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Space–Time/Space–Frequency/Space–Time–Frequency Block Coded MIMO-OFDM System with Equalizers in Quasi Static Mobile Radio Channels Using Higher Tap Order

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Abstract In 4G broadband wireless communications, multiple transmit and receive antennas are used to form multiple input multiple output (MIMO) channels to increase the capacity (by a factor of the minimum number of transmit and receive antennas) and data rate. In this paper, the combination of MIMO technology and orthogonal frequency division multiplexing (OFDM) systems is analyzed for wideband transmission which mitigates the intersymbol interference and hence enhances system capacity. In MIMO-OFDM systems, the coding is done over space, time, and frequency domains to provide reliable and robust transmission in harsh wireless environment. Also, the performance of space time frequency (STF) coded MIMO-OFDM is analyzed with space time and space frequency coding as special cases. The maximum achievable diversity of STF coded MIMO-OFDM is analyzed and bit error rate performance improvement is verified by simulation results. Simulations are carried out in harsh wireless environment, whose effect is mitigated by using higher tap order channels. The complexity is resolved by employing sphere decoder at the receiver.

Keywords MIMO-OFDM · STF coding · Diversity analysis · DFE · ML equalizer · Sphere decoder

1 Introduction

Next generation wireless systems require high system capacity, high voice quality and high data rate compared to current cellular mobile radio standards. In addition, they should operate reliably in various practical environments like macro, micro and picocellular, urban, sub-urban and rural, indoor and outdoor. In previous systems, the use of higher bandwidth

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achieved these goals. Due to its scarcity, the increase in bandwidth is an impractical method. In the last decade, research is done towards efficient coding and modulation schemes that improve the quality and bandwidth efficiency of wireless systems. MIMO [1] antenna systems is a powerful tool to achieve these requirements in current wireless systems. The key advantages of employing multiple antennas are, (a) the improvement in reliability performance through diversity, and (b) increase in data rate through spatial multiplexing. In wireless environment the signal propagates through different paths referred as multipath. Diversity describes the available degrees of freedom present in the MIMO channel. To obtain diversity, the signal is transmitted through multiple independent fading paths in time, frequency or space domains and combined constructively at the receiver. Further, in MIMO systems, same information can be transmitted from multiple antennas and received at multiple antennas simultaneously.

In order to take advantage of the spatial and temporal diversity, a number of ST coding [2–7] and modulation methods have been proposed. In ST coding, the maximum achievable diversity is equal to the product of number of transmit and receive antennas. It is proved that ST coding has simple implementation that provides minimal decoding complexity, but it does not provide multipath diversity or high rate. To exploit frequency diversity, MIMO system is combined with OFDM [8]. In OFDM, a high data rate stream is split into a number of low rate streams, and each stream is modulated with orthogonal subcarrier. The number of subcarriers is decided in such a way that each subcarrier has bandwidth much less than the coherence bandwidth of channel. Therefore, the intersymbol interference (ISI) on each subcarrier is very small. ISI can be further mitigated by adding cyclic prefix (CP) to each OFDM symbol.

In MIMO-OFDM systems [9], it is desirable to have multipath propagation along with space diversity gain. SF codes have been proposed to exploit the spatial and frequency diversity present in frequency selective MIMO channels. SF coding [10,11] distributes the channel symbols over different transmit antennas and OFDM tones within one OFDM block. If longer decoding delay and higher decoding complexity are allowed, one may consider coding over several OFDM block periods, resulting in STF codes [12]. It is proved that a MIMO-OFDM system can achieve a maximum diversity gain equal to the product of transmitting antennas, receiving antennas and multiple paths present in the frequency selective channel if the channel correlation matrix is full rank [10]. In [13,14] it is proved that STF codes can achieve a diversity order equal to product of transmitting antennas, receiving antennas, independent channel taps and the rank of temporal correlation matrix of channel.

