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RESEARCH ARTICLE

# Hybrid Technique for Robust and Imperceptible Image Watermarking in DWT–DCT–SVD Domain

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**Abstract** In this paper an algorithm for digital watermarking based on discrete wavelet transforms (DWT), discrete cosine transforms (DCT), and singular value decomposition (SVD) has been proposed. In the embedding process, the host image is decomposed into first level DWTs. Low frequency band (LL) is transformed by DCT and SVD. The watermark image is also transformed by DCT and SVD. The S vector of watermark information is embedded in the S component of the host image. Watermarked image is generated by inverse SVD on modified S vector and original U, V vectors followed by inverse DCT and inverse DWT. Watermark is extracted using an extraction algorithm. The proposed method has been extensively tested against numerous known attacks and has been found to be giving superior performance for robustness and imperceptibility compared to existing methods suggested by other authors.

**Keywords** Image watermarking · Steganography · Discrete wavelet transforms · Discrete cosine transforms · Singular value decomposition

## Introduction

Information Technology has eased the duplication, manipulation and distribution of digital data in recent times which has resulted in the demand for safe ownership of digital images. A very crucial concern for the content owners and distributors is copyright protection and content authentication [1]. The solution to these problems is digital watermarking, which is a technique for inserting information into an image and later extracted or detected for variety of purposes including identification and authentication. According to the domain in which the watermark is inserted, these techniques are classified into two categories, i.e., spatial-domain and transform-domain methods [2, 3]. In spatial domain methods [4, 5], the data is embedded directly by manipulating the pixel values, bit stream or code values of the host signal. This is much simple, straightforward, less complex, but is not robust against attacks, whereas transform domain watermarking techniques are more robust. This is because in transform domain watermarking, when the image is transformed, the watermarks are irregularly distributed over the whole image which makes it difficult for the attacker to read or decode it. In this method the data is embedded by modulating the coefficients in transform domain like discrete Fourier transform, discrete cosine transform and discrete wavelet transform, Singular value decomposition etc. The computational cost of transform domain method is higher than spatial domain. The wavelet transforms provides excellent spatial-frequency localization properties [6, 7].

This paper introduces an algorithm of digital watermarking based on discrete wavelet transforms (DWT), discrete cosine transforms (DCT), and singular value decomposition (SVD). In the embedding process, the host image is decomposed into first level DWTs. Low frequency band (LL) is transformed by DCT and SVD. The watermark image is also transformed by DCT and SVD. The S vector of watermark information is embedded in the S component of the host image. Watermarked image is

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generated by inverse SVD on modified S vector and original U, V vectors followed by inverse DCT and inverse DWT. Watermark is extracted using an extraction algorithm. The proposed method has been extensively tested against numerous known attacks and has been found to be giving superior performance for robustness and imperceptibility compared to existing methods suggested by other authors. The DWT has excellent spatio-frequency localization properties that are suitable to identify areas in the cover image where a watermark can be imperceptibly embedded [8]. The DCT information of the watermark image contain the low frequency information, as long as these information do not lose or lose little then the watermarking image can be extracted well. However, the SVD has two main properties from the view point of image processing applications: (1) the singular values of an image have very good stability, that is, when a small perturbation is added to an image, its singular values do not change significantly; and (2) singular values represent intrinsic algebraic image properties [9]. Consequently, many image-watermarking techniques combining these three transform methods have been proposed [10–19]. For a detailed description on these combined approaches, interested readers may directly refer to them.

## Theoretical Background

The proposed work based on DWT, DCT, SVD requires certain theoretical considerations related to and their application in image processing. Hence, a brief description of these concepts is included in the given below sections.

Discrete Wavelet Transform (DWT)

Discrete wavelet transform is nothing but a system of filters that decomposes an image into a set of four non-overlapping multi-resolution sub bands [20] denoted as LL (approximation sub band), LH (horizontal sub-band), HL (vertical sub-band) and HH (diagonal sub-band), where LH, HL, and HH represent the finest scale wavelet coefficients and LL stands for the coarse-level coefficients. The process can be repeated to obtain multiple scale wavelet decomposition.

