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Threshold selection analysis of spectrum sensing for cognitive radio network with censoring based imperfect reporting channels

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Abstract

An appropriate threshold selection scheme is one of the main components to adjudicate the performance of energy detection spectrum sensing (EDSS) technique for cognitive radio network. In this paper, we have employed two different threshold selection approaches namely, the constant false-alarm rate (CFAR) and minimized error probability (MEP) and analyzed the threshold selection effects on the performance of cognitive user (CU) communication systems particularly, the total spectrum sensing error probability and throughput. We have derived the expressions and analyzed these performance parameters by considering an imperfect spectrum sensing and reporting channels in the cooperative spectrum sensing scenarios for additive white Gaussian noise (AWGN), Rayleigh and Nakagami-m fading environments. In addition, the censoring concept has been applied to the proposed system and compared its effect with that of the non-censoring based cognitive radio network (CRN) system under the perfect reporting (PR) and imperfect reporting (IR) channel. With the help of simulation, we have illustrated that the role of threshold selection approach is crucial to maximize the throughput and minimize the spectrum sensing error while considering the amount of error in the reporting channel. Further, from the results, the existence of trade-off between the spectrum sensing error probability and throughput is presented with threshold selection approaches. Moreover, it is also shown that there is need to switch among CFAR and MEP threshold selection approaches in the censoring scenario, to enhance the throughput and decrease the spectrum sensing error probability.

Keywords Cognitive radio · Energy detector spectrum sensing · Threshold selection approach · Imperfect reporting channels · Censoring · Sensing performance parameter

1 Introduction

Cognitive radio (CR), which allows the cognitive users (CUs) to probe the underutilized licensed spectrum and opportunistically use them such that the interference with primary user (PU) is below the interference temperature limit [1], has been considered as a promising technology to overcome the spectrum scarcity problem. In the CR system, CUs attempts to access the licensed channel under different approaches such as interweave, underlay, overlay and hybrid scenarios [2, 3] and the accurate detection of spectrum holes is the most essential step for an efficient utilization of radio frequency spectrum. Various spectrum sensing (SS) approaches are employed by different researchers [4–6] to realize the spectrum holes. However, each approach has its own merits and demerits with respect to the spectrum sensing performance, implementation complexity, sensing time, information of the PU (blind or non-blind) etc. [4, 5]. With this context, the energy detector spectrum sensing (EDSS) technique is widely employed by various researchers [7–9] due to its low computational and implementation complexity [4, 5]. In EDSS, the binary hypothesis (H_0 and H_1) is considered to know the current status of the licensed channel, whether it is occupied by PU or not, for which the energy of received signal ($T(x)$) in a specific band of interest is computed by collecting the number (N) of samples of the received signal. When the

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energy of received signal is greater than that of the specific value of threshold (λ), the PU's presence is detected by the CU i.e. hypothesis H_1 is true and vice-versa [2]. In this context, an appropriate selection of λ is indispensable in EDSS technique for better spectrum sensing performance [10]. Further, various researchers have employed different threshold selection approaches for EDSS which are as follows: (1) Constant false alarm rate (CFAR) [6, 11, 12], (2) Minimized error probability (MEP) [10, 13] and (3) Constant detection rate (CDR) [12, 14]. In CFAR approach, the threshold (λ_f) is computed for predefined targeted value of the false-alarm probability ($\overline{P_f}$) while in MEP approach the threshold (λ_e) is achieved by minimizing the error probability (P_e) with respect to threshold $\left(\frac{\partial P_e}{\partial \lambda} = 0\right)$. However, in CDR approach, the threshold (λ_d) is computed for predefined value of targeted detection probability ($\overline{P_d}$) [12]. Moreover, the spectrum sensing performance of CU degrades under the multipath fading, shadowing and hidden node scenario in the non-cooperative SS while employing EDSS technique [15, 16]. Therefore, to improve the spectrum sensing results in the aforementioned scenarios, the cooperative spectrum sensing (CSS) technique has been employed by different researchers [17, 18]. In the CSS technique, the spectrum sensing decisions of each CU are sent to the fusion center (FC) via the reporting channels where FC apply different cooperative rules (OR, AND, Majority and K-out-of-M rules) [17, 18] to take global decision about the status of PU channel. In practice, the reporting channels are imperfect which leads to an inaccurate spectrum sensing decision by the FC [19]. However, in the CSS technique, the energy consumption increases due to cooperation overhead bits [20], therefore to reduce the energy consumption and to save the bandwidth of reporting channels, the censoring approach [21, 22] is commonly employed by different researchers in which the spectrum sensing results of only those CUs are sent to FC which are reliable [21, 22].

Moreover, with the aforementioned scenarios, various researchers have selected the threshold with different approaches (CFAR, CDR, MEP) in EDSS technique for diverse channels and has illustrated the effect of threshold selection over several performance parameters of CU. Numerous researchers have reported in this context which are briefed below along with the insufficiency of their proposed approach.

- In [23], the authors have suggested that to improve the throughput of CU, the CFAR based threshold selection approach is better although to provide sufficient protection to PU from CU, the threshold selection with CDR approach is better.
- Further, Verma and Sahu [12] have employed the threshold selection with combination of CFAR and CDR approaches and improved the throughput of CU while providing sufficient protection to PU in only the non-cooperative AWGN scenario.
- Atapattu et al. [10] have proposed an approach in the fading environment to select the optimal threshold in order to minimize the total error probability of CU in the cooperative spectrum sensing scenario. Further, the OR cooperative fusion rule has been employed in [10] which is not the optimal rule for minimizing the error probability as presented by Zhang et al. in [24]. However, the analysis of the effect of threshold selection on throughput of CU was missing in the literature [10].
- Moreover, the threshold selection effect on both the throughput and error probability of CU is analyzed by Kumar et al. [25] in only the non-cooperative AWGN environment. Further, in [26], the authors have considered fading environment and analyzed the effect of threshold selection on both the throughput and error probability of CU under the Majority cooperative rule at FC. However, their analysis is limited to only perfect reporting channels.
- Recently, Li et al. [22] have assumed the predefined value of threshold in EDSS technique and observed the individual effect on both the false-alarm and detection probability (P_f and P_d) of CU at high SNR under CSS technique (OR rule) in the Rayleigh fading environment. The censoring approach with imperfect reporting channel is considered, and the effect of number of CUs and antennas on the false-alarm probability and throughput in the Rayleigh fading channel is analyzed. However, the method of computation of threshold for different fading channels presented in [22] and its effect on total spectrum sensing error and throughput for diverse channels at low SNR is missing.

Motivated by aforementioned reported literatures [10, 12, 22, 25, 26] and to address their stated limitations, the authors have attempted to integrate them in a single framework in this paper. In this context, the author's contributions of the proposed system are stated as follows.

- We have considered more realistic low SNR scenario for the spectrum sensing in AWGN, Rayleigh and Nakagami-m channels.
- The imperfect reporting channel is employed which affects the spectrum sensing decision at FC and two scenarios are considered in such a manner where the spectrum sensing decision of CUs is sent to the FC with and without the censoring approach. The comparison among the censoring and non-censoring scenarios are further described.

- Furthermore, we have derived the mathematical expressions for the total spectrum sensing error and throughput for the proposed fading channel while employing the perfect/imperfect reporting channel in the censoring and non-censoring scenarios.
- Moreover, we have also analyzed the effect of threshold selection approaches on the throughput and total spectrum sensing error probability for the proposed spectrum sensing scenario.

This article is arranged as follows. The related work and problem formulation are presented in Section 2. System model and performance analysis for the proposed CRN is presented in Section 3. Section 4 describes simulation results for the proposed system model. Finally, Section 5 concludes the work.

2 Related work and problem formulation

In the CSS technique, the advantages of spatial diversity are employed in cognitive radio network (CRN) to enhance the spectrum sensing performance of CUs when the sensing channels suffers from fading. Various researchers employed several methods in CSS for CU's performance improvement in terms of throughput, total error probability and energy efficiency (EE) which is illustrated briefly in Fig. 1.

Further, in the CSS technique, two phases namely, the sensing and reporting [22] are considered. Various researchers have considered perfect reporting (PR) or/and imperfect reporting (IR) channels and have employed censoring/non-censoring approaches to analyze the performance of CRN which is described below.

2.1 Perfect reporting channels

In the perfect reporting channels, Zhang et al. [24] have minimized the total spectrum sensing error probability by employing the Majority rule at FC when P_f (probability of false-alarm) and P_m (probability miss-detection) are nearly same. Moreover, the authors in [10] have illustrated the effect of number of samples (*N*) on the total spectrum sensing error probability for perfect reporting channel under OR and Majority fusion rules for MEP threshold selection approach in the fading environment. It has been observed by Kumar et al. [26] that at SNR = 20 dB, the Majority cooperative rule provides less total spectrum sensing error probability as compare to that of OR cooperative rule for same number of CUs (M = 10). Further, by employing different cooperative rules (OR, AND, Majority), Liang et al. [14] have presented the sensing-throughput trade-off problem and maximized the throughput by computing the optimal spectrum sensing time. Moreover, to improve the throughput of CU with the optimal spectrum sensing time and number of cooperative CUs are reported by Peh et al. [27] as well as Liu and Tan [28]. However, the improvement in throughput is achieved by Tuan and Koo [29] employing simultaneous spectrum sensing and data transmission technique. Further, the throughput enhancement is also performed by Lu et al. [30] while considering adaptive spectrum sensing window for CR. Furthermore, the multi-objective optimization models are employed by Li and Liu [31] to maximize the throughput of CU by addressing joint CSS and power allocation technique in CRN. However, to maximize the throughput in multichannel CSS, Fan and Jiang [32] have computed the total spectrum sensing duration in the CU's frame structure and have proposed the approach to allocate this optimal total sensing duration among different channels. Moreover, Tang et al. [33] have considered both the perfect and imperfect spectrum sensing techniques and achieved the

