Ain Shams Engineering Journal 9 (2018) 1791–1797

Contents lists available at ScienceDirect

Ain Shams Engineering Journal

journal homepage: www.sciencedirect.com

Electrical Engineering

Comparative analysis on pulse compression with classical orthogonal polynomials for optimized time-bandwidth product

Ankur Thakur, Salman Raju Talluri *

Jaypee University of Information Technology, Solan, Himachal Pradesh, India

ARTICLE INFO

Article history:
Received 21 December 2015
Revised 1 October 2016
Accepted 4 January 2017
Available online 27 January 2017

Keywords:
Radar
Pulse compression
Classical orthogonal polynomials
Range resolution
Angle modulation
Time-bandwidth product

ABSTRACT

The theme of this paper is to analyze and compare the pulse compression with classical orthogonal polynomials (Chebyshev, Laguerre, Legendre and Hermite polynomials) of different orders. Pulse compression is used in radar systems to improve the range resolution by increasing the time-bandwidth product of the transmitted pulse. It is done by modulating the instantaneous angle of the transmitted pulse. Three types of angle modulations are considered in this paper. Initially, the angle is varied in proportional to the original polynomials. Secondly, the angle is proportional to integral of the polynomial and thirdly, the angle is proportional to derivative the polynomial. The main purpose of this analysis is to obtain and use the best of all these polynomials in pulse compression. This is done by comparing the quantitative parameter of pulse compression - time-bandwidth product. Optimization to maximize the time-bandwidth product is also considered in the analysis.

1. Introduction

In radar systems, for the determination of range and velocity of the target, a narrow pulse is transmitted into the space and then the reflected signal from the target is captured [1–3]. The time difference between the instant at which the initial pulse is transmitted and reflected pulse has arrived is used in the determination of the range of the target. If the transmitted pulse is having any carrier frequency which is the general mode of carrier transmission by modulation of the carrier, there will be a change in the frequency of the pulse that is being transmitted due to relative velocity between radar receiver and target. The change in the frequency which is called as Dopplershift is proportional to the relative velocity between the observing system and the moving target. This Doppler shift will be used in determination of the velocity of the target.

If the pulse transmitted is very narrow in time domain, there is a possibility that the reflected signal will be having small signal strength and in the detection process it may fall into the noise floor of the system giving a chance to miss the target due to narrow width in the time domain and small signal strength [2]. To avoid this problem, the peak power of the signal has to be increased. This is not practically a viable solution as most of the radar transmitters operate in the saturation level of the high power amplifier before the antenna [3]. To balance this, a long pulse has to be transmitted which reasonably avoids the missing of target due to narrow pulse width and small signal strength. But the longer pulse has a limitation in determining the very closely spaced two targets in a collinear orientation [3]. This happens due to the overlapping of the reflected pulse from the first target by the reflected pulse from the second target there by giving scope for the ambiguity in identifying them as two different targets. The ability of the radar system to identify two closely spaced objects is defined as the range resolution and for sophisticated radar systems this must be as small as possible. Hence very short pulses are to be transmitted for high range resolution radar systems.

In summary, for good radar system which has the better capability of resolving the range should have a narrow pulse in time domain and wideband signal in frequency domain. These two requirements cannot be attained with a simple single tone pulse; hence the modulation has been incorporated into the pulse which alters the spectral distribution without changing the duration of the pulse in time domain. The objective of the pulse compression is to de-spread the spectral content of the fixed duration pulse with some modulation [4].

The block diagram for a radar system that uses the pulse compression is shown in Fig. 1. Initially a continuous carrier signal is generated by a microwave source and this continuous carrier is

* Corresponding author.
E-mail addresses: thakur9569@gmail.com (A. Thakur), salmanraju.talluri@juit.ac.in (S.R. Talluri).
Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.asej.2017.01.003
2090-4479/© 2017 Ain Shams University. Production and hosting by Elsevier B.V.

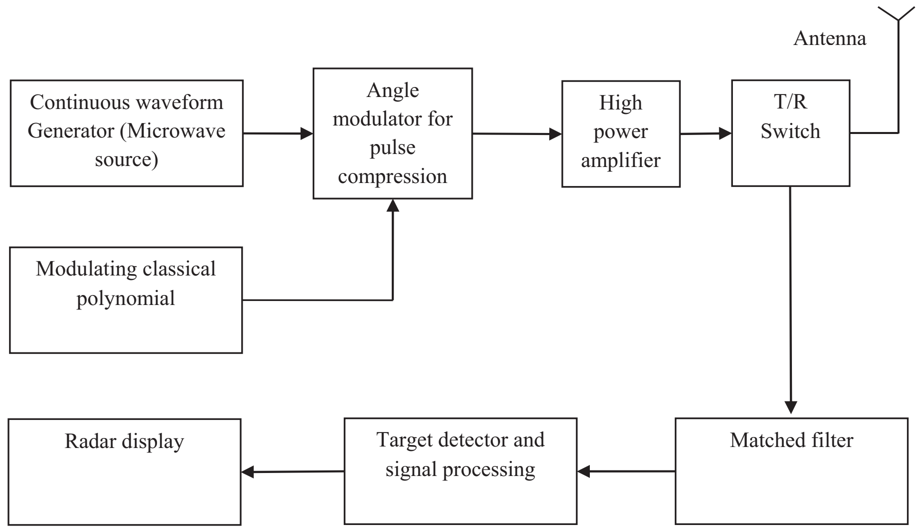

Fig. 1. Simplified block diagram of the radar system with pulse compression.