Discrete Cosine Transform (DCT)

The discrete cosine transform (DCT) works by separating image into parts of different frequencies, low, high and middle frequency coefficients [21], makes it much easier to embed the watermark information into middle frequency band that give additional resistance to the lossy compression techniques, while avoiding significant modification of the cover image. The DCT has a very good energy compaction property. For an input image, I, of size N × N the DCT coefficients for the transformed output image, D, are computed according to Eq. (1). I (x,y) is the intensity of the pixel in row x and column y of the image, and D (i, j) is the DCT coefficient in row i and column j of the DCT matrix.

$$D(i,j) = \frac{1}{\sqrt{2N}} C(i)C(j) \times \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} I(x,y) \cos\frac{(2x+1)i\pi}{2N} \times \cos\frac{(2y+1)i\pi}{2N} \quad (1)$$

$$C(i), C(j) = \frac{1}{\sqrt{N}} \quad \text{for } i,j = 0 \quad \text{and}$$

$$C(i), C(j) = \sqrt{\frac{2}{N}} \quad \text{for } i,j = 1,2,\ldots,N-1$$

Singular Value Decomposition (SVD)

The singular value decomposition of a rectangular matrix $R_m$ is a decomposition of the form

$$R_m = USV^T \quad (2)$$

where $R_m$ is an M × N matrix, U and V are orthonormal matrices, S is a diagonal matrix comprised of singular values of $R_m$. The singular values s1 ≥ s2 ≥ … ≥ sn ≥ 0 appear in descending order along the main diagonal of S. These singular value are obtained by taking the square root of the eigen values of $R_m R_m^T$ and $R_m^T R_m$. The singular values are unique; however the matrices U and V are not unique.

## Performance Measures

The performance of the watermarking algorithm can be evaluated on the basis of its robustness and imperceptibility. A larger peak signal to noise ratio (PSNR) indicate that the watermarked image more closely resembles the original image meaning that the watermark is more imperceptible. Generally, watermarked image with PSNR value greater than 28 is acceptable [22]. The PSNR is defined as

$$PSNR = 10 \log \frac{(255)^2}{MSE} \quad (3)$$

where the mean square error (MSE) is defined as

$$MSE = \frac{1}{X \times Y} \sum_{i=1}^{X} \sum_{j=1}^{Y} (I_{ij} - W_{ij})^2 \quad (4)$$

where $I_{ij}$ is a pixel of the original image of size X × Y and $W_{ij}$ is a pixel of the watermarked image of size X × Y. The robustness of the algorithm determined in term of correlation factor. The similarity and differences between original ‘watermark and extracted watermark is measured by the normalized correlation (NC). Its value is generally 0–1. Ideally it should be 1 but the value 0.7 is acceptable [15].

$$NC = \sum_{i=1}^{X}\sum_{j=1}^{Y}\left(W_{original\,ij} \times W_{recovered\,ij}\right) \Bigg/ \sum_{i=1}^{X}\sum_{j=1}^{Y} W^2_{original\,ij} \quad (5)$$

where $W_{original\,ij}$ is a pixel of the original watermark of size $X \times Y$ and $W_{recovered\,ij}$ is a pixel of the recovered watermark of size $X \times Y$.

## Proposed Algorithm

The algorithm proposed is a combined DWT, DCT and SVD based process. The proposed algorithm increases the robustness without much degradation of image quality against the signal processing attacks. The proposed algorithm has two parts, watermark embedding and watermark extraction method, as given below