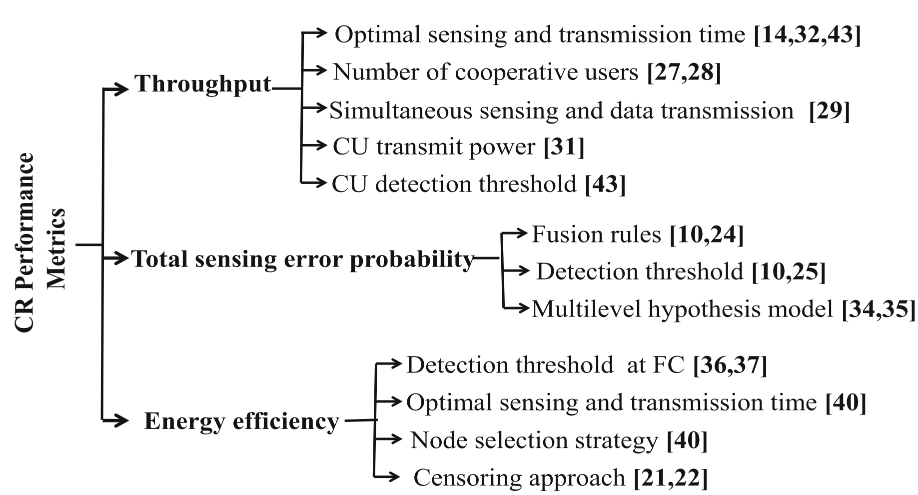

Fig. 1 Diverse approaches employed by various researchers to improve the spectrum sensing performance parameters of CRN

closed-form expression for normalized throughput in multi-PU cognitive radio network. In addition, they have analyzed the effect of frame duration of CU and traffic pattern of PU on the normalized throughput. Moreover, Yadav et al. in [34] and Sharifi et al. in [35] have considered the multi-level hypothesis in CSS technique to improve the spectrum sensing accuracy of CU under the primary user emulation attack. Since the energy consumption increases with the number of cooperative users therefore, Althunibat et al. [36, 37] have enhanced the energy efficiency (EE) by obtaining the optimal detection threshold at FC. Further, as EE is enhanced, there is degradation in the spectral efficiency (SE) and this trade-off between SE and EE is illustrated by Hu et al. in [38].

2.2 Imperfect reporting channels

In the CRN, the CSS is promising technique to mitigate the effect of fading, shadowing and hidden node problems on spectrum sensing performance of CR however it degrades the EE of network. In the context of EE, Bhowmick et al. [39] have shown the trade-off between the spectrum sensing time and throughput of CU while considering the imperfect sensing along with imperfect reporting channels under the Rayleigh fading environment. Further, in the CSS, Najimi [40] have proposed a scheme to improve the EE of multichannel, multi-antenna CRN by optimizing the spectrum sensing time and node selection strategy in the imperfect reporting channel. Moreover, Zhao et al. [41] have considered multichannel CSS with imperfect reporting channel and has maximized the throughput of CU under the constraints on PU interference and sensing overhead without considering the mobility of CUs. In this context, Gahane et al. [42] have presented the effect of CU mobility on the spectrum sensing performance in the multi-antenna environment at CU in the improved energy detector spectrum sensing (IEDSS). Moreover, the consideration of reporting time is of great importance as it affects the throughput of CRN therefore, Firouzabadi and Rabiei [43] have considered the multi-channel spectrum sensing and maximized the throughput of CRN by optimizing the sensing time, reporting time, sensing threshold and overall sensing plus reporting time in each sub-band.

Furthermore, the performance of hard-and soft-decision based CSS is analyzed by Chaudhari et al. [44] by considering the error in reporting channels. Further, they demonstrated that the soft-decision based CSS shows better performance. However, the effect of number of bits employed for sending the sensing decision to FC on detection probability is shown by Sakran and Shokair [45] and presented that there is degradation in the detection probability of 16.5% and 12.2% when reporting is performed by employing one bit and two bits, respectively under the imperfect reporting channel. In addition, Sakran and Shokair [45] have employed amplify and forward relaying schemes to improve the detection probability. Moreover, the improvement in the false-alarm and detection probabilities of CSS with respect to the hard-decision logic is achieved by Yilmaz et al. [46] while employing uniform quantization and weight vector in the global decision logic under imperfect reporting channels at the cost of increased number of overhead bits. Further, Mi et al. [47] also illustrated that the spectrum sensing performance of soft-decision fusion rule for CSS is better than that of the hard-decision fusion rule at the cost of increased bandwidth of reporting of channels. Furthermore, they also proposed a sensing approach and have shown the tradeoff between the reporting bandwidth and sensing performance. Moreover, the reduction of cooperation overhead bits and energy consumption is achieved by Bae and Kim [48] with on/off reporting schemes as well as reduction of the number of reporting CUs over the imperfect reporting channels. However, Liu et al. [49] have employed intelligent clustering technique to decrease the cooperative overhead bits and improved the spectrum sensing performance. Additionally, Oh and Lee [50] have maximized the detection probability in an imperfect reporting channels when selected CUs are sending the spectrum sensing results to the FC. Moreover, Ghorbel et al. [51] have formulated the detection and false-alarm probabilities in CSS for un-identical spectrum sensing and reporting channels.

2.3 Censoring approach

One of the main advantages of CSS technique is to enhance the spectrum sensing performance of CU however at the same time, the energy consumption also increases with the number of cooperative users. Therefore, this issue is more critical for battery powered users. In order to reduce the energy consumption in CSS technique, various researchers [21, 22] have employed censoring approach where the spectrum sensing decision of some limited CUs are transmitted over the reporting channels with the purpose of energy saving. However, Nallagonda et al. [21] have suggested that when the reporting channels are deeply faded, the decision received at FC due to these channels will be erroneous, therefore the authors have employed censoring approach by preventing the reporting of spectrum sensing results through these erroneous reporting channels. However, Li et al. [22] have used the censoring approach by transmitting decisions of only those CUs which has sensed active PUs on the channel. Subsequently, the authors in [52] have improved the throughput of network by reducing the signaling cost with censoring based CSS. Further, Sun et al. [19] and Jiang et al. [53] have employed the censoring approach under the perfect/imperfect reporting channels and analyzed the detection performance of CU. However, in [22], Li et al. have considered that each CU perform spectrum sensing through multiple antennas and send the sensing results to FC through single antenna with the imperfect sensing and reporting channel. Further, they have derived mathematical expressions for false-alarm probability, detection probability and normalized throughput for the Rayleigh faded channels. In addition, Li et al. [22] have presented the variation in detection probability of CU with predefined threshold for different number of cooperative CUs and number of antennas (L) while employing the censoring and non-censoring approaches. Further, they concluded that for the same number of CUs and number of antennas, the non-censoring approach outperform censoring approach in terms of detection probability. However, in the non-censoring approach, degradation in the detection probability (sensing tail problem) occurs due to high error probability in the reporting channel.

Based on the aforementioned literature on censoring and perfect/imperfect reporting channel, in the the proposed paper, we have derived the mathematical expressions for total spectrum sensing error and throughput in the fading environment while employing the perfect/imperfect reporting channel with the censoring and non-censoring approaches. In addition to this, we have also analyzed the effect of CFAR and MEP threshold selection approaches on throughput and total spectrum sensing error probability of CU.

In summary, the comparative analysis of various researchers' reported works with the proposed CRN system model on CUs performance parameters has been presented in Table1. From Table 1, it is clear that Verma and Sahu in [12] and Charan and Pande in [59] have also presented the threshold selection approaches. In [12], Verma and Sahu have employed the threshold selection approach with CFAR and CDR for AWGN channel under the non-cooperative environment to maximize the throughput. However, the performance of their proposed approach [12] on the total spectrum sensing error probability in AWGN channel as well as the effects of threshold selection approaches, cooperation on the throughput and total spectrum sensing error probability for fading environments has not reported [12]. Further, a covariance-based channel sensing method is presented in [59] in which the adaptive threshold is selected to minimize the total spectrum sensing error probability in the non-cooperative scenario however, the throughput analysis is missing. Moreover, the comparative study presented in Table 1 also clarify novelty of the proposed approach.

3 Proposed system model with performance analysis

3.1 System model

In the proposed interweave based CRN system model, we have considered one PU transmitter, M cognitive users' nodes employing single antenna and one fusion center (FC) as shown in Fig. 2a. Each CU employed EDSS and sends the spectrum sensing results to FC via the reporting channels where the sensing results of CU are affected due to additive white Gaussian noise (AWGN), Rayleigh or Nakagami-m fading channel. Further, the reporting channels are considered imperfect with different reporting error probabilities (P_e^r) and at FC, the MAJORITY cooperative rule is applied to take the global final decision about the presence/absence of PU on the channel. Moreover, the periodic frame structure of CU is assumed as shown in Fig. 2b where each CU senses the channel after T frame duration in which the spectrum sensing plus reporting time (T_{sr}) is fixed.

