non-censoring and censoring scenario.

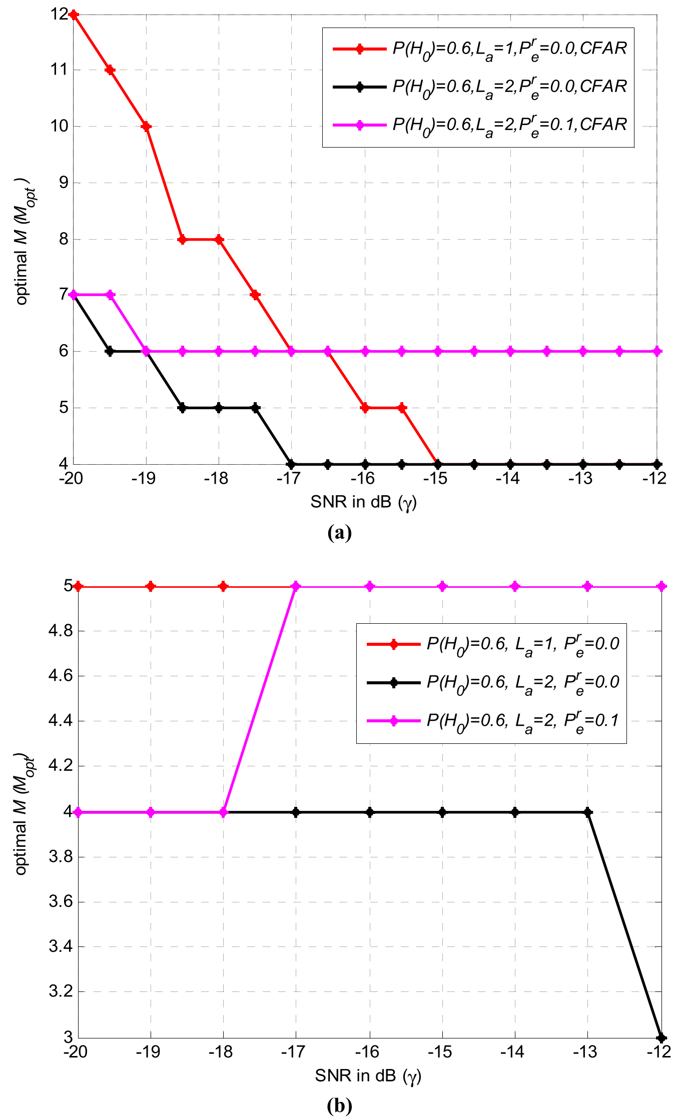


Fig. 7. Variation in the M_{opt} value with SNR to minimize the Bayes risk for fixed k ($k = 4$) and $C_{00} = 0.1$, $C_{11} = 0.2$, $C_{01} = 2$, $C_{10} = 1.5$ (a) CFAR (b) MEP.

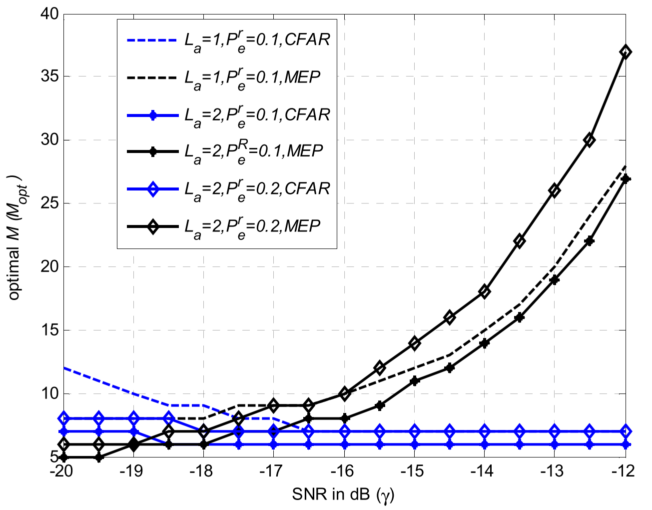


Fig. 8. Variation in the M_{opt} value with SNR to minimize the Bayes risk for fixed k ($k = 4$), $P(H_0) = 0.8$, and $C_{00} = 0.1$, $C_{11} = 0.2$, $C_{01} = 2$, $C_{10} = 1.5$ while employing CFAR and MEP threshold selection approaches in censoring scenario.