Watermark Embedding Algorithm

**start:**

**STEP 1: Variable Declaration**

Barbara Image: cover image
Medical Image (Thorax): watermark image
C_w: read the cover image
W_w: read the watermark image
α: scale factor
DWT, DCT and SVD: Transform Domain Techniques
Wavelet filters: Haar
$LL_c, LH_c, HL_c, and\, HH_c$: First level DWT coefficients for cover image
D: DCT coefficients of watermark image
$D_c^1$: DCT coefficients matrix for $HH_c$
$U_c\, and\, V_c^T$: orthonormal matrices for $D_c^1$
$S_c$: diagonal matrix for $D_c^1$
$U_w\, and\, V_w^T$: orthonormal matrices for D
$S_w$: diagonal matrix for D
$W_w^k$: modified value of $S_c$
$U_{ww}\, and\, V_{ww}^T$: orthonormal matrices for $W_w^k$
$S_{ww}$: diagonal matrix for $W_w^k$
$W_{modi}$: Modified DWT coefficient
$W_{idct}$: InverseDCT coefficients matrix
$W_d$: Watermarked Image

**STEP 2: Read the Images**

C_w ← MRI.bmp (Cover image of size 512*512)
W_w ← Thorax.bmp (Watermark image of size 256*256)

**STEP 3: Perform DWT on Cover and DCT on Watermark image**

Apply first level DWT on cover image
$[LL_c, LH_c, HL_c, HH_c]$ ← DWT(C_w, wavelet filter);
D = DCT(W_w);

**STEP 4: Choice of subands in Cover and obtain the DCT coefficients for the same**

//Choose subband $HH_c$ from cover image
**if** (DCT on $LL_c$)**then**
$D_c^1 \leftarrow$ DCT($LL_c$);
**endif;**

**STEP 5: Compute the singular values of DCT coefficients for Cover and Watermark image**

**if** (SVD on $D_c^1$) **then**
$U_c S_c V_c^T \leftarrow SVD(D_c^1)$
**endif;**
**if** (SVD on D)**then**
$U_w S_w V_w^T \leftarrow SVD(D)$
**endif;**

**STEP 6: Watermark Embedding**

**for** α ← 0.1:0.9
$S_c + \alpha S_w = W_w^k$;
***end;***

**STEP 7: Compute the singular values for $W_w^k$ and obtain the modified DWT coefficients**

**if** (SVD on $W_w^k$)**then**
$U_{ww} S_{ww} V_{ww}^T \leftarrow SVD(W_w^k)$
**endif;**
//modified DWT coefficient
$W_{modi} \leftarrow U_c S_{ww} V_c^T$

**Step 8: Obtain the Watermarked Image.**

$W_{idct} \leftarrow inverse(W_{modi})$;
//Apply InverseDWT to $LL_c, LH_c, HL_c and HH_c$ with modified coefficient
$W_d$ ← InverseDWT($W_{idct}, LH_c, HL_c, HH_c$wavelet filter);
**end:**

Watermark Extraction Algorithm

**start:**

**STEP 1: Variable Declaration**

α : scale factor
$LL_c, LH_c, HL_c, HH_c$: subbands for watermarked image
$D_w^*$: DCT coefficients matrix for $HH_c$
$U_w^*\, and\, V_w^{*T}$: orthonormal matrices for $D_w^*$
$S_w^*$: diagonal matrix for $D_w^*$
$S^{*k}$: modified values
$U_w^{*1} and V_w^{*1T}$: orthonormal matrices for $S^{*k}$

$S_w^{*1}$: diagonal matrix for $S^{*k}$
$I_{cc}^*$: modified DWT coefficients
$W_{EW}$: Extracted watermark image

**STEP 2: Perform DWT on Watermarked image (possibly distorted)**
$[LL_c, LH_c, HL_c, HH_c] \leftarrow$ DWT ($W_d$, wavelet filter);

**STEP 3: obtain the DCT coefficients for $HH_c$**
**if** (DCT on $LL_c$)**then**
$D_w^* \leftarrow$ DCT ($LL_c$);
**endif;**

**STEP 4: Compute the singular values for $D_w^*$**
$U_w^* S_w^* V_w^{*T} \leftarrow SVD(D_w^*)$
**end;**

**STEP 5: Perform the operation and then apply SVD**
**for** $\alpha = 0.1{:}0.9$
$S^{*k} = \frac{S_w^* - S_c}{\alpha}$
**end;**
$U_w^{*1} S_w^{*1} V_w^{*1T} \leftarrow SVD(S^{*k})$