The false-alarm and detection probability of *ith* CU is assumed to be P_{f_i} and P_{d_i}, respectively (where i = 1,2…M). In addition, the sensing decision of each CU is sent to the FC via the imperfect reporting channels. The censoring effect has been considered in the system model and comparison with the non-censoring based system has been made in the further sections of the paper. In the non-censoring approach, the spectrum sensing results of all M cognitive users are sent to the FC for which the received false-alarm and detection probability of *ith* CU at FC due to the imperfect reporting is presented as: $P_{f_i}^r$ and $P_{d_i}^r$, respectively. However, for the censoring approach (where limited number of CUs send their sensing results to FC), FC receives sensing decision of only M_c (where $M_c \leq M$) CUs for which the received false-alarm and detection probability of CU at FC with imperfect reporting channel is presented as: $P_{f_i}^{rc}$ and $P_{d_i}^{rc}$, respectively. In the proposed analysis presented further, we have assumed that each CU has same false-alarm and detection probability due to particular considered threshold detection approach and the spectrum sensing decision of each CU is affected equally at FC in the reporting channel. Therefore, for simplicity, we have removed the subscript i eg P_{f_i} and P_{d_i} are represented simply as: P_f and P_d in further analysis and same for the other symbolic representations. The detailed discussion about the censoring approach and method for the computation of M_c is presented in section 3.3.

However, we have assumed $S(n)$ and $W(n)$ being the transmitted modulated signal and AWGN noise in the channel, respectively. $S(n)$ and $W(n)$ are the independent and identically distributed random variable. Further, the

Table 1 Comparative study between different threshold selection approaches under different scenario

Reference no.	Channel	Threshold selection approach	CSS	Reporting channels	Censoring approach	Throughput	Sensing error probability
[34]	Rayleigh	CDR	✔	✘	✘	✔	✘
[35]	Rayleigh	Predefined	✔	✘	✘	✘	✔
[54]	–	CDR	✔ (K out of M)	✘	✘	✔	✘
[55]	–	CFAR + CDR	✔ (OR)	✘	✘	✔	✘
[32]	–	CFAR +CDR	✔ (AND)	✘	✘	✔	✘
[56]	Nakagami-*m*	CDR	✔ (Soft decision)	✘	✘	✔	✘
[36]	–	MEP	✔ (K out of M)	Imperfect	✘	✘	✔
[29]	–	Selected threshold maximized the throughput	✔	✘	✘	✔	✘
[57]	AWGN	Double threshold	✔ (OR)	Imperfect	✘	✔	✘
[58]	Rayleigh	Double threshold	✔ (OR)	Imperfect	✘	✔	✔
[14]	AWGN	CDR	✔ (AND)	✘	✘	✔	✘
[12]	AWGN	CFAR, CDR, CFAR + CDR	✘	✘	✘	✔	✘
[59]	AWGN	Adaptive Threshold (MEP)	✘	✘	✘	✘	✔
[25]	AWGN	CFAR, CDR, MEP	✘	✘	✘	✔	✔
[26]	AWGN, Rayleigh, Nakagami-*m*	CFAR, MEP	Majority	Perfect	✘	✔	✔
[10]	AWGN, Rayleigh, Nakagami-*m*	CFAR, CDR, MEP	✔ (OR)	✘	✘	✘	✔
[43]	–	–	✔ (OR)	Imperfect	✘	✔	✘
[39]	Rayleigh	Predefined	✔ (OR, AND, Majority)	Imperfect	✘	✔	✘
[22]	Rayleigh	Predefined	✔ (OR rule)	Perfect, imperfect	✔	✔	✘
Proposed	AWGN, Rayleigh, Nakagami-*m*	CFAR, MEP	✔ (Majority)	Perfect, imperfect	✔	✔	✔

received signal $X(n)$ under binary hypothesis at CU is presented as follows [2]:

$$X(n) = \left\{ \begin{array}{c} W(n) : H_0 \\ h.S(n) + W(n) : H_1 \end{array} \right\} \tag{1}$$

where n is the sample number and n ranges from 1, 2,….N. h represents the channel gain coefficient between PU and CU, therefore the energy of the received signal $X(n)$ or test statistics $T(x)$ of the energy detector is given as [10]:

$$T(x) = \frac{1}{N}\sum_{n=1}^{N}|X(n)|^2 \tag{2}$$

For $N > 250$, the probability density function of $T(x)$ can be considered as Gaussian distributed [7]. Further, in the proposed analysis, we have assumed the mean and variance of hypothesis H_0 and H_1 represented as follows [13]: $H_0 : \mathcal{N}\left(N\sigma_n^2, N\sigma_n^4\right)$ & $H_1 : \mathcal{N}\left(N\sigma_n^2(1+\gamma), N\sigma_n^4(1+\gamma)^2\right)$, where σ_n^2 is the noise variance and γ is the SNR received at each CU from PU. Moreover, the probability of false-alarm (P_f) and probability of detection (P_d) at each CU for a given threshold (λ) is given by Eq. (3) and (4) such as [13]:

$$P_f = \frac{1}{2}Erfc\left(\frac{\lambda - N\sigma_n^2}{\sqrt{2N\sigma_n^4}}\right) \tag{3}$$

$$P_d = \frac{1}{2}Erfc\left(\frac{\lambda - N\sigma_n^2(1+\gamma)}{\sqrt{2N\sigma_n^4(1+\gamma)^2}}\right) \tag{4}$$

$$P_m = 1 - P_d \tag{5}$$

$$P_e = P(H_0)P_f + P(H_1)P_m \tag{6}$$

where Erfc(.) is the error function, $P(H_0)$ and $P(H_1)$ are the probability of licensed channel being idle and active, respectively in a selected frequency band. In the proposed system model, we have considered that CU can transmit

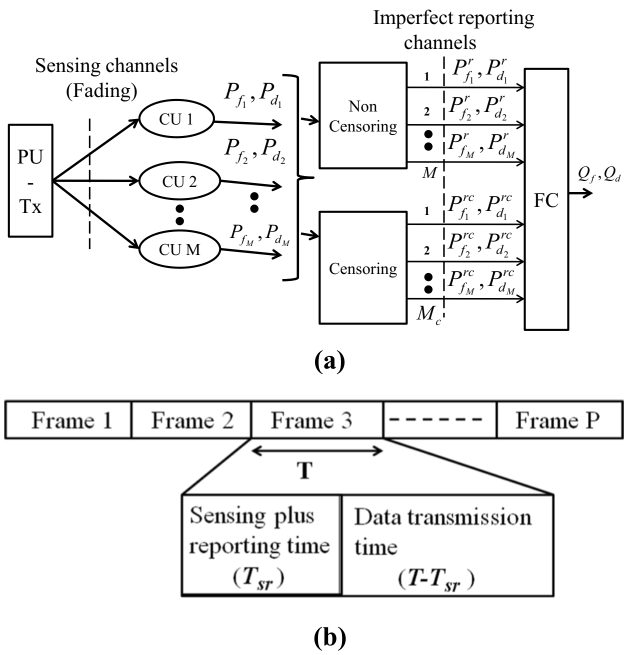

Fig. 2 Schematic of the proposed cognitive radio network **a** system model and **b** frame structure

the data on the licensed channels only in the following two States. In State-1, the licensed channel is assumed to be idle and no false-alarm is generated by CU while in State-2, the licensed channel is assumed to be occupied by PU but CU has missed its detection. Therefore, the total throughput (R) of CU in the non-cooperative spectrum sensing while considering two States is given as: $R = R_1 + R_2$, where $R_1 = P(H_0)\left(\frac{T-T_{sr}}{T}\right)(1-P_f)log_2(1+\gamma_s)$, is the throughput for 1st State and $R_2 = P(H_1)\left(\frac{T-T_{sr}}{T}\right)(1-P_d)log_2\left(1+\frac{\gamma_s}{1+\gamma}\right)$, is the throughput for 2nd state. Therefore, the total throughput (R) is [14]:

$$R = P(H_0)\left(\frac{T-T_{sr}}{T}\right)(1-P_f)log_2(1+\gamma_s) + P(H_1)\left(\frac{T-T_{sr}}{T}\right)(1-P_d)log_2\left(1+\frac{\gamma_s}{1+\gamma}\right) \tag{7}$$

3.2 Analysis and computation of threshold for different channels

The spectrum sensing performance of CRN is generally, measured in terms of P_d and P_f. However, for the fading channel, the average detection probability ($\overline{P_d}$) is computed as [17]:

$$\overline{P}_d = \int_0^\infty P_d f(\gamma)d\gamma \tag{8}$$

where $f(\gamma)$ is the SNR distribution function over fading channel. Therefore, the detection probability over the Rayleigh and Nakagami-m fading channels is given as [10]:

$$\overline{p_d^{ray}} = 1 - \frac{1}{2}\left[Erfc\left(\frac{N\sigma_n^2-\lambda}{\sqrt{2N}\sigma_n^2}\right) - \left(e^{\left(\frac{\frac{1}{\bar{\gamma}^2}+\frac{4}{\bar{\gamma}}\left(\frac{N\sigma_n^2-\lambda}{\sqrt{2N}\sigma_n^2}\right)\sqrt{\frac{N}{2}}}{2N}\right)} Erfc\left(\frac{N\sigma_n^2-\lambda}{\sqrt{2N}\sigma_n^2}+\frac{1}{\bar{\gamma}\sqrt{2N}}\right)\right)\right] \tag{9}$$

$$\overline{P_d^{Naka}} = 1 - \frac{\left(\frac{m}{\bar{\gamma}}\right)^m}{2\Gamma(m)}\int_0^\infty (x^{m-1})\left(e^{-\frac{mx}{\bar{\gamma}}}\right)Erfc\left(\sqrt{\frac{N}{2}}x+\frac{N\sigma_n^2-\lambda}{\sqrt{2N}\sigma_n^2}\right)dx \tag{10}$$

where $\Gamma(.)$ is the complete Gamma function, m is the shape parameter and $\bar{\gamma}$ is average SNR. However, the false-alarm probability is independent on the SNR value because the false-alarm probability is computed under the hypothesis H_0 [10]. Therefore, its value remains same for all the channels (fading or non-fading) [10]. Further, we have computed the threshold with CFAR approach from Eq. (3) for all the channels as:

$$\lambda_f = \left\{\sqrt{\frac{2}{N}}Erfc^{-1}(2P_{f\_fixed}) + 1\right\}N\sigma_n^2 \tag{11}$$