modulated by a pulse modulator in order to transmit the pulses at regular intervals of time which is also called as pulse repetitive interval. In the absence of the pulse compression, this pulse is directly applied to the high power amplifier and then to the transmitting antenna. If the pulse compression is required, then the carrier is modulated in angle and then applied to the high power amplifier and then to the transmitting antenna. Pulse compression is done by modulating the angle of the transmitted pulse in different ways. The figure of merit of the pulse compression is the time-bandwidth product [5,6]. It is a known fact that time duration of the pulse and bandwidth of the same pulse are inversely proportional. Hence it is not possible to improve the time-bandwidth product by simply stretching the pulse in time domain. This has to be done only with the modulation of the carrier pulse which leads to pulse compression.

Many pulse compression techniques have been developed in the literature [7–10]. Baghel and Panda [7], have proposed a hybrid model for the phase coded waveforms in which matched filter output is modulated by the output of radial function for different Barker codes. In addition to this, the hardware requirement is also significantly less to implement this hybrid model without any training iterations as in the neural network. In [8], the authors have developed a model in which a mismatched filter, comprised of a matched filter is cascaded with a parameterized multiplicative finite-duration impulse response filter. For a given main lobe to sidelobe ratio, the proposed filter is longer than the length-optimal filters but uses fewer multipliers and adders. Vizitiu [9] has produced a technique to overcome the problems of linear frequency modulated signal that is stretching of the main lobe width which disturb the range resolution by using nonlinear laws and recently developed Woo filters [10] are much better choices for the pulse compression if the hardware requirements are of no constraint.

In the literature there is little comparative analysis on pulse compression with classical orthogonal polynomials [11–13] and this paper addresses the pulse compression with classical orthogonal polynomials for different orders and a detailed analysis is carried out.

This paper has been organized as follows. The section II gives the problem formulation while section III gives the simulation and results. Section IV gives the conclusion based on the detailed study.

2. Problem formulation

The range resolution can be expressed as

$$R = c\tau/2 \tag{1}$$

where c is the propagation velocity of the pulse and τ is the duration of the pulse. In practice, rather than sending a single pulse, multiple pulses will be sent at some intervals of time. This interval sometimes can be regular or irregular depending on the application. If it is regular interval, then it is called as pulse repetitive interval (p.r.i). For a better range resolution, τ must be as small as possible. A rectangular pulse with duration τ has resolution bandwidth (BW) as $1/\tau$. Hence the range resolution can be expressed in terms of bandwidth as

$$R = \frac{C}{2BW} \tag{2}$$

For better resolution in range, the bandwidth of the pulse has to be very large which indicates a shorter pulse. This shorter pulse makes difficulty in decision of the target. Hence the BW of the pulse has to be increased as much as possible while maintaining the duration of the pulse fixed. To satisfy this constraint, modulation can be applied on the pulse with the equation

$$x(t) = A = \cos(\theta(t)) \times Rect\left(\frac{t}{\tau}\right) = A\cos(2\pi ft + \varphi(t)) \times Rect\left(\frac{t}{\tau}\right) \tag{3}$$

where $Rect(t/\tau)$ is a rectangular pulse of duration τ. Here τ has been fixed and the search has to be conducted for the best possible function $\varphi(t)$, such that the spectrum of $x(t)$ has to spread flatly over the large band of frequencies.

There are many functions possible for $\varphi(t)$, but in this paper, the functions are confined to the classical polynomials due to the wide area applications of these classical polynomials in engineering domain [12]. This paper presents the detailed analysis of the pulse compression with respect to classical orthogonal polynomials. In order to observe time-bandwidth product, spectrum of different classical orthogonal polynomials has been obtained. Depending on the values of α (optimizing factor), the spectrum is expanding smoothly up to a certain value after that spectrum get distorted. Hence this variable α has to be selected such that the time-bandwidth can be improved without any distortion in the signal spectrum.