**STEP 6: Compute modified DWT coefficients**
$I_{cc}^* \leftarrow U_w S_w^{*1} V_w^T$

**STEP 7: Extract the watermark image.**
$W_{EW} \leftarrow$ InverseDCT ($I_{cc}^*$);
**end:**

## Experimental Results

We describe the performance of the combined DWT–DCT–SVD watermarking algorithm. The gray-level images "Barbara Image" of size 512 × 512 as cover image and "Thorax" of size 256 × 256 are used as watermark image as shown in Fig. 1a, b respectively. We implemented our algorithm in MATLAB. Also, evaluate the quality of watermarked image (as shown in Fig. 1c) by the parameter PSNR and robustness of the proposed algorithm by the NC. We compare the performance of the proposed algorithm with Singh and Tayal [10], Khan et al. [14] and Harish et al. [15] against different kind of attacks. It can be seen that the results from Tables 1, 2, 3, 4 and 5, proposed method gives better result than the other reported methods [10, 14, 15]. In the experiments, we use the scale factor ($\alpha$) as 0.5–0.9 and the value of NC are illustrated in Table 1, 2, 3, 4 to 5. Table 1, 2, 3, 4 and 5 shows behavior of NC. Without any noise attack the PSNR obtained is 55.01 dB and 38.08 at scale factor ($\alpha$) = 0.01 and 0.1 respectively, NC value is 1 at $\alpha = 0.1$. We found that the larger the scale factor, stronger the robustness and smaller the scale factor, better the image quality. However, the graphical representation of the performance by the proposed method as shown in Figs. 2, 3 and 4 reveals better performance compared to other reported methods [10, 14, 15]. In Table 1, NC value have been evaluated for LL band at scale factor ($\alpha$) = 0.5, 0.7 and 0.9. Also, the NC value for HH band at scale factor ($\alpha$) = 0.5 with the proposed method have been compared to that reported by Singh and Tayal [10]. However, the better results have been obtained with LL band at scale factor ($\alpha$) = 0.9. The NC value of 0.9999 has been obtained against Histogram equalization

In Table 2, NC values have been evaluated for salt and pepper noise at noise densities from 0.01 to 0.08 and the obtained result demonstrate better performance in LL band. However, the proposed method with HH band also provides better performance than that of the results reported by Singh