Moreover, with the help of Eqs. (4), (10), and (11), the thresholds with MEP approach $\left(\frac{\partial P_e}{\partial \lambda} = 0\right)$ under AWGN, Rayleigh, and Nakagami-m environment are computed as follows [10].

$$\lambda_e(AWGN) = \frac{N\sigma_n^2}{2}\left\{1+\sqrt{1+\frac{2(2+\gamma)ln(1+\gamma)}{N\gamma}}\right\}\left(\frac{1+\gamma}{1+\frac{\gamma}{2}}\right) \tag{12}$$

$$\lambda_e(Ray) = \left(1+\frac{1}{N\bar{\gamma}} - \sqrt{\frac{2}{N\pi}} + \sqrt{\frac{2}{N}\left(\frac{1}{\pi}+\sqrt{\frac{2N}{\pi}}\bar{\gamma}-1\right)}\right)N\sigma_n^2 \tag{13}$$

$$\lambda_e(Naka) = \left(1+\frac{2}{N\bar{\gamma}} - \frac{1}{2\sqrt{2N}}\left(\sqrt{\pi}-\sqrt{\pi-8+2N\bar{\gamma}^2}\right)\right)N\sigma_n^2 \tag{14}$$

With the help of Eq. (11–14) for individual channels, we have computed the critical SNR (γ_c) by finding the SNR value where CFAR and MEP approaches have same value of threshold. Further, we have employed critical SNR in the proposed system model to maximize the throughput and minimize the spectrum sensing error probability in the fading environment.

3.3 Effect of reporting error and censoring approach on sensing and throughput performance

In Section IIIC, the mathematical expressions for the total spectrum sensing error and throughput under the perfect and imperfect reporting channels as well as with and without censoring approach is analyzed. Moreover, we have derived the Eqs. (20) and (27) in the this paper which has illustrated the effect of non-censoring and censoring scenarios under the imperfect reporting channel on the throughput of CU. Further, in the result and discussion Section, we have described the consequence of different threshold selection approaches on the total spectrum sensing error and throughput for the above considered scenario under the AWGN, Rayleigh and Nakagami-m *fading* channels, which was lacking in the earlier reported literature.

3.3.1 Imperfect reporting channels

When the imperfect reporting channel is considered between CUs and FC, then the spectrum sensing results of CUs received at FC is affected with the amount of error probability of the reporting channel (P_e^r). Each CU send the spectrum sensing decision to FC in favour of either PU is active or idle on the channel. Moreover, by considering these above scenarios following states has to be considered i.e. (i) true status of PU, (ii) status of PU detected by CU (due to the perfect/imperfect spectrum sensing), and (iii) status of PU received at FC (due to perfect/imperfect reporting). Therefore, there are total eight possible cases based on the states of above scenarios out of which we have considered only those four cases in Table 2 for computation of combined false-alarm and detection probabilities at FC after cooperation, where PU status received at FC is active. The reason for selecting only these four cases is that since at FC, the K-out-of M rule is employed in which the final decision of the FC comes in favor of active PU only when at least K CUs decision received at the FC is in favour of active PU [60].

Moreover, from the Table 2, in the non censoring scenario, Case I considered active status of PU and status of PU detected by CU is also active so its corresponding probability is P_d. Further, the PU status received at FC is also active means there is no error in the reporting channel, therefore combined probability of case I in non-censoring scenarios will be $P_d\left(1-P_e^r\right)$ which is shown in second last column of Table 2.

Further, in Case II, the status of PU is idle however its status detected by CU is active, therefore the false alarm (P_f) is generated. Further, the PU status received at FC is active means there is no error in the reporting channel, therefore the combined probability of Case II is $P_f\left(1-P_e^r\right)$ in the non-censoring scenario. In the same manner, we can compute the detection/false alarm probability of CU at FC for the remaining two cases i.e. Cases III and Case IV in the non-censoring secnrio. In Case III, the status of PU is active however it is detected idle by CU, therefore CU has missed the detection and its corresponding probability is $(1-P_d)$. In addition, for Case III, the PU status received at FC is active means there is error in the reporting channel, therefore combined probability of Case III is $(1-P_d)P_e^r$. However, in Case IV, the status of PU is idle and it is also detected idle by CU, therefore its corresponding probability is $(1-P_f)$. Further, in Case IV, the PU status received at FC is active means there is error in the reporting channel, therefore the combined probability of Case IV is $(1-P_f)P_e^r$. However, in the censoring scenario, the corresponding false alarm or detection probability of each above mentioned four cases is shown in the last column of Table 2. In the censoring scenario, only those CUs send their spectrum sensing results to FC who finds the active status of PU, therefore in Case I and Case II, the detection/false alarm probability of CU at FC is same as in the non-censoring scenario. However, in Case III and Case IV, the detection/false alarm probability is zero because no sensing result is sent by CU to FC. From Table 2, the combined false-alarm\detection probabilities received at FC for the above considered four cases under the imperfect reporting channel in the non-censoring and censoring scenario, is given by Eqs. (15), (16), (22) and (23).

$$P_f^r = (1-P_f)P_e^r + P_f\left(1-P_e^r\right) \tag{15}$$

$$P_d^r = (1-P_d)P_e^r + P_d\left(1-P_e^r\right) \tag{16}$$

where P_f^r and P_d^r are the received false-alarm and detection probability, respectively at FC by each CU under the imperfect reporting channels as already described in system model of the proposed scheme. Afterwards FC applies the cooperative rules to take the global single decision about the status of licensed channel sensed by the multiple CUs. Since the individual CU has particular false-alarm (P_f) and detection probability (P_d), and the FC measures the collective (total) false-alarm (Q_f^r) and detection probability (Q_d^r) by taking into account P_f^r and P_d^r of each CUs, which is represented as follows:

Table 2 Detection and false alarm probability received at FC by CU for censoring and non-censoring approach

Case	True status of PU	Status of PU detected by CU	PU status received at FC (due to perfect/imperfect reporting)	Test Statistics at FC	Detection/false alarm probability of CU at FC (non-censoring)	Detection/false alarm probability of CU at FC (censoring)
I	Active	Active	Active	$P_d = (T > \lambda / H1)$	$P_d\left(1 - P_e^r\right)$	$P_d\left(1 - P_e^r\right)$
II	Idle	Active	Active	$P_f = (T > \lambda / H0)$	$P_f\left(1 - P_e^r\right)$	$P_f\left(1 - P_e^r\right)$
III	Active	Idle	Active	$P_d = (T > \lambda / H1)$	$(1 - P_d)P_e^r$	0
IV	Idle	Idle	Active	$P_f = (T > \lambda / H0)$	$(1 - P_f)P_e^r$	0

$$Q_f^r = \sum_{l=k}^{M} \binom{M}{l} \left(P_f^r\right)^l \left(1 - P_f^r\right)^{M-l} \tag{17}$$

$$Q_d^r = \sum_{l=k}^{M} \binom{M}{l} \left(P_d^r\right)^l \left(1 - P_d^r\right)^{M-l} \tag{18}$$

$$Q_e^r = Q_f^r + \left(1 - Q_d^r\right) \tag{19}$$

where M is the total number of CUs employed for cooperation and k are the number of CU reporting active status of PU. In the expressions (17) or (18), FC follows the Majority cooperative rule at $k = M/2$. In addition, the total spectrum sensing error probability (Q_e^r) provides the measure for sensing performance of CU.

Furthermore, the Eq. (7) represent the throughput (R) of CU in the non-cooperative spectrum sensing while considering two States. In State-1 (Case IV from Table 2), the licensed channel is assumed to be idle and no false-alarm is generated by CU while in State-2 (Case III from Table 2), the licensed channel is assumed to be occupied by PU but CU has missed its detection. Therefore, the total throughput (R) of CU in the non-cooperative spectrum sensing while considering two states is given as: $R = R_1 + R_2$, where $R_1 = P(H_0)\left(\frac{T-T_{sr}}{T}\right)\left(1 - P_f\right)log_2(1 + \gamma_s)$, is the throughput for 1st state and $R_2 = P(H_1)\left(\frac{T-T_{sr}}{T}\right)(1 - P_d)log_2\left(1 + \frac{\gamma_s}{1+\gamma}\right)$, is the throughput for 2nd state. Therefore, the total throughput (R) is given by the Eq. (7). Furthermore, the throughput of CU under the imperfect reporting channels in non-censoring scenario after cooperation (R_I) is computed with the help of Eqs. (7), (17), (18) and Table 2. After replacing the false-alarm probability (P_f) in Eq. (7) with the collective false-alarm probability while employing majority rule under the imperfect reporting channel (Q_f^r) and detection probability (P_d) in Eq. (7) with collective detection probability while employing majority rule under the imperfect reporting channel (Q_f^r), we achieved the throughput of CU under imperfect reporting channels presented in Eq. (20).

$$R_I = P(H_0)\left(\frac{T - T_{sr}}{T}\right)\left(1 - Q_f^r\right)log_2(1 + \gamma_s) + P(H_1)\left(\frac{T - T_{sr}}{T}\right)\left(1 - Q_d^r\right)log_2\left(1 + \frac{\gamma_s}{1 + \gamma}\right) \tag{20}$$

3.3.2 Perfect reporting channel

In the perfect reporting channels, it is assumed that whatever the decision is sent by CUs over the reporting channel, is received same at FC. Therefore, it is a special case of imperfect reporting channel where $P_e^r = 0$. Further, the total error probability and throughput of perfect reporting channel can be computed with the help of (19) and (20), respectively by placing $P_e^r = 0$ in Eq. (15) and (16).