3. Simulations and results

The transmitted pulse can be expressed in mathematical as $x(t)$. This precisely represents a carrier of duration τ seconds whose angle is varied in accordance with $\varphi(t)$. Variations in the function $\varphi(t)$ give the modulation in the transmitted pulse. In this paper, three types of variations are considered. Firstly, $\varphi(t)$ is varied in proportion to the classical orthogonal polynomial $p_n(t)$. Here $p_n(t)$ can be any orthogonal polynomial of order n. Secondly, $\varphi(t)$ is varied in proportion to the integral of the classical orthogonal polynomial and finally $\varphi(t)$ is varied in proportion to the derivative of the classical orthogonal polynomial. First case is considered as the phase modulation of the carrier with $\varphi(t)$

$$\theta(t) = 2\pi ft + \alpha p_n(t) \tag{4}$$

Second case is considered as frequency modulation.

$$\theta(t) = 2\pi ft + \alpha \int p_n(t)dt \tag{5}$$

Third case can be treated as general angle modulation of carrier with $\varphi(t)$.

$$\theta(t) = 2\pi ft + \alpha dp_n(t)/dt \tag{6}$$

The maximum variations in the instantaneous frequency and the maximum phase deviation for the transmitted pulse can be controlled with the parameter α and after the analysis is carried out, it is possible to come up with the maximization of time-bandwidth product. Hence α is considered as optimizing parameter for maximum bandwidth.

Table 1
Recursive equations for classical orthogonal polynomials of order $n, (n \geqslant 0)$ with $T_0 = P_0 = L_0 = H_0 = 1$.

Chebyshev (T_n)	$T_{n+1} = 2xT_n - T_{n-1}$
Legendre (P_n)	$P_{n+1} = \frac{1}{n+1}((2n+1)xP_n - nP_{n-1})$
Laguerre (L_n)	$L_{n+1} = \frac{1}{n+1}((2n+1-x)L_n - nL_{n-1})$
Hermite (H_n)	$H_{n+1} = 2xH_n - 2nH_{n-1}$

In the simulations, the above mentioned three variations are considered with first four order polynomials of all classical orthogonal polynomials. The duration of the pulse has been fixed constant for all simulations and the carrier frequency f has been taken as 61 Hz. This is arbitrary and the conclusions are not going to change because of this choice, as the frequency can be scaled up according to the requirements in practical applications. All four types of the classical polynomials are given in Table 1. Fig. 2 represents the variation of the polynomials as a function of time. Fig. 3 represents the transmitted signals with different polynomials for the same optimizing parameter α with $\varphi(t)$ proportional to $p_n(t)$ With the help of FFT [14] the spectrum for the transmitted pulse is obtained and Fig. 4 represents the frequency spectrum for different optimizing parameters for all four types of polynomials. All simulations are carried out in MATLAB [15]. The color of the traces has been preserved in all the succeeding figures for comparison.

From Fig. 4 it is observed that by increasing the α from small number to large number, the spectrum of the signal is spreading smoothly from narrow band to large band and then there is a distortion in the spectrum distribution. This can be observed in the column-wise plots. This indicates that there is an optimal value of α which maximizes the time-bandwidth product which is the main requirement in pulse compression. This has been calculated for all types of polynomials and tabulated in Table 2.

The optimal value is possible for all the classical polynomials except for the Laguerre polynomials which are deviated from the rest of the polynomials. This can be attributed to the fact that Laguerre polynomials are monotonic in the entire time duration and the variations in the arguments are too fast (very high frequency). If there is a practical device which can support such a huge frequency variations with high accuracy in a short period of time (restriction on the physical reliability of the source), the Laguerre polynomials are a better choice as far as the pulse compression is require. A comparative analysis on all the spectral properties of these transmitted pulses have been carried out with the