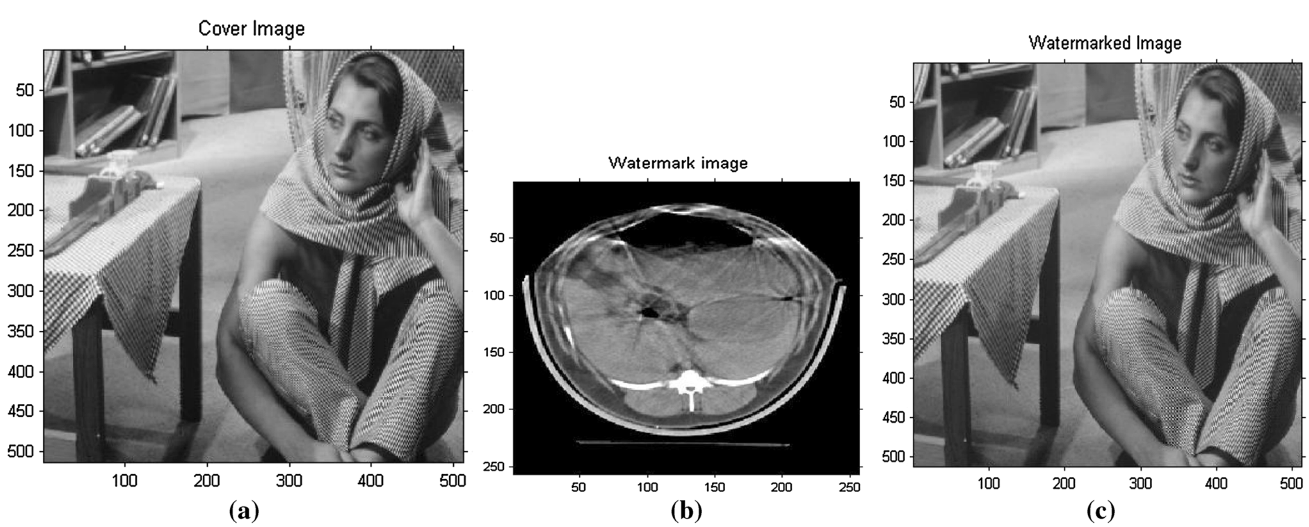


**Fig. 1** **a** Cover image. **b** Watermark image. **c** Watermarked image

**Table 1** Experimental results showing NC values against different attacks

| Various attack | NC values (proposed method using Haar wavelet) LL subband α = 0.9 | NC values (proposed method using Haar wavelet) LL subband α = 0.7 | NC values (proposed method using Haar wavelet) LL subband α = 0.5 | NC values (proposed method using Haar wavelet) HH subband α = 0.5 | NC values (Singh and Tayal [10] method using Haar Wavelet) HH subband α = 0.5 |
|---|---|---|---|---|---|
| Sobel horizontal edge emphasizing filter | 0.9996 | 0.9994 | 0.9994 | 0.9992 | 0.9992 |
| Linear motion | 0.9979 | 0.9985 | 0.9996 | 0.9927 | 0.9912 |
| Disk(circular averaging filter) | 0.9979 | 0.9989 | 0.9996 | 0.9887 | 0.9887 |
| Average filter[3 3], | 0.9979, | 0.9934, | 0.9926, | 0.9914, | 0.9909, |
| Average filter[5 5], | 0.9909, | 0.9900, | 0.9898, | 0.9897, | 0.9896, |
| Average filter[7 7] | 0.9896 | 0.9894 | 0.9893 | 0.9891 | 0.9891 |
| Poisson noise | 0.9981 | 0.9974 | 0.9973 | 0.9811 | 0.9754 |
| Contrast adjustment | 0.9915 | 0.9886 | 0.9812 | 0.9558 | 0.9338 |
| Histogram equalization | 0.9999 | 0.9998 | 0.9996 | 0.9994 | 0.9991 |

**Table 2** Experimental results showing NC values against salt and pepper noise

| Noise density | NC values (proposed method using Haar wavelet) LL subband α = 0.9 | NC values (proposed method using Haar wavelet) LL subband α = 0.7 | NC values (proposed method using Haar wavelet) LL subband α = 0.5 | NC values (proposed method using Haar wavelet) HH subband α = 0.5 | NC values (Singh and Tayal [10] method using Haar wavelet) HH subband α = 0.5 |
|---|---|---|---|---|---|
| 0.01 | 0.9962 | 0.9961 | 0.9948 | 0.9636 | 0.9636 |
| 0.02 | 0.9917 | 0.9910 | 0.9855 | 0.9295 | 0.9289 |
| 0.03 | 0.9869 | 0.9823 | 0.9719 | 0.9043 | 0.8971 |
| 0.04 | 0.9799 | 0.9734 | 0.9565 | 0.8804 | 0.8685 |
| 0.05 | 0.9729 | 0.9635 | 0.9439 | 0.8612 | 0.8420 |
| 0.06 | 0.9641 | 0.9508 | 0.9226 | 0.8430 | 0.8209 |
| 0.07 | 0.9551 | 0.9389 | 0.9059 | 0.8271 | 0.7980 |
| 0.08 | 0.9468 | 0.9277 | 0.8892 | 0.8171 | 0.7809 |