3.3.3 Censoring with imperfect reporting

In the censoring approach, the spectrum sensing results of only those CUs are sent to FC through the reporting channel who detected the presence of PU on the channel (i.e. PU is active on the channel). Therefore, the number of cooperative users (M_c) who have sent the sensing results to FC with censoring approach are computed with the help of Table 2.

$$M_c = \lceil \left(M\left\{P(H_0)P_f + P(H_1)P_d\right\}\right) \rceil \tag{21}$$

where $\lceil . \rceil$ indicate the ceiling function. Further, in the censoring approach, the received false-alarm and detection probability at FC while considering the imperfect reporting channels is computed from Table 2 and given as:

$$P_f^{rc} = P_f\left(1 - P_e^r\right) \tag{22}$$

$$P_d^{rc} = P_d\left(1 - P_e^r\right) \tag{23}$$

Moreover, the total false-alarm (Q_f^{rc}), detection (Q_d^{rc}) and error probability (Q_e^{rc}) with censoring under the imperfect reporting channelis given as:

$$Q_f^{rc} = \sum_{l=k}^{M_c} \binom{M_c}{l} \left(P_f^{rc}\right)^l \left(1 - P_f^{rc}\right)^{M_c - l} \tag{24}$$

$$Q_d^{rc} = \sum_{l=k}^{M_c} \binom{M_c}{l} (P_d^{rc})^l (1 - P_d^{rc})^{M_c - l} \tag{25}$$

$$Q_e^{rc} = Q_f^{rc} + \left(1 - Q_d^{rc}\right) \tag{26}$$

Moreover, the throughput of CU after cooperation in the censoring scenario under the imperfect reporting channels (R_{IC}) is computed with the help of Eq. (7), (24), (25) and Table 2. Similarly, as described above, we replace the false-alarm probability (P_f) in Eq. (7) with collective false-alarm probability while employing majority rule under the imperfect reporting channel with censoring (Q_f^{rc}) and the detection probability (P_d) in Eq. (7) with collective detection probability while employing the majority rule under the imperfect reporting channel with the censoring (Q_{rc}^{d}), then the R_{IC} is given as:

$$\begin{aligned} R_{IC} = P(H_0)\left(\frac{T - T_{sr}}{T}\right)\left(1 - Q_f^{rc}\right) log_2(1 + \gamma_s) \\ + P(H_1)\left(\frac{T - T_{sr}}{T}\right)(1 - Q_d^{rc}) log_2\left(1 + \frac{\gamma_s}{1 + \gamma}\right) \end{aligned} \tag{27}$$

3.3.4 Censoring with perfect reporting

It is a special case of censoring with imperfect reporting channel where $P_e^r = 0$. Further, in this scenario, the total spectrum sensing error probability and throughput of CU is computed with the help of (26) and (27) respectively, by placing $P_e^r = 0$ in Eq. (22) and (23). Further, we have presented an Algorithm-1 to compute the throughput and total spectrum sensing error in the considered scenario.

3.4 Complexity analysis

In the proposed algorithm, the performance parameters such as total spectrum sensing error probability and throughput, while employing either CFAR or MEP threshold selection approach under the perfect/imperfect reporting channel and with/without the censoring scenario, which depends on the P_f and P_d. Further, these values (P_f and P_d) depends up on the Erfc function.

Algorithm-1: Total sensing error & Throughput Computation

Input: Reporting channel (RC) ={Perfect, Imperfect}, Threshold selection approach (TSA) = {CFAR, MEP}, Sensing channel (SC) ={AWGN, Rayleigh, Nakagami-m}, Event sequence (ES) = {Censoring, Non-censoring}, γ

Output: R and Q_e

```
Initialization: N, σn², Pf_fixed , γs, M , T, Ts , P(H0), γset = [- 20,-8], p ϵ (0,1]
if γ ϵ γset
    if RC = = Imperfect
        Pe^r ← p
    else
        Pe^r ← 0
    end if
    if TSA = = CFAR
        find the value of λf using (11)
        λ ← λf
        Compute Pf using (3)
        if SC = =AWGN
            compute Pd using (4)
        else if SC = = Rayleigh
            compute Pd^ray (overbar) using (9)
        else
            compute Pd^Naka (overbar) using (10)
        end if
    else
        if SC = = AWGN
            find out λe(AWGN) using (12)
            λ ← λe
            compute Pf and Pd using (3) and (4) respectively
        else if SC = = Rayleigh
            find out λe(Ray) using (13)
            λ ← λe
            compute Pf and Pd^ray (overbar) using (3) and (9) respectively
        else
            find out λe(Naka) using (14)
            λ ← λe
            compute Pf and Pd^Naka (overbar) using (3) and (10) respectively
        end if
    end if
    if ES = = Censoring
        compute Mc using (21)
        find Pf^rc from (22) and Pd^rc using (23)
        find Qe^rc from (26) and RIC using (27)
    else
        find Pf^r from (15) and Pd^r using (16)
        find Qe^r from (19) and RI using (20)
    end if
end if
```

However, the Erfc function and Q function are correlated and expressed as $Q(x) = \frac{1}{2} Erfc\left(\frac{x}{\sqrt{2}}\right)$. Since $Q(x)$ contains infinite terms and can be approximated as follows [61]:

$$Q(x) = \frac{e^{-\frac{x^2}{2}} \sum_{n=1}^{n_a} (-1)^{n+1} (A)^n}{B\sqrt{\pi}\left(\sqrt{2}\right)^{n+1} n!} \tag{28}$$

where A and B are constants. Further, the complexity of $Q(x)$ depends on the term n_a and given as $O(n_a)$. Moreover, the value of n_a is chosen as per the accuracy requirement.

Further, the steps involved in the proposed algorithm, which consist computation of false-alarm and detection probabilities under the perfect/imperfect reporting channel with/without the censoring involves error function, therefore the complexity of these steps is given as $O(n_a)$. In addition, the majority rule which is employed in the proposed algorithm steps consists of summation of $M/2$ terms, therefore its complexity is $O(M/2)$ and remaining steps have complexity $O(1)$. Hence, we have observed that the complexity in the algorithm for observing the performance parameters (total spectrum sensing error probability and throughput) remains same either we employ CFAR or MEP threshold selection approach and is given as Max($O(M/2)$, $O(n_a)$).

4 Results and discussion

In this section, we have illustrated numerically simulated results of the proposed CRN system model. In the proposed system model, we have considered CSS technique in which FC employed Majority cooperative rule. Further, the parameters employed for simulation of the results are presented in Table 3. We have chosen the value of $N = 25{,}000$, so that the threshold computed with CFAR and MEP approach from Eq. (11) to (14) for different channels remain positive.

The variation in normalized threshold $\left(\lambda^* = \lambda/N\right)$ [10] with SNR under different channels while employing CFAR and MEP are presented in Fig. 3. As illustrated in Fig. 3, the normalized threshold (λ^*) with CFAR approach remains constant for all considered channels however its value increases with increase in SNR for MEP approach. The constant nature of λ^* with CFAR approach for all the considered channels is due to the fact that P_f remains unaffected with variation in SNR as discussed in section 3.2. Further, we have illustrated the critical SNR values for different channels which are achieved at that particular SNR where the normalized threshold curve with MEP and CFAR intersect each other. Therefore, the critical SNR (γ_c) values achieved for the AWGN, Rayleigh, and Nakagami-m channels are given as: − 17.9 dB, − 17.7 dB,

Table 3 The parameters used for simulation in the proposed CRN

Parameter	Value	Parameter	Value
N	25,000	$P\ (H_0)$	0.8
γ_s	20 dB	$P\ (H_1)$	0.2
T_{sr}	2.5 m.sec	m	2
T	100 m.sec	M	10
P_{f_fixed}	0.1		

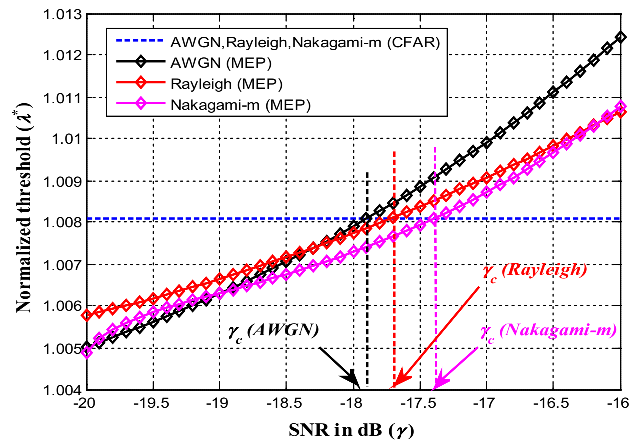

Fig. 3 The normalized threshold selection curve with SNR for AWGN, Rayleigh and Nakagami-m fading channels

− 17.4 dB, respectively. For the results which we have obtained in Fig. 4–7, initially the reporting channel condition is checked as per line 3 and corresponding reporting error probability will be assigned either from line 4 or 6 of the algorithm. Afterwards the threshold selection method is checked whether it is CFAR or MEP based and also the sensing channel is AWGN, Rayleigh or Nakagami, which provides the corresponding false-and detection-probability values. For the CFAR approach false-alarm is given by line 11 for all above mentioned sensing channels and detection probability will be computed from line 13, 15 and 17 for AWGN, Rayleigh and Nakagami channels, respectively. However, for MEP approach, the false-alarm and detection probabilities will be computed from line 23, 27 and 31 for AWGN, Rayleigh and Nakagami channels, respectively. Finally, the end result of the simulations are obtained from the algorithm for the censoring and non-censoring scenario with the help of line 37 and 40 which have employed the results of the equations used in the above lines of the algorithm.