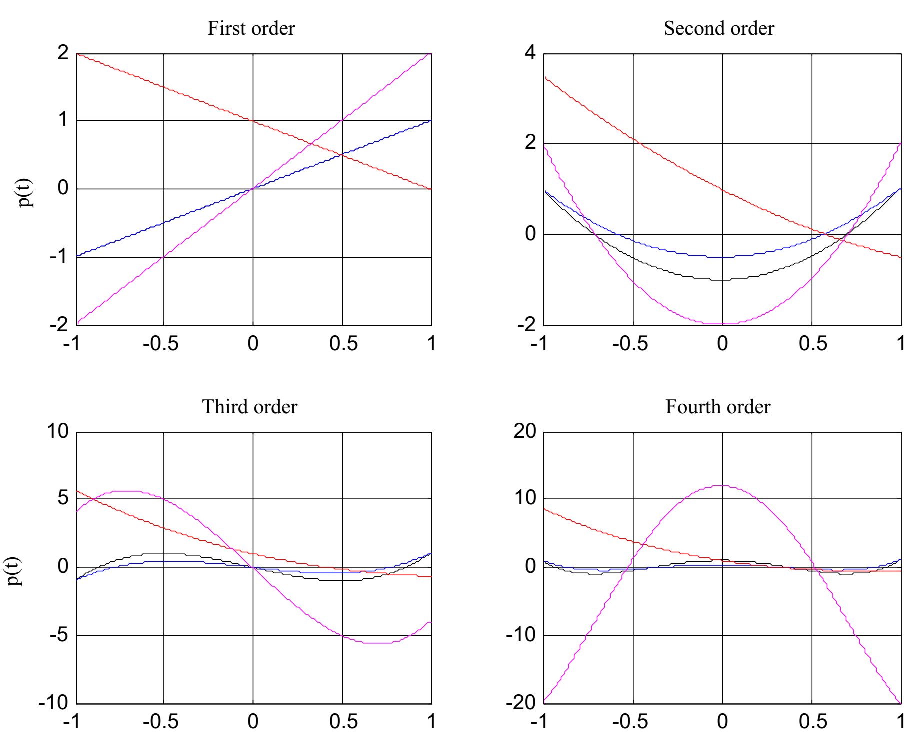

Fig. 2. Classical orthogonal polynomials of different orders. (Black, Blue, Red and Magenta: Chebyshev, Legendre, Laguerre and Hermite polynomials respectively.)

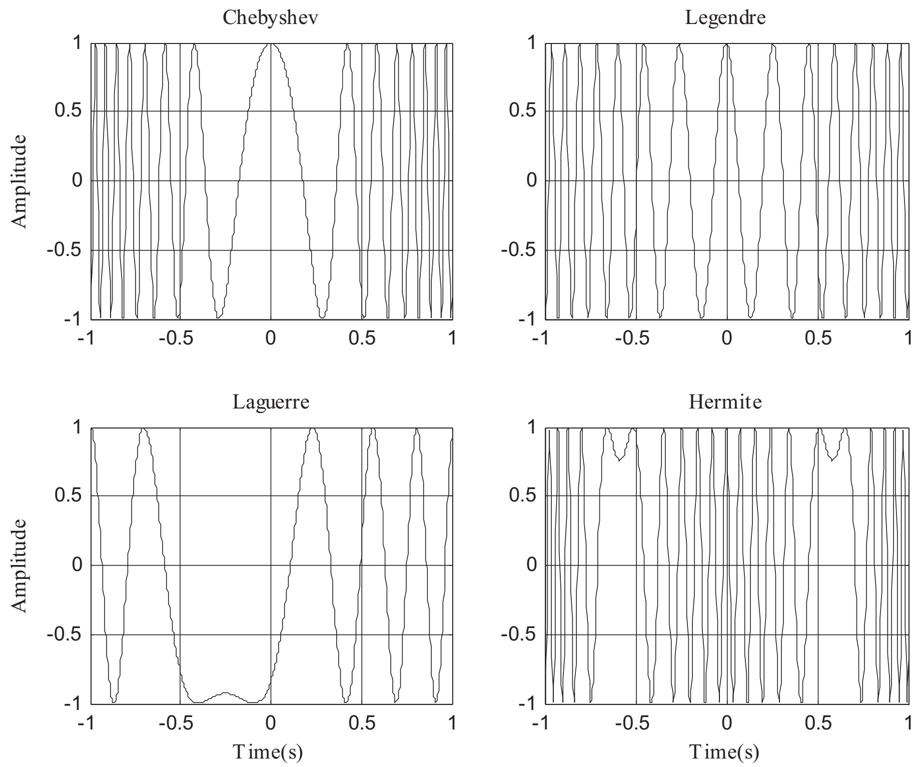

Fig. 3. Transmitted signal with different classical orthogonal polynomials.