**Table 3** Experimental results showing NC values against Gaussian noise

| Variance | NC values (proposed method using Haar wavelet) LL subband α = 0.9 | NC values (proposed method using Haar wavelet) LL subband α = 0.7 | NC values (proposed method using Haar wavelet) LL subband α = 0.5 | NC values (proposed method using Haar wavelet) HH subband α = 0.5 | NC values (Singh and Tayal [10] method using Haar wavelet) HH subband α = 0.5 |
|---|---|---|---|---|---|
| 0.01 | 0.9872 | 0.9841 | 0.9752 | 0.8939 | 0..8893 |
| 0.02 | 0.9841 | 0.9818 | 0.9727 | 0.8937 | 0.8116 |
| 0.03 | 0.9803 | 0.9770 | 0.9689 | 0.8930 | 0.7563 |
| 0.04 | 0.9760 | 0.9730 | 0.9617 | 0.8925 | 0.7139 |
| 0.05 | 0.9709 | 0.9661 | 0.9531 | 0.8923 | 0.6814 |
| 0.06 | 0.9649 | 0.9593 | 0.9452 | 0.8919 | 0.6565 |
| 0.07 | 0.9587 | 0.9513 | 0.9336 | 0.8914 | 0.6323 |
| 0.08 | 0.9523 | 0.9441 | 0.9282 | 0.8894 | 0.6142 |

and Tayal [10]. Even though at high noise density of 0.08, the HH band performance with proposed method give NC of 0.8171 against 0.7809 obtained by Singh and Tayal [10] at scale factor (α) = 0.5. However, in proposed method same has increased to 0.8892 with LL band at scale factor (α) = 0.5. Further increase in the scale factor (α) to 0.9, increases NC to 0.9468. So that, under the Salt and pepper noise the NC is decreased with the increment in the noise

Table 4 Experimental results showing NC values against Speckle noise

| Variance | NC values (proposed method using Haar wavelet) LL subband α = 0.9 | NC values (proposed method using Haar wavelet) LL Subband α = 0.7 | NC values (proposed method using Haar wavelet) LL subband α = 0.5 | NC values (proposed method using Haar wavelet) HH subband α = 0.5 | NC values (Singh and Tayal [10] method using Haar wavelet) HH subband α = 0.5 |
|---|---|---|---|---|---|
| 0.01 | 0.9981 | 0.9948 | 0.9944 | 0.9662 | 0.9662 |
| 0.02 | 0.9906 | 0.9896 | 0.9848 | 0.9660 | 0.9659 |
| 0.03 | 0.9849 | 0.9818 | 0.9724 | 0.9116 | 0.9057 |
| 0.04 | 0.9786 | 0.9727 | 0.9570 | 0.8903 | 0.8808 |
| 0.05 | 0.9718 | 0.9626 | 0.9405 | 0.8725 | 0.8569 |
| 0.06 | 0.9631 | 0.9496 | 0.9262 | 0.8557 | 0.8370 |
| 0.07 | 0.9540 | 0.9409 | 0.9078 | 0.8441 | 0.8191 |
| 0.08 | 0.9458 | 0.9275 | 0.8918 | 0.8323 | 0.8013 |

Table 5 Experimental results showing NC values against Khan et al. [14] and Harish et al. [15]

| Various attack | NC values (proposed method) | Max NC values (Khan et al. [14]) | NC values (Harish et al. [15]) |
|---|---|---|---|
| Gaussion noise | 0.9872 | 0.9762 | 0.969 |
| Histogram equalization | 0.9999 | 0.9979 | 0.918 |
| Salt and pepper noise | 0.9962 | 0.9894 | 0.894 |
| Poisson noise | 0.9981 | 0.9981 | 0.939 |
| Speckle noise | 0.9981 | 0.9981 | 0.989 |