ST coded OFDM was first introduced in [2]. It uses space time trellis codes over frequency tones. The resulting codes achieve spatial diversity instead of full diversity. In [14] space frequency time method over MIMO-OFDM channels is introduced, but with more than two transmitting antennas it provides a rate of only 3/4. To reduce the complexity of code design, a grouping method with precoding and bit-interleaving is proposed in [15,16]. In [17], the repetition mapping technique to transform existing ST codes to full diversity SF codes is proposed. This method provides a trade off in diversity and symbol rate. A rate 1 SF codes are proposed in [10] where the target diversity was obtained but decoding complexity increases exponentially with diversity.

This paper gives a general performance analysis for MIMO-OFDM systems with ST, SF and STF coding schemes based on Alamouti coding. MIMO-OFDM system is analyzed in quasi static Rayleigh frequency flat and selective channel with higher tap order. We simulated the space–time (ST)/ space–frequency (SF)/ space–time–frequency (STF) codes in severely faded channel environment. In such channel environments, channel coefficients are close to signal amplitudes and the signal is severely faded. It is very difficult to decode signal in such a channel scenario. In this paper, the decoding is done by increasing the diversity order without employing precoding as done in existing literature. The results plotted in the results section verify this for higher SNR region. We determine the maximum achievable diversity with Alamouti based ST, SF and STF based coding with repetition techniques but in more severely faded environment. It is shown that if channel changes independently from one block to another, Then STF coding will show a significant improvement compared to the SF coding approach. Also we resolved the increased decoder complexity as in [14] by employing SD on receiver side. This work can be extended to more practical high altitude platform (HAP) [18] MIMO channels along with more number of users. In such cases, we have to employ multi-user detection algorithms [19,20].

The major advantage of using MIMO technology in any wireless system includes, increase in its array gain, diversity gain, multiplexing gain and also reduction in co-channel interference. Increase in array gain and diversity gain increases the coverage distance and improves quality of service (QoS) of a system. Multiple antenna system also increases data rate and system capacity (by a factor of the minimum number of transmit and receive antennas). In MIMO systems, interference is quite less as compared to other techniques and it further decreases after combining MIMO with OFDM. There are some standards based upon MIMO technology like IEEE 802.11n, 3GPP LTE and IEEE 802.16e (Mobile Wi-MAX) etc. All upcoming 4G systems will employ MIMO technology. MIMO applications can be extended to multi-users also like cross-layer MIMO, cognitive MIMO, cooperative MIMO etc. Its target applications include large files backup, HD streams, online interactive gaming, home entertainment, in Ad-Hoc networks (to get high capacity links) and in RFID technology (to increase read range and throughput).

The rest of the paper is organized as follows. In Sect. 2, a general MIMO-OFDM structure is introduced and ST, SF and STF code design criteria are reviewed in Sect. 3. In Sect. 4, performance design criteria and formulation for diversity analysis are derived for STF coding with ST and SF as its special cases. In Sect. 5, various equalizers and decoders are presented. Simulation results of ST, SF and STF coding with equalizers are given in Sect. 6 and paper is concluded in Sect. 7.

2 MIMO-OFDM Transceiver Model

Before introducing the system model, the list of notations used in this paper are tabulated in Table 1.

A general systematic transceiver model of $M_T \times M_R$ MIMO-OFDM system is shown in Fig. 1, where M_T is number of transmit antenna and M_R is number of receive antennas. Initially, the incoming bit stream is mapped into data symbols via modulation technique like BPSK. Then a block of data symbols is encoded into a codeword matrix C of size $N_c T \times M_{T,}$ which will then be sent through M_T transmit antennas in T OFDM blocks i.e. $c_i^1, c_i^2, \ldots c_i^T$. Each OFDM block consists of N_C subcarriers which will be transmitted from ith transmitting antenna in OFDM blocks 1, 2,…,T. The codeword matrix C [21] can be expressed as

$$C_{ST} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,M_T} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,M_T} \\ \cdots & \cdots & \cdots & \cdots \\ c_{T,1} & c_{T,2} & \cdots & c_{T,M_T} \end{bmatrix} \in C^{N_C T \times M_T} \quad (1)$$