Moreover, the variation in total spectrum sensing error probability (Q_e^r) of CU with different probability of error in the reporting channel (P_e^r) while employing CFAR and MEP threshold selection approaches at different SNR is illustrated in Fig. 4a–c for AWGN, Rayleigh and Nakagami-m channels, respectively. From Fig. 4, it is clear that Q_e^r increases with increase in P_e^r for a particular value of SNR, and it reduces with increase in SNR for a fixed P_e^r, under both the CFAR and MEP approaches. For AWGN channel at all SNR, Q_e^r is less in MEP approach as compare to that of CFAR approach for any particular value of P_e^r as illustrated in Fig. 4a.

However, in the Rayleigh and Nakagami-m fading channels, Q_e^r is less with MEP approach at low SNR ($\gamma \le \gamma_c$ e.g. ($\gamma = -20$ dB)), and at high SNR ($\gamma > \gamma_c$ e.g. ($\gamma = -17, -14$ dB)) Q_e^r is less with CFAR approach. This

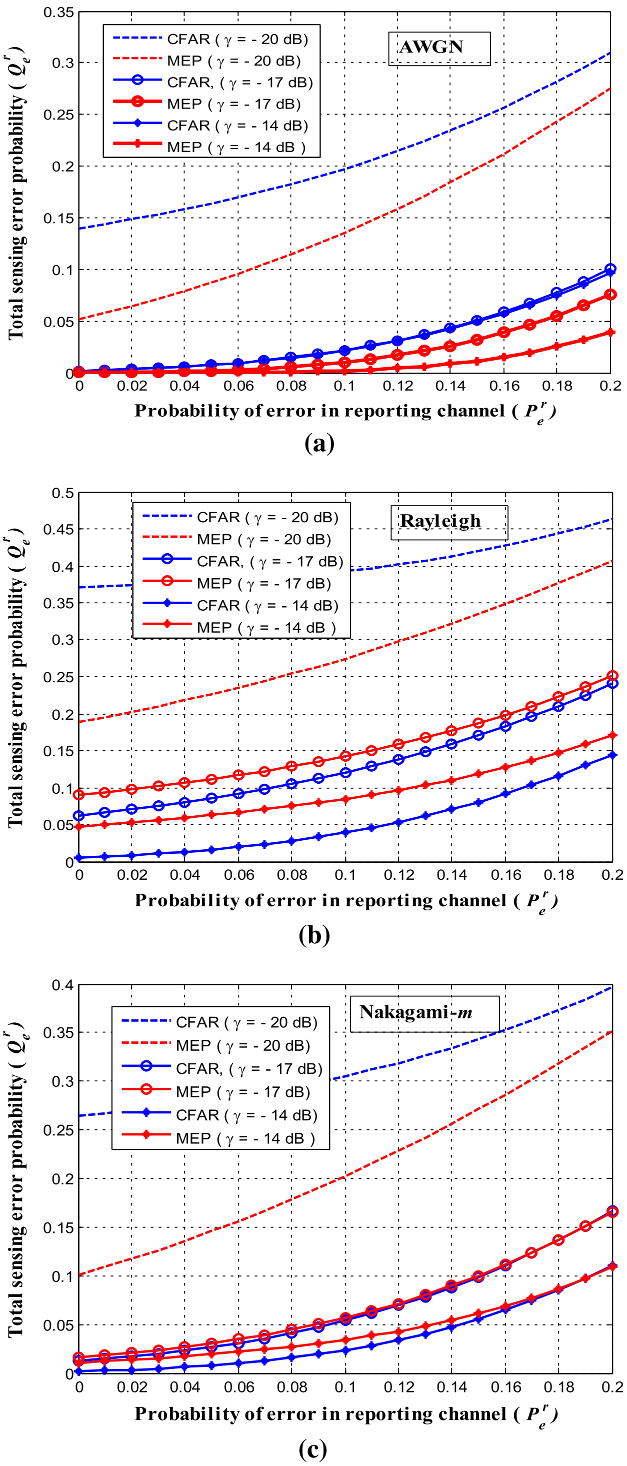

Fig. 4 Variations in the total spectrum sensing error probability with probability of error in reporting channel for different threshold selection approaches under **a** AWGN **b** Rayleigh, and **c** Nakagami-*m* fading channel

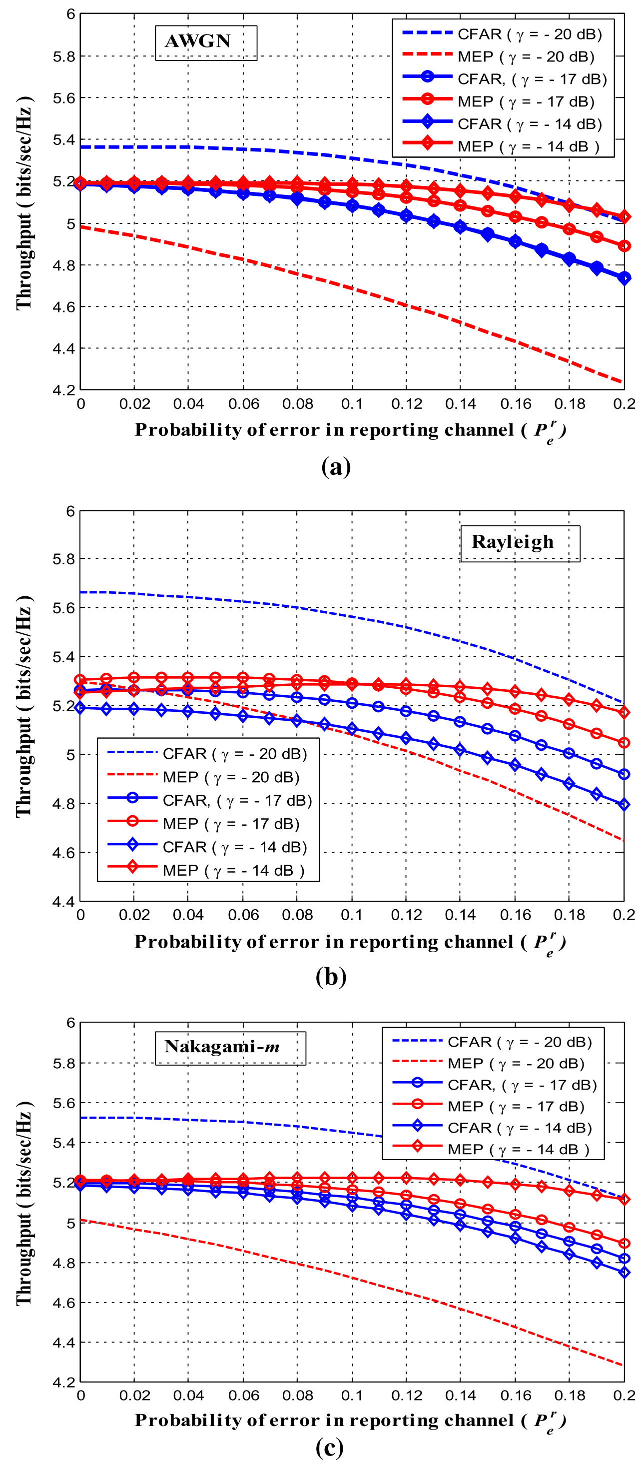

Fig. 5 Variations in the throughput with probability of error in reporting channel for different threshold selection approaches in CSS technique under **a** AWGN, **b** Rayleigh, and **c** Nakagami-*m* fading channel

is because for high SNR, the miss-detection probability is less with CFAR threshold as compare to that of MEP approach which leads to low Q_e^r with CFAR.

Further, the variation in throughput of CU with probability of error in reporting channel while employing both CFAR and MEP threshold selection approaches at different SNR for AWGN, Rayleigh and Nakagami-*m* channels are presented in Fig. 5a–c, respectively. From Fig. 5, it is clear that under all the considered channels for fixed SNR, the throughput of CU decreases with increases in P_e^r for both CFAR and MEP approaches. As already described in the proposed system model (presented in section 3), the total throughput is computed with the combination of throughputs of two cases (i.e. $R = R_1 + R_2$). With the increase in reporting error P_e^r, the throughput of Case-1 (R_1) decreases and it increases for Case-2 (R_2). However, the effect of decrease in throughput of R_1 is more dominating than that of increase of R_2, resulting overall throughput reduction with increase in P_e^r.

Further, under all the considered channels with increase in SNR, the throughput of CU decreases for CFAR and increases for MEP approach for a particular value of P_e^r. It is depicted that the increased SNR provide better total detection probability (Q_d^r).

Further, from Eq. (20), it is clear that the increase in Q_d^r results the reduction of throughput. Further, it is illustrated from Fig. 5a to Fig. 5c, under all the considered channels, the throughput is enhanced with CFAR approach for $\gamma \le \gamma_c$ ($\gamma = -20$ dB) while for $\gamma > \gamma_c$ ($\gamma = -17$ dB or -14 dB), it is maximized with MEP approach at a fixed P_e^r. The significantly high throughput with CFAR in comparison to MEP at $\gamma \le \gamma_c$ is resulting due to increased value of R_1 and R_2, whereas increased R_1 with MEP results higher throughput at $\gamma > \gamma_c$ as compare to that of CFAR. Moreover, in Fig. 6a–c, we have illustrated the variation in total spectrum sensing error probability with γ for AWGN, Rayleigh and Nakagami-m channels while employing CFAR/MEP threshold selection approaches in the non-censoring/censoring scenarios under perfect/imperfect reporting (PR/IR). For the non-censoring scenario, the effect of γ and reporting error on the total spectrum sensing error probability is already presented in Fig. 4. Moreover, for the fixed value of γ while employing either CFAR or MEP approach, the total spectrum sensing error probability is high in the censoring approach as compare to that of the non-censoring approach because at the FC, the false-alarm and miss-detection probability is significantly high in the censoring approach as compared to that of the non-censoring approach as is obvious from the Eq. (17)–(19), (21), and (24)–(26). Moreover, from Fig. 6, it is clear that there is switching between CFAR and MEP threshold approach to achieve less total spectrum sensing error probability with variation in γ in the censoring scenario under either PR or IR. This is because from Eq. (21)–(26), the total spectrum sensing error probability changes with number of cooperative CUs in the censoring (M_c) and Q_f and Q_m, where M_c, P_f and P_m varies with γ and threshold selection approaches.