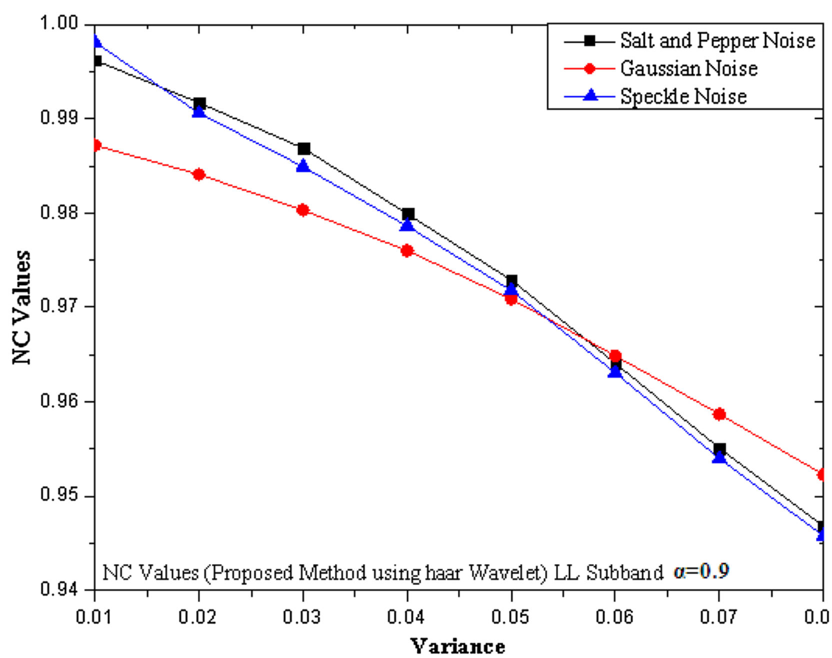


Fig. 2 Performance of the proposed method against salt and pepper noise, Gaussian noise and Speckle noise at α = 0.9

density. In Table 3, NC values have been evaluated for Gaussian noise at noise densities from 0.01 to 0.08. The result show better performance with LL band. However, the proposed method with HH band also gives better performance than the results of Singh and Tayal [10] method. Even at high noise density of 0.08 the HH band performance with proposed method give NC of 0.8894 against 0.6142 obtained by Singh and Tayal [10] at scale factor (α) = 0.5. However, in the proposed method same has increased to 0.9282 with LL band at scale factor (α) = 0.5. Further increase in scale factor (α) to 0.9 increases NC to 0.9523. So that, under the Gaussian noise attack the NC is decreased with the increment

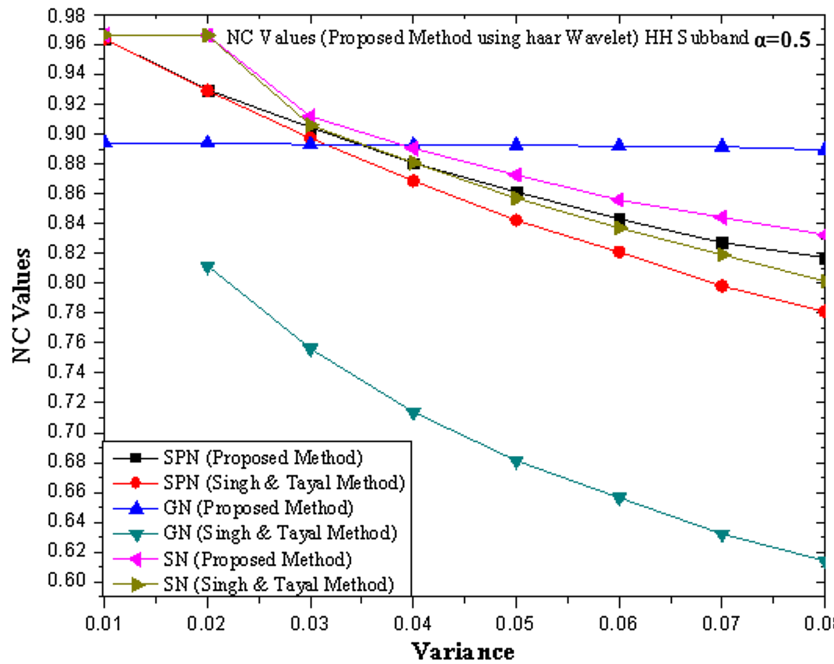


**Fig. 3** Comparision of proposed method with Singh and Tayal method [10] against salt and pepper noise (SPN), Gaussian noise (GN) and Speckle noise (SN) at $\alpha = 0.5$