Table 1 Notations and symbols

Notation	Meaning	Notation	Meaning
$(\cdot)^*$	Complex conjugation	L	Number of channel taps
$(\cdot)^T$	Transposition	W	Matrix of $w = e^{-j2\pi\Delta f}$ of size $N_C \times L$
$(\,)^H$	Hermitian transposition.	$A_{i,j}$	Matrix of channel gain coefficients α between transmitting antenna i and receive antenna j
$\|\| \cdot \|\|_F$	Frobenius norm	$\Re_T$	Time correlation matrix
M_T, M_R	Number of transmitting and receiving antennas	$\Re_F$	Frequency correlation matrix
N_C	Number of subcarriers	ν	Rank of $\Delta o \ \mathcal{R}$
A o B	Hadamard product of matrices A and B	$\tilde{w}$	Weight vector
$A \otimes B$	Kroneckor product of A and B	N_r	Number of iterations
C_{ST}, C_{SF}, C_{STF}	Codeword matrix in space–time, space–frequency and space–timer-frequency domain	μ	Step size of equalization algorithm
$H^k_{i,j}$	Channel between transmitting antenna i and receiving antenna j during kth block	d	Desired response
y^k_j	Received signal at jthreceive antenna for nth subcarrier during kth block	$H\downarrow$	Pseudo inverse of channel matrix H
S0, S1	Simultaneously transmitted symbols from antenna 1 and 2	r_S	Radius of search hyper-sphere
$\hat{S}0, \hat{S}1$	Receiver estimates of S0 and S1	Λ	Complex lattice
Z	Additive complex Gaussian noise vector	U	Upper triangular matrix
ρ	Average signal to noise ratio	Z_S	Unconstrained solution of Frobenius norm of Y- CH
I_{M_R}	Identity matrix of size M_R and M_R	$d^2_{K_S}$	Distance between codeword and centre of k_S- dimensional sphere
D	Diagonal matrix constructed from	$\Re$	Correaltion matrix of channel H
λ	Non-zero eigenvalues	$P(D \to \tilde{D})$	Pairwise error probability(PEP) between two codeword's D and $\tilde{D}$
r	Rank of matrix $\left(D - \tilde{D}\right) R \left(D - \tilde{D}\right)^H$	E	Expectation operator

The codeword matrix in (1) encodes the data symbols in space–time (ST) domain. It can be modified to form space–frequency (SF) [21] codeword matrix C_{SF} as

$$C_{SF} = \begin{bmatrix} c_1^{(0)} & c_2^{(0)} & \cdots & c_{M_T}^{(0)} \\ c_1^{(1)} & c_2^{(1)} & \cdots & c_{M_T}^{(1)} \\ \cdots & \cdots & \cdots & \cdots \\ c_1(Nc-1) & c_2(Nc-1) & \cdots & c_{M_T}(Nc-1) \end{bmatrix} \in C^{N_C \times M_T} \quad (2)$$