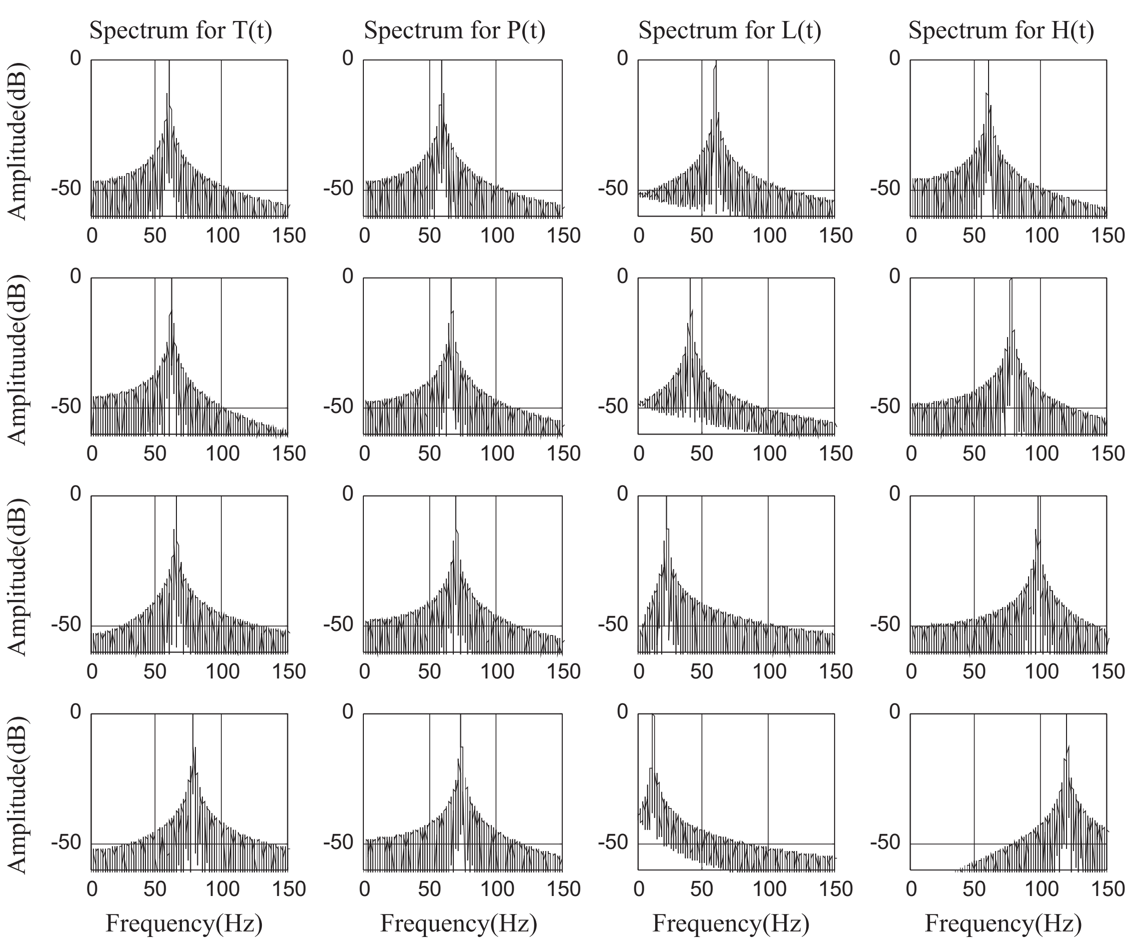

Fig. 4. Spectrum for classical orthogonal polynomials having order one and different values of α. First, second, third and fourth columns are for Chebyshev, Legendre, Laguerre and Hermite polynomials respectively.

Table 2
Optimal values (α) and bandwidth for classical orthogonal polynomials in phase modulation.

Polynomials	2nd order		4th order	
	α	B.W.	α	B.W.
Chebyshev	85	110	21	82*
Legendre	130	117	50	116*
Laguerre	600**	178**	21	35
Hermite	48	117	3	50*

three abovementioned cases. Figs. 5–7 represent the spectral variations of different order classical polynomials in phase modulation (first case), frequency modulation (second case) and finally the general case of angle modulation. The spectrums are observed for orders two, three and four for proportional, derivative and integral variations and the results are tabulated in Tables 2–4 for phase, frequency and general angle modulations.

From the tables and figures, it is observed that, when the order of the polynomial is increased, the spectral distribution is not smooth and the first order polynomials are giving better properties in case of frequency modulation compared to all other polynomials. At the same time, it is also observed that the frequency modulation with first order is similar to phase modulation with second order which is a known fact that the frequency modulation and phase modulations are inter-related. Even though they appear to be almost same in spectrum, there is a slight difference in the spectrum due to the difference in the functions in the arguments. Of these two options, frequency modulation is better than the phase modulation.

In frequency modulation, by comparing time-bandwidth product in polynomials to polynomials, Except Laguerre, all other polynomials have good bandwidth in first order. Second ordered Laguerre polynomial has good bandwidth of 500 Hz when α is equal to 760. Polynomials having order three, Laguerre and Hermite gives flat spectrum but Laguerre have better result that is bandwidth of 560 Hz at α is equal to 570. Laguerre polynomial having order four gives better bandwidth than other order polynomials that is 880 Hz at α is equal to 610. So Laguerre polynomials are better in order to improve time-bandwidth product than other polynomials. It is also observed that when the value of α is increased then spectrum of the signal is spreading but there are some ripples in the response along with attenuation in the spectrum. These ripples can be averaged in order to make the response smooth. In phase modulation, polynomials having order one have not any optimal bandwidth. For order two, all the polynomials have good bandwidth for different α values. Polynomials having order four, Hermite have better bandwidth that is 50 Hz at α is equal to 3.

Finally, the spectral contents are observed for higher order polynomials and they are not suitable for the pulse compression as the spectral distribution is highly non-uniform as represented in Fig. 8. Here the orders of the polynomials (for Chebyshev) are taken as 2 and 31. Similar kind of non-uniform spectral distribution is happening for other higher order classical polynomials.