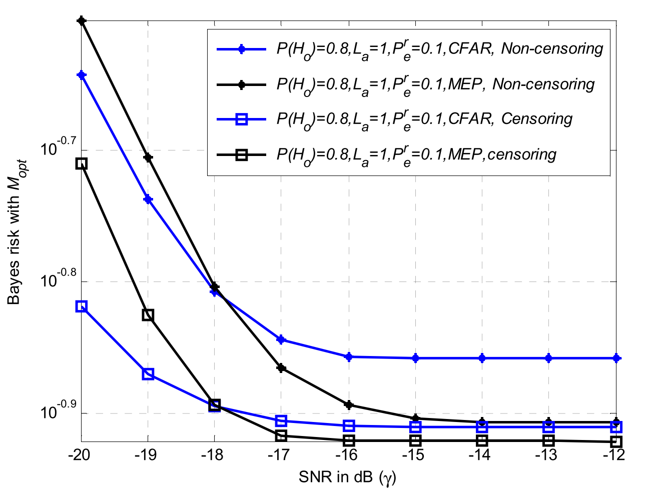


Fig. 9. Variation in Bayes risk with SNR while employing different threshold selection approach in non-censoring and censoring scenario for optimal value of M, $C_{00} = 0.1$, $C_{11} = 0.2$, $C_{01} = 2$, $C_{10} = 1.5$, and $k = 4$.

From Fig. 9, it is also observed that Bayes risk is less with censoring approach with respect to non-censoring approach due to decrement in both P_f^r and P_d^r and increment in M_{opt} while employing any threshold selection approach.

Moreover, in Fig. 10 we have presented the variation in M_{opt} with high SNR (high SNR i.e., $\gamma > -12$dB) while considering different threshold selection approaches under imperfect reporting channel. From Fig. 10, we have observed that M_{opt} decreases with increase in SNR while employing any threshold selection approaches. Further in Fig. 10, we have shown that when the sensing threshold is fixed and equal to $\lambda = 12$, the results achieved in [10] is the special case of proposed approach. However, in [10], the sensing threshold ($\lambda = 12$) is chosen randomly and have not provided any criterion for its selection which is less feasible. However, we have considered two different threshold selection approaches (CFAR and MEP) for computation of threshold and have shown the required value of M_{opt} at different SNR. Further, from Fig. 10, it is clear that required optimal value of M affect with the threshold selection approach.

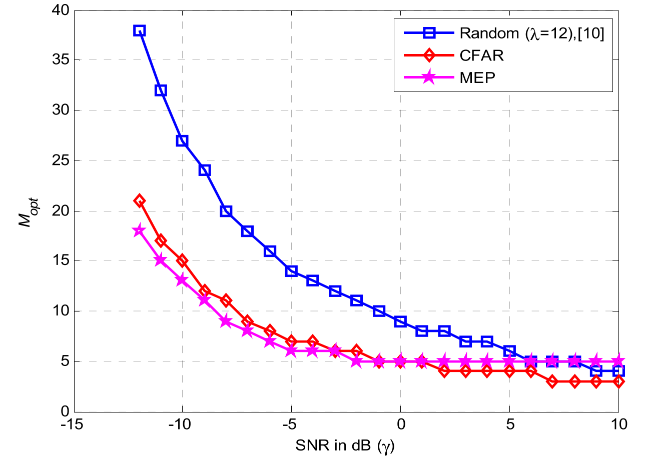


Fig. 10. Variation in the M_{opt} value with SNR to minimize the Bayes risk for $k = 3$, $C_{00} = 0.1$, $C_{11} = 0.2$, $C_{01} = 3$, $C_{10} = 2$, $P_e^r = 0.5$, and $L_a = 1$ [10].

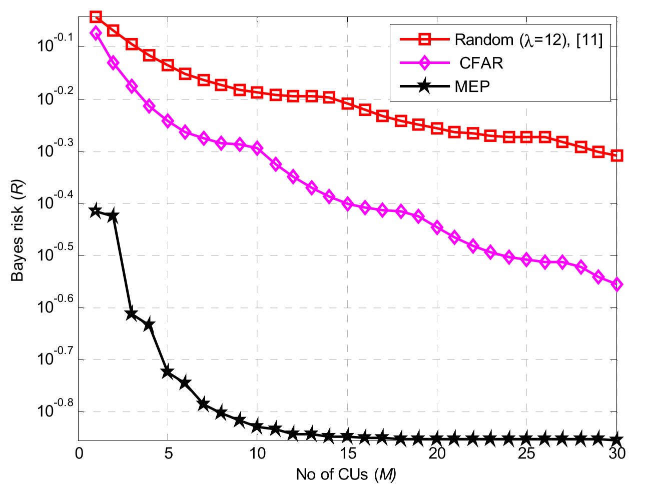


Fig. 11. Variation in Bayes risk with M for optimal k value with different threshold selection approach for $C_{00} = 0.1$, $C_{11} = 0.2$, $C_{01} = 2$, $C_{10} = 1.5$, $P_e^r = 0.05$, $L_a = 1$, $\gamma = 10$ dB in non-censoring scenario.