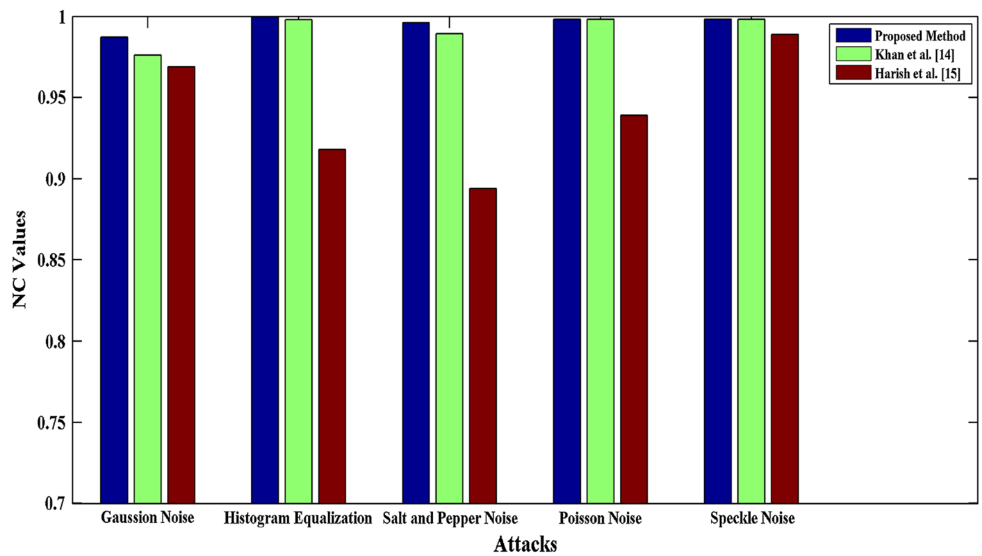


**Fig. 4** Comparison of proposed method with Khan et al. [14] and Harish et al. [15] against various attack

in variance and mean zero. In Table 4, NC values have been evaluated for Speckle Noise at noise densities from 0.01 to 0.08. The result shows the better performance with LL band. However, the proposed method with HH band also gives better performance than the results of Singh and Tayal [10]. Even though at high noise density of 0.08 the HH band performance with proposed method give NC of 0.8323 against 0.8013 obtained by Singh and Tayal [10] at scale factor ($\alpha$) = 0.5. However, in proposed method same has increased to 0.8918 with LL band at scale factor ($\alpha$) = 0.5. Further increase in scale factor ($\alpha$) to 0.9 increases NC to 0.9458. So that, under the speckle noise attack the NC is decreased with the increment in variance. In Table 5, the maximum values of NC have been compared with those obtained by Khan et al. [14] and Harish et al. [15]. The maximum NC value with proposed method has been obtained as 0.9999 for Histogram equalization noise against 0.9979 and 0.9180 obtained by Khan et al. [14] and Harish et al. [15], respectively. The proposed method is better to that of the existing method [14, 15] even in maximum NC.

## Conclusions

In this paper, a hybrid image-watermarking technique based on DWT, DCT and SVD has been presented, where the watermark is embedded on the singular values of the cover image DWT sub bands. The main properties of this work can be identified as follows: (1) Proposed algorithm combines the advantages and remove the disadvantages of these three most popular transforms. (2) Our approach needs less SVD computation than other methods. (3) The DCT information of the watermark image contain the low frequency information, as long as these information do not lose or lose little then the watermarking image can be extracted well. (4) Also, the first level decomposition has some advantages such as the watermark embedding is maximized, and the extracted watermarks are more textured with better visual quality. We would like to further improve the performance, which will be reported in future communication.

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