4. Conclusions

The complete detailed analysis on pulse compression with the classical orthogonal polynomials is carried out and it is concluded that the Laguerre polynomials are a better choice for pulse compression if there is no restrictions on the physical implementation of the source. After Laguerre polynomials, Legendre polynomials

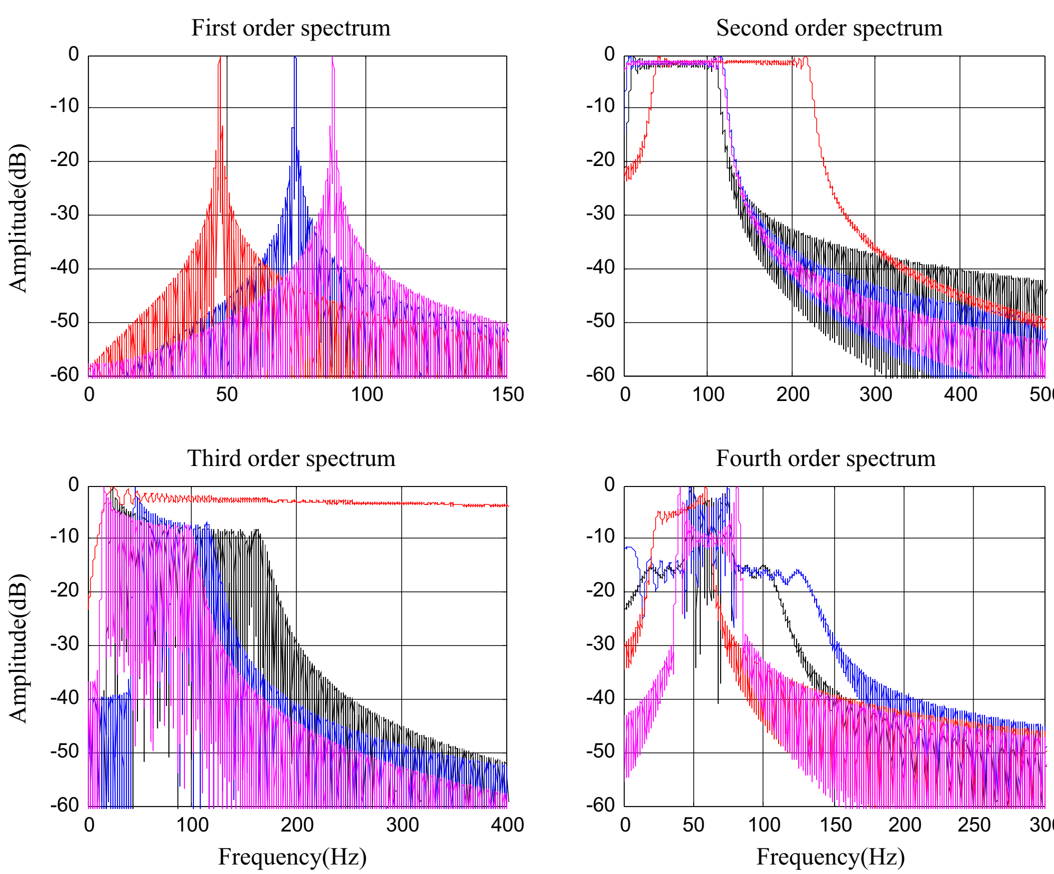

Fig. 5. Spectrum for classical orthogonal polynomials having different order in phase modulation.