Further, we have analyzed that the selection of sensing threshold plays a major role to minimize the Bayes risk. In this context, Banavathu and Khan in [11] shows the variation in Bayes risk with M for fixed value of randomly selected threshold ($\lambda = 12$) at $\gamma = 10$dB. However, the random selection of sensing threshold is less feasible at different SNR. Therefore, in Fig. 11, we have illustrated the variation in Bayes risk with M by finding the optimal value of k while employing different threshold selection approaches. From Fig. 11, it is clear that for fixed value of SNR ($\gamma = 10$dB), the Bayes risk decreases with increase in M while employing any threshold selection approaches due to increment in k_{opt}, Q_M^r and decrement in Q_F^r at FC. Further from Fig. 11, we have analyzed that for fixed value of M, MEP threshold selection approach provides less Bayes risk with respect to CFAR or randomly selected threshold ($\lambda = 12$) approach due to less value of k_{opt}, Q_F^r, and Q_M^r at FC.

Moreover, in Fig. 12 we have illustrated the variation in Bayes risk with high SNR (i.e $\gamma \geq -12$dB) for optimal value of k (k_{opt}) while employing different threshold selection approaches. From Fig. 12, we have observed that the Bayes risk decreases with increase in SNR. Further, MEP threshold approach provide less Bayes risk at higher range of high SNR

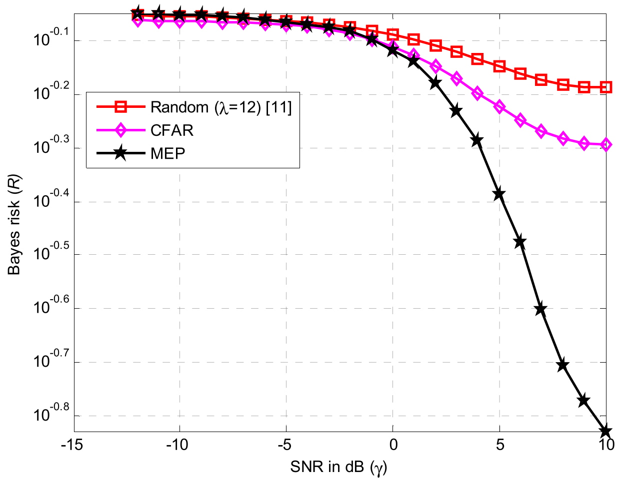


Fig. 12. Variation in Bayes risk with SNR while employing different threshold selection approach for k_{opt}, and $C_{00} = 0.1$, $C_{11} = 0.2$, $C_{01} = 2$, $C_{10} = 1.5$, $P_e^r = 0.05$, $L_a = 1$, and $M = 10$.

($\gamma > -1$dB) while CFAR threshold provide less Bayes risk at lower range of high SNR ($\gamma < -1$dB). This is happened because MEP threshold provide less value of k_{opt}, Q_F^r and Q_M^r at FC while CFAR approach provide less Q_F^r and high value of k_{opt} and Q_M^r at FC. Moreover, from the result we have concluded that single threshold selection approach at all SNR is not suitable to achieve the minimum value of Bayes risk.

V. Conclusion

In this article, we have computed the optimal value of k for fixed value of M^r, and optimal value of M^r for fixed value of k to minimize the Bayes risk by considering the combined effect of threshold selection approach, licensed channel's active/idle state probability, and multiple antennas employed at each CU under imperfect reporting channel in the non-censoring/censoring scenario. From the results, we have concluded that under the imperfect reporting channel and in the non-censoring scenario, the Bayes risk decreases with increase in L_a when k_{opt} is employed at FC while employing any threshold selection approach. Further, MEP threshold selection approach provides less value of Bayes risk over CFAR approach except at very low SNR in the non-censoring scenario. Moreover, when the optimal value of $M^r (M_{opt})$ is employed at FC, then Bayes risk is less with CFAR approach at very low SNR while at high SNR, MEP threshold is better in non-censoring/censoring scenario. Therefore, single sensing threshold approach is not suitable to minimize the Bayes risk at all SNR under non-censoring/censoring scenario. Further, censoring approach reduces Bayes risk over non-censoring scenario. The computation of optimal k and M^r for maximizing the throughput and energy efficiency is a potential challenge which will report in future communication.