However, Fig. 7a–c demonstrate the variation in throughput with SNR for AWGN, Rayleigh and Nakagami-

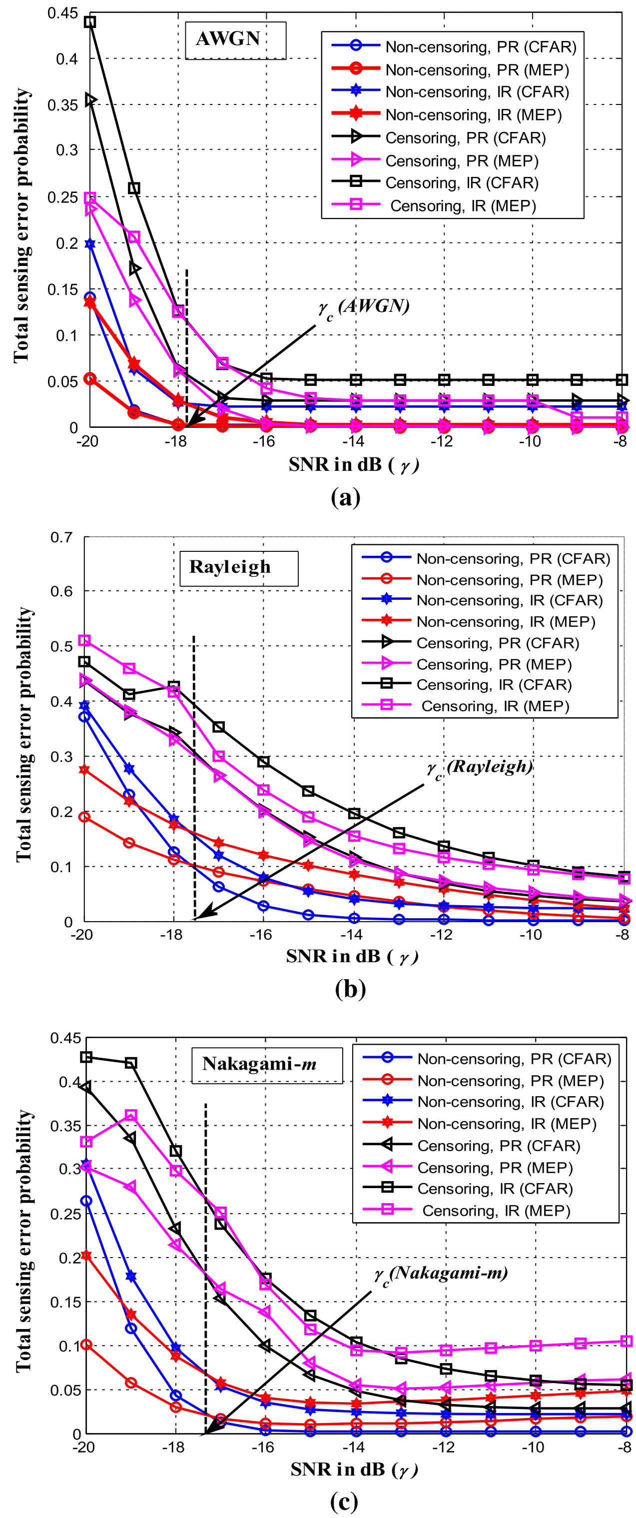

Fig. 6 Variations in the total spectrum sensing error probability with SNR for different threshold selection approaches at imperfect reporting error (P_e^r) of 0.1 for **a** AWGN, **b** Rayleigh and **c** Nakagami-m fading channel

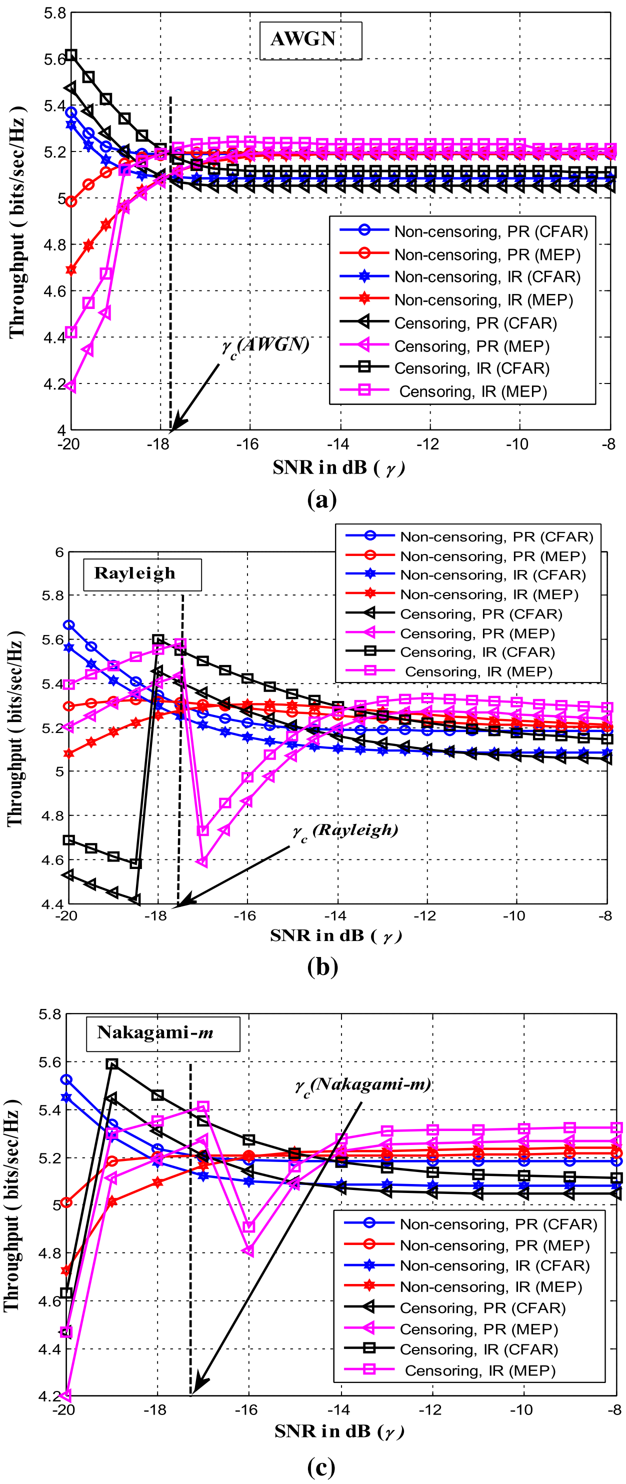

Fig. 7 Variations in the throughput with SNR for different threshold selection approaches under the perfect and imperfect reporting ($P_e^r = 0.1$) channel for **a** AWGN, **b** Rayleigh and **c** Nakagami-m fading

m channels, respectively while employing CFAR/MEP threshold selection approaches under the non-censoring/censoring and perfect/imperfect reporting (PR/IR).

It is clear from Fig. 7 that in the non-censoring scenario, initially the throughput decreases with increase in SNR for CFAR approach however it increases with MEP approach and then becomes nearly constant. From Eq. (20), it is clear that the throughput is high for low values of Q_f^r and Q_d^r. Therefore, with increase in γ, initially the throughput reduction with CFAR is due to Q_d^r increase and afterwards Q_d^r is remain constant causing constant throughput with increase of γ. Whereas in the MEP approach, initially with increase in γ, there is reduction in Q_f^r and increase in Q_d^r causing throughput increase due to more prominent effect of decreased Q_f^r and afterwards throughput is nearly constant due to minor change in Q_f^r and Q_d^r with change in γ. When further comparison is made between CFAR and MEP approaches, at $\gamma \le \gamma_c$ the throughput is higher with CFAR approach as compare to that of MEP approach since the false-alarm and miss-detection probabilities are less in CFAR. In addition, at $\gamma \le \gamma_c$ in the non-censoring scenario, the throughput is higher with perfect reporting (PR) channel as compare to that of the imperfect reporting (IR) channel while employing MEP threshold selection approach. The higher throughput with perfect reporting in MEP approach is achieved because the throughput of Case-1(i.e. R_1) is more and for Case-2 (i.e. R_2) is less in perfect reporting channel as compare to that of the imperfect reporting channel.