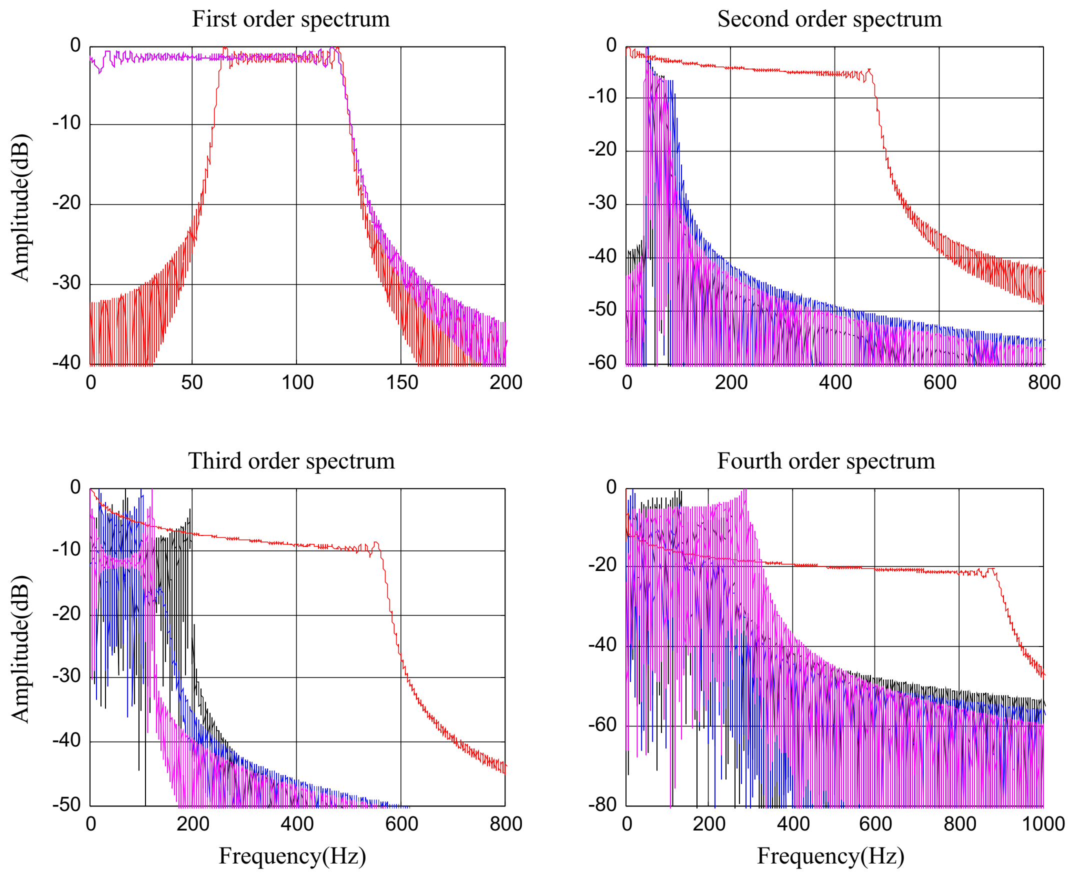

Fig. 6. Spectrum for classical orthogonal polynomials having different order in frequency modulation.

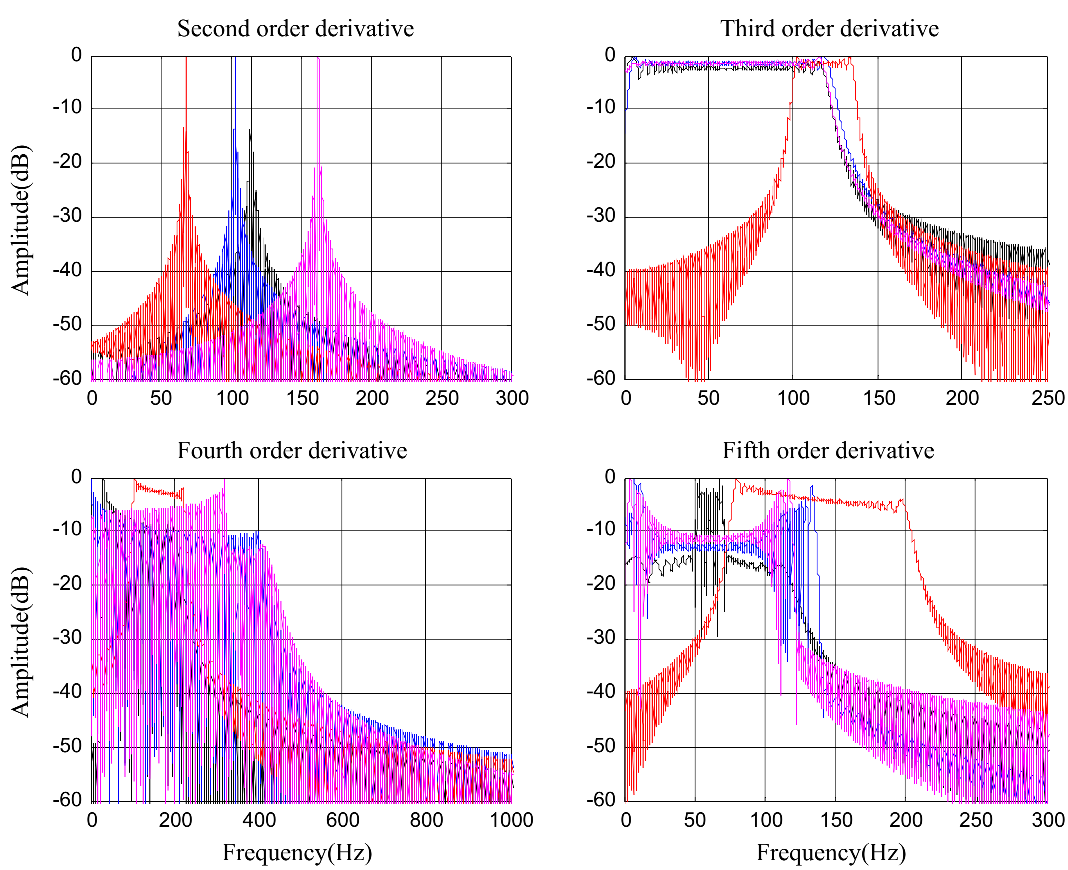

Fig. 7. Spectrum for differentiated classical orthogonal polynomials having different order.