Appendix A
Computation of Bayes Risk

Given cost value, the Bayes risk of CRN is given in (22) and can be rearranged as follows:

$$\begin{aligned} R = {} & C_{00} P(H_0) P(Decision/H_0) + C_{01} P(H_1) \\ & \times P(Decision/H_1) + C_{10} P(H_0) P(Decision/H_0) \\ & + C_{11} P(H_1) P(Decision/H_1) \end{aligned} \tag{29}$$

where, decision is the judgment of the licensed channel state at FC from (22). Further, (29) can be solved with the help of [27] and is written as follows:

$$\begin{aligned} R = {} & C_{00} P(H_0) + C_{01} P(H_1) + [(C_{10} - C_{00}) P(H_0)] Q_F^r \\ & + [(C_{11} - C_{01}) P(H_1)] Q_D^r \end{aligned} \tag{30}$$

where, Q_F^r is the global false alarm and Q_D^r is the global detection probability at FC in non-censoring/censoring scenario. In (30), put $Q_D^r = 1 - Q_M^r$ and rearrange the expression for Bayes risk (R) which is expressed in (23).

$$\begin{aligned} R = {} & C_{00} P(H_0) + C_{11} P(H_1) + (C_{10} - C_{00}) P(H_0) Q_F^r \\ & + (C_{01} - C_{11}) P(H_1) Q_M^r \end{aligned}$$

Appendix B
Computation of Optimal k (k_{opt})

For optimal k, $k_{opt} = arg_k^{min} R(k, M^r)$ is achieved when, $\frac{\partial R(k,M^r)}{\partial k} = 0$.

$$\frac{\partial R(k, M^r)}{\partial k} = R(k+1, M^r) - R(k, M^r) = 0 \tag{31}$$

Since, $R(k, M^r)$ is the function of Q_F^r and Q_M^r in non-censoring/censoring scenario.

Q_F^r and Q_M^r are given by $\sum_{l=k}^{M^r} \binom{M^r}{l} (P^r)^l (1 - P^r)^{M^r - l}$ where, Q_F^r is computed by putting $P^r = P_f^r$ and Q_M^r is computed by substituting $P^r = P_m^r$.

Moreover, put the value of Bayes risk in (31), we get,

$$\begin{aligned} & (C_{10} - C_{00}) P(H_0) \left[Q_F^r (k+1, M^r) - Q_F^r (k, M^r)\right] \\ & \quad + (C_{01} - C_{11}) P(H_1) \left[Q_M^r (k+1, M^r) - Q_M^r (k, M^r)\right] \\ & = 0 \end{aligned} \tag{32}$$

Further, by placing the value of Q_F^r and Q_M^r in (32) and applying some basic mathematical operation, we got (33), as shown at the top of the next page.

Further by solving the (33), we get,

$$\left(\frac{1 - P_f^r}{P_m^r}\right)^{M^r - k} \left(\frac{P_f^r}{1 - P_m^r}\right)^k = \left(\frac{(C_{01} - C_{11})}{(C_{10} - C_{00})} \frac{P(H_1)}{P(H_0)}\right) \tag{34}$$

In (34), take log on both sides and rearranging it, we get the exact value of k at which Bayes risk is minimum and is given as:

$$K = \frac{log\left[\left(\frac{C_{01} - C_{11}}{C_{10} - C_{00}}\right)\left(\frac{P(H_1)}{P(H_0)}\right)\right]}{log\left(\frac{P_f^r}{1 - P_m^r}\right) - log\left(\frac{1 - P_f^r}{P_m^r}\right)} + \frac{M^r . log\left(\frac{P_m^r}{1 - P_f^r}\right)}{log\left(\frac{P_f^r}{1 - P_m^r}\right) - log\left(\frac{1 - P_f^r}{P_m^r}\right)}$$

where, $P_f^r = P_f^{r,SLS}$ and $P_d^r = P_d^{r,SLS}$ in non-censoring scenario while in censoring scenario $P_f^r = P_f^{rc,SLS}$ and $P_d^r = P_d^{rc,SLS}$. Since k is integer quantity so take ceiling of K and which is the optimal value of k and given as:

$$k_{opt} = \lceil K \rceil$$

$$(C_{10}-C_{00})\,P(H_0)\left[\left(\sum_{l=k+1}^{M^r}\binom{M^r}{l}\left(P_f^r\right)^l\left(1-P_f^r\right)^{M^r-l}\right)-\left(\sum_{l=k}^{M^r}\binom{M^r}{l}\left(P_f^r\right)^l\left(1-P_f^r\right)^{M^r-l}\right)\right]+(C_{01}-C_{11})$$
$$\times P(H_1)\left[\left(1-\sum_{l=k+1}^{M^r}\binom{M^r}{l}\left(1-P_m^r\right)^l\left(P_m^r\right)^{M^r-l}\right)-\left(1-\sum_{l=k}^{M^r}\binom{M^r}{l}\left(1-P_m^r\right)^l\left(P_m^r\right)^{M^r-l}\right)\right]=0 \quad (33)$$

Appendix C
Computation of Optimal M^r (M_{opt})

For optimal M^r, $M_{opt}=arg_{M^r}^{min} R(k,M^r)$ is achieved when, $\frac{\partial R(k,M^r)}{\partial M^r}=0$.

$$\frac{\partial R(k,M^r)}{\partial M^r}=R\left(k,M^r+1\right)-R\left(k,M^r\right)=0 \quad (35)$$

where, $R(k,M^r)$ is the function of Q_F^r and Q_M^r in non-censoring/censoring scenario.

Q_F^r and Q_M^r are given by $\left(\sum_{l=k}^{M^r}\binom{M^r}{l}(P^r)^l(1-P^r)^{M^r-l}\right)$ where, Q_F^r is computed by putting $P^r=P_f^r$ and Q_M^r is computed by substituting $P^r=P_m^r$.

Moreover, put the value of Bayes risk in (35), we get

$$\begin{aligned}&(C_{10}-C_{00})\,P(H_0)\left[Q_F^r\left(k,M^r+1\right)-Q_F^r\left(k,M^r\right)\right]\\&\quad+(C_{01}-C_{11})\,P(H_1)\left[Q_M^r\left(k,M^r+1\right)-Q_M^r\left(k,M^r\right)\right]\\&=0\end{aligned} \quad (36)$$

Further, by placing the values of Q_F^r and Q_M^r and applying basic mathematical operation, we get

$$\begin{aligned}&(C_{10}-C_{00})\,P(H_0)\left[\binom{M^r}{k-1}\left(P_f^r\right)^k\left(1-P_f^r\right)^{M^r+1-k}\right]\\&\quad-(C_{01}-C_{11})\,P(H_1)\left[\binom{M^r}{k-1}\left(1-P_m^r\right)^k\left(P_m^r\right)^{M^r+1-k}\right]\\&=0\end{aligned} \quad (37)$$

After solving the (37), we get

$$\left(\frac{1-P_f^r}{P_m^r}\right)^{M^r+1-k}=\left(\frac{(C_{01}-C_{11})}{(C_{10}-C_{00})}\frac{P(H_1)}{P(H_0)}\right)\left(\frac{1-P_m^r}{P_f^r}\right)^k \quad (38)$$

Further, by taking log on both sides of (38) and rearranging it, we get the exact value of M^r at which Bayes risk is minimum and is given as:

$$m=(k-1)+\frac{log\left[\left(\frac{P(H_1)}{P(H_0)}\right)\left(\frac{C_{01}-C_{11}}{C_{10}-C_{00}}\right)\right]}{log\left(\frac{1-P_f^r}{P_m^r}\right)}+\frac{k.log\left(\frac{1-P_m^r}{P_f^r}\right)}{log\left(\frac{1-P_f^r}{P_m^r}\right)}$$

where, $P_f^r=P_f^{r,SLS}$ and $P_d^r=P_d^{r,SLS}$ in non-censoring scenario while in censoring scenario $P_f^r=P_f^{rc,SLS}$ and $P_d^r=P_d^{rc,SLS}$. Since m is integer quantity so by taking the ceiling of m, we get the optimal value of M^r as below:

$$M_{opt}=\lceil m\rceil$$

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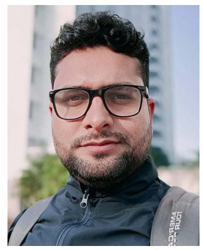

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