However, the total throughput is affected more by Case-1 throughput i.e. R_1, results an increased throughput with perfect reporting. Further, in the censoring scenario, the throughput of imperfect reporting is higher than that of the perfect reporting channel while employing any threshold selection approaches. It is because throughput of both cases i.e. R_1 and R_2 is high in the imperfect reporting as compare to that of the perfect reporting for censoring scenario. Further, the abrupt change in throughput is occurring at particular SNR in the censoring scenario under perfect/imperfect reporting channel in the Rayleigh and Nakagami fading environments while employing either CFAR or MEP threshold selection approach. The reason for this is that in the censoring scenario, less number of CU nodes ($M_c \le M$) are reporting to FC and M_c value depend up on the value P_f and P_d as it is revealed from Eq. (21). In Fig. 7b, at SNR = − 18 dB, while employing CFAR threshold selection approach, the number of CU nodes (M_c) reporting to FC are increase from 2 to 3 and result degradation of Q_f^{rc} and Q_d^{rc} which causes sudden increase in the value of throughput as we can also observe from Eq. (27) and Table 4. Further, while employing MEP threshold selection approach in Fig. 7b, at SNR = − 17 dB or in Fig. 7c at SNR = − 16 dB, the number of CU nodes (M_c) reporting to FC is decreases and then becomes constant, which resulted into increase in Q_f^{rc} and Q_d^{rc} and degradation

Table 4 The achieved value of various parameters while employing CFAR and MEP threshold selection approach in censoring and perfect reporting scenario for Rayleigh fading channel

Parameters SNR (in dB)	CFAR						MEP					
	P_f	P_d	M_c	Q_f^{rc}	Q_d^{rc}	Throughput	P_f	P_d	M_c	Q_f^{rc}	Q_d^{rc}	Throughput
− 20.0	0.1	0.502	2	0.19	0.752	4.52	0.18	0.599	3	0.08	0.647	5.201
− 19.5	0.1	0.534	2	0.19	0.783	4.48	0.16	0.610	3	0.07	0.663	5.256
− 19.0	0.1	0.586	2	0.19	0.811	4.45	0.14	0.620	3	0.05	0.677	5.309
− 18.5	0.1	0.596	2	0.19	0.837	4.41	0.12	0.630	3	0.04	0.690	5.358
− 18.0	0.1	0.626	**3**	**0.028**	**0.686**	**5.45**	0.11	0.638	3	0.03	0.702	5.402
− 17.5	0.1	0.655	3	0.028	0.725	5.40	0.09	0.647	3	0.02	0.714	5.438
− 17.0	0.1	0.683	3	0.028	0.762	5.35	**0.07**	**0.654**	**2**	**0.14**	**0.880**	**4.591**
− 16.5	0.1	0.709	3	0.028	0.795	5.31	0.06	0.662	2	0.11	0.886	4.736
− 16.0	0.1	0.734	3	0.028	0.825	5.27	0.04	0.670	2	0.09	0.891	4.866
− 15.5	0.1	0.757	3	0.028	0.852	5.23	0.03	0.677	2	0.06	0.896	4.979
− 15.0	0.1	0.779	3	0.028	0.875	5.20	0.02	0.685	2	0.04	0.901	5.073

in throughput which is also obvious from Eq. (24), (27) and Table 4. In Table 4, we have presented the achieved value of different parameters while employing CFAR and MEP threshold selection approach for Rayleigh fading channel under perfect reporting channel in the censoring scenario.

Moreover, from Fig. 7, it is clear that in the censoring scenario in order to yield high throughput for all γ, there is need of switching between CFAR and MEP threshold selection approaches for both the perfect and imperfect reporting channels. This is because with variation in γ under the censoring scenario, the CFAR and MEP threshold selection approaches varies with the number of CUs (M_c) and hence accordingly Q_f^{rc} and Q_d^{rc} values are updated at FC. Subsequent results present the validation of MEP cooperative approach, which we have considered earlier in this paper for analysis, as a suitable method for minimizing the total spectrum sensing error probability in AWGN channel. In this context, we have analyzed the outcomes of [22] and [24]. At γ = 10 dB and N = 10, the variation of total spectrum sensing error probability with threshold for different cooperative rules under the perfect reporting channel is shown by Li et al. [22] and Zhang et al. [24] for AWGN channel which has been presented in Fig. 8a. Further, the authors concluded that when the order of P_f and P_m are nearly same, the Majority rule provides least total spectrum sensing error probability as is illustrated in Fig. 8a. While considering the same number of samples (N = 10), we have represented the variation of total spectrum sensing error probability with threshold under the Majority rule at low SNR value of − 20 dB in Fig. 8b.

It is clear from Fig. 8b that the higher spectrum sensing error probability results due to a smaller number of samples (N = 10). In this context, to achieve satisfactory total

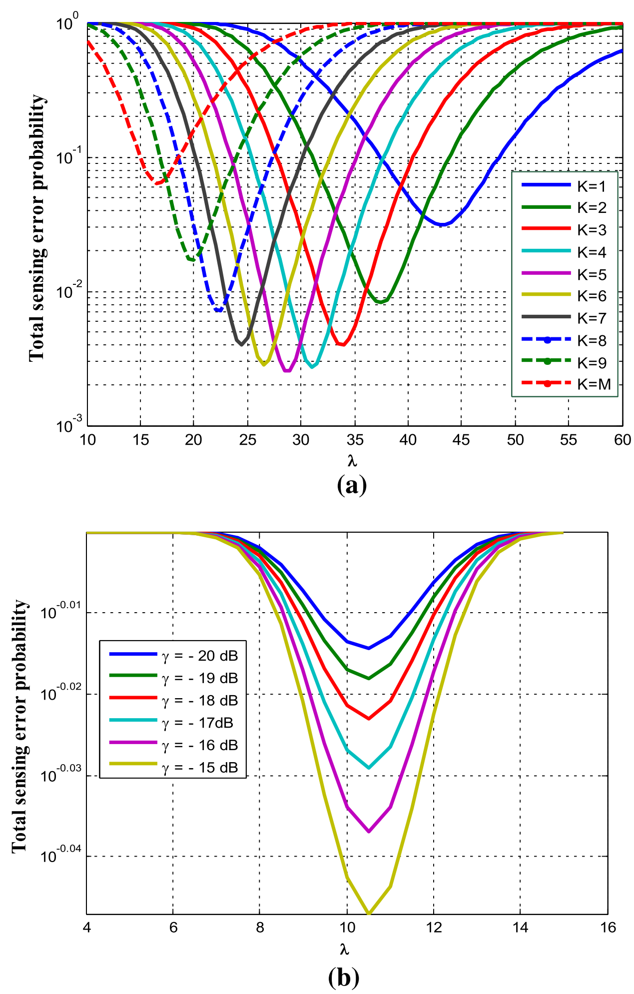

Fig. 8 Variations in the total error probability with threshold for Majority rule under the perfect reporting channel for AWGN channel at **a** high SNR (γ = 10 dB) [24] and **b** low SNR (γ = − 20 dB)

spectrum sensing error probability, we have increased the number of samples (N) to 25,000, the same value which we have considered for the proposed system. In addition, while analyzing the performance at $\gamma = -20$ dB, the mathematical expression employed in [24] is inappropriate for computing P_f and P_m of CU with high number of samples ($N = 25{,}000$). The reason for the same is that the total spectrum sensing error probability reaches 1 with high number of samples in the formula described in [24]. Therefore, we have employed different formula to compute P_f and P_m at $\gamma = -20$ dB for their approach [24] in Fig. 9, which was used by Atapattu et al. [10] under AWGN channel at low SNR (γ). Figure 9 illustrates the variation in total spectrum sensing error probability at low SNR with $N = 25{,}000$ under the non-cooperative/Majority cooperative rule with CFAR and MEP approach and compared total sensing error probability with their approach [24]. Moreover, it is clear from Fig. 9 that the MEP approach in cooperative spectrum sensing technique provides nearly same performance as the approach used by Li et al. [22] and Zhang et al. [24]. Hence the conclusion is made from Fig. 9 that the MEP approach for CSS is also suitable to compute the threshold and to reduce total spectrum sensing error probability in AWGN environment.

5 Conclusion

In this paper, we have illustrated the effect of CFAR and MEP threshold selection approaches on the total spectrum sensing error probability and throughput of CU under the perfect/imperfect reporting and censoring/non-censoring based CRN. From the results of non-censoring and imperfect reporting channel scenario, we have concluded that in the Rayleigh and Nakagami-m fading channels at low SNR ($\gamma \leq \gamma_c$), the MEP approach has provided better total spectrum spectrum sensing error probability performance however for high SNR ($\gamma > \gamma_c$), the CFAR approach perform better. Further, for the throughput enhancement, the reverse is true this means for $\gamma \leq \gamma_c$, the throughput is significantly high with CFAR approach however for $\gamma > \gamma_c$, its value is higher with MEP approach. Hence, there exist a trade-off between the spectrum sensing error probability and throughput with threshold selection approaches. The censoring scenario has although reduced the spectrum sensing overhead information but at the cost of increased total spectrum sensing error probability as compare to that of the non-censoring scenario due to the smaller number of users' reporting to the FC. In the censoring scenario, we have to change the CFAR and MEP threshold selection approaches according to γ, to enhance the throughput and decrease the spectrum sensing error probability as illustrated in the results. However, to compute the optimal number of CUs for different γ and appropriate threshold selection approach to enhance the throughput and minimize the total sensing error probability in the censoring and non-censoring scenario is a challenging task which will be reported in the future communication.

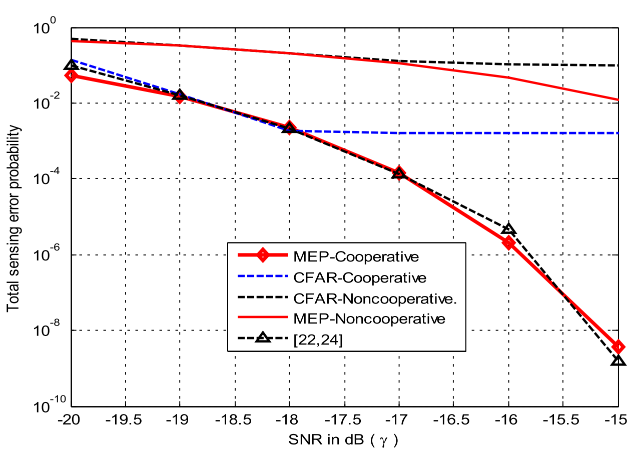

Fig. 9 Variations in the total spectrum sensing error probability with SNR for AWGN channel with different threshold selection approaches

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