Table 3
Optimal values (α) and bandwidth for classical orthogonal polynomials in frequency modulation.

Polynomials	1st order		2nd order		3rd order		4th order	
	α	B.W.	α	B.W.	α	B.W.	α	B.W.
Chebyshev	400	120	–	–	–	–	–	–
Legendre	400	120	–	–	–	–	–	–
Laguerre	200**	55**	760	471	570	560	610	880
Hermite	200	120	–	–	70	120	–	–

Table 4
Optimal values (α) and bandwidth for differentiated classical orthogonal polynomials.

Polynomials	3rd order derivative		5th order derivative	
	α	B.W.	α	B.W.
Chebyshev	16	118	2	115*
Legendre	27	118	5	135*
Laguerre	120**	35**	40**	120**
Hermite	8	118	0.8	120*

* Represents averaging the spectrum is required.
** Laguerre polynomials whose spectrum is increasing up to any optimising value.

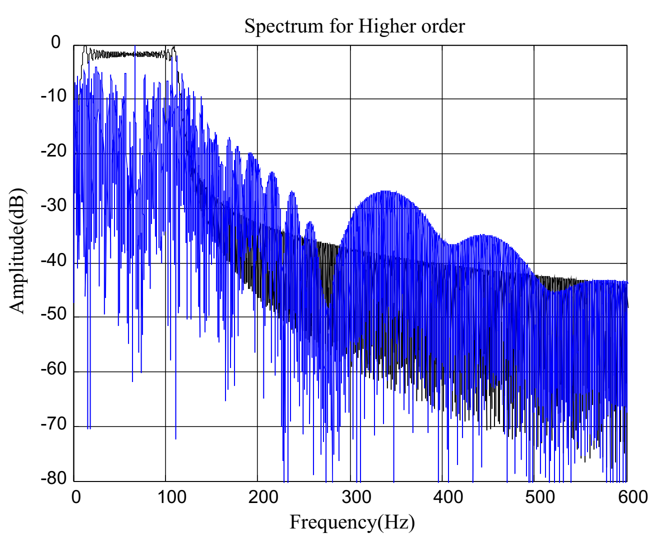

Fig. 8. Spectrum for Chebyshev polynomial of n = 2 (Black) and n = 31 (Blue).

are giving better time-bandwidth product for frequency modulation with orderone. It is also observed that frequency modulation gives better time-bandwidth product for these polynomials than phase modulation and higher order polynomials are not suitable for pulse compression.

References

[1] Skolnik MI. Introduction to radar systems. New York: McGraw-Hill; 2001.
[2] Mahafza RM. Radar systems analysis and design using matlab. Boca Raton: CRC Press; 2000.
[3] Levanon N, Mozeson E. Radar signals. Hoboken: Wiley; 2004.
[4] Barton DK. Pulse Compression. Artech House; 1975.
[5] Lee W, Griffith H. A new pulse compression technique generating optimal uniform range side lobe and reducing integrated sidelobe level. IEEE Int Radar Conf 2001:441–6.
[6] Cilliers J, Smit J. Pulse compression sidelobe reduction by minimization of *Lp*-norms. IEEE Trans Aerospace Electronic Syst 2007;43:1238–47.
[7] Baghel V, Panda G. Development of an efficient hybrid model for range sidelobe suppression in pulse compression radar. Aerospace Sci Technol 2013;27:156–62.
[8] Indranil S, AdlyFam T. Multiplicative mismatched filters for sidelobe suppression in Barker Codes. IEEE Trans Aerospace Electron Syst 2008;44:349–59.
[9] Vizitiu IC. Some aspects of sidelobe reduction in pulse compression radars using NLFM signal processing. Progr Electromag Res 2014;47:119–29.
[10] Zakeri BG, Zahabi MR, Alighale S. Side lobes level improvement by using a new scheme used in microwave pulse compression radars. Progr Electromag Res Lett 2012;30:81–90.
[11] Andrews GE, Askey R, Roy R. Special functions. Cambridge University Press; 1999.
[12] Nevai Paul. Orthogonal polynomials: theory and practice. Boston: Kluwer Academic; 1989.
[13] Apostal Tom M. Calculus, vol. 2. New York: Wiley; 1968.
[14] Oppenheim Alan V, Willsky Alan S, Hamid S. Signals and systems. Prentice-Hall International; 1996.
[15] MATLAB version 7.10.0.499 (R2010a). Natick (MA): The Math Works Inc.; 2010.

Ankur Thakur has completed him B.Tech from LPU Jalandhar (Pb.) in 2012 and now pursuing M.Tech from Jaypee University of Information Technology, Solan, Himachal Pradesh, India.

Salman Raju Talluri has completed him M.Tech from IITK in 2009 and working towards Ph.D from Jaypee University of Information Technology, Solan, Himachal Pradesh, India. His areas of interst include microwave engineering and antenna arrays.