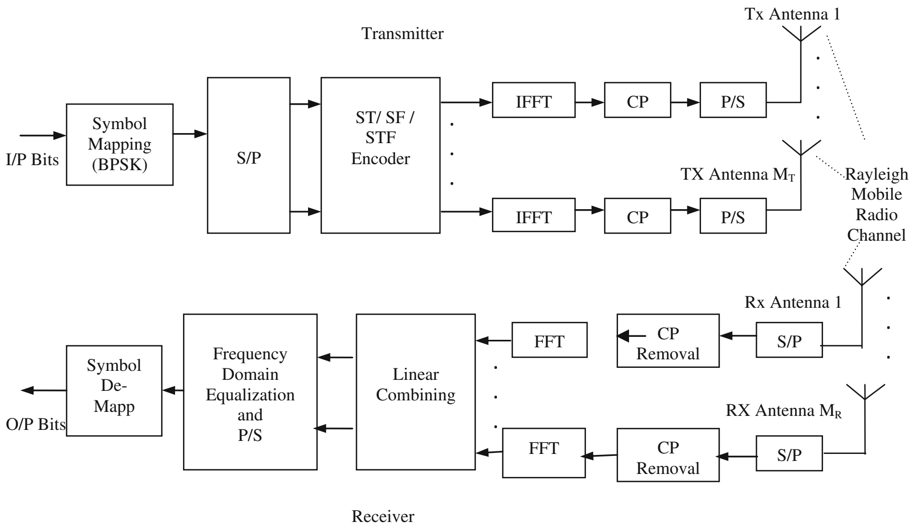

Fig. 1 ST/ SF / STF coded $M_T \times M_R$ MIMO-OFDM transceiver structure

A space–time–frequency codeword [22,23] has an additional dimension of time diversity added to the above SF codeword as shown below

$$C^K_{STF} = \begin{bmatrix} c^K_1(0) & c^K_2(0) & \dots & c^K_{M_T}(0) \\ c^K_1(1) & c^K_2(1) & \dots & c^K_{M_T}(1) \\ \dots & \dots & \dots & \dots \\ c^K_1(N_C-1) & c^K_2(N_C-1) & \dots & c^K_{M_T}(N_C-1) \end{bmatrix} \in C_T^{N_C K \times M_T} \quad (3)$$

The OFDM transmitter performs an N_C—point inverse fast fourier transform (IFFT) to each column of matrix C. ISI caused due to multiple delays of channel is removed by addition of CP to each OFDM symbol but its addition reduces spectral efficiency. The length of CP should be equal to or greater than delay spread of channel. The OFDM symbol corresponding to the ith $(i = 1, 2, \dots, M_T)$ column of C is transmitted by transmit antenna i. The information is then passed through MIMO channel which is characterized by Jake's model [24] for both Rayleigh frequency flat and selective channels. After removing the CP and applying FFT on frequency tones, the received signal at jth receive antenna for nth subcarrier during kth block is given by

$$y^k_j(n) = \sqrt{\frac{\rho}{M_T}} \sum_{i=1}^{M_T} H^k_{i,j}(n) C^k_{i,j}(n) + Z^k_j(n) \quad (4)$$

For $1 \le k \le K$, $0 \le n \le N_C - 1$, $1 \le i \le M_T$, and $1 \le j \le M_R$ also $H^k_{i,j}(n)$ given by

$$H^k_{i,j}(n) = \sum_{l=0}^{L-1} \alpha^k_{i,j}(l) e^{-j2\pi n \Delta f \zeta_L} \quad (5)$$

represents channel frequency response at the nth subcarrier between transmit antenna i and receive antenna j, $\Delta f = 1/T$ is the subcarrier separation in frequency domain and T is the

OFDM symbol period. L is the number of channel taps in time domain which are 12 for flat and 34 for selective channels. Such a higher tap order channel is chosen to combat deep fades due to harsh wireless channels. It is assumed that channel state information (CSI) is perfectly known at receiver but not at the transmitter. Due to this assumption, there is no need of estimating channel coefficients. Otherwise, channel coefficients can be estimated by techniques like pilot symbols, decision directed via mean square criterion [25,26] in MIMO-OFDM systems. System performance can be improved further if we estimate the carrier frequency offsets [27] in OFDM systems. The channels between different transmit and receive antenna pairs are assumed to have same power delay profile. Another assumption is to consider quasi-static channel, where the path gains are constant over a frame of time T and change from frame to frame. In (4) $Z_j^k(n)$ denotes the additive complex Gaussian noise with zero mean and unit variance at the nth subcarrier and at jth receive antenna. The noise samples are assumed to be uncorrelated for different j's and n's. The factor $\sqrt{\frac{\rho}{M_T}}$ ensures that, ρ is the average signal to noise ratio (SNR) at each receive antenna, and is independent of the number of transmit antennas. The linear combiner combines the output from FFT's to form composite output signal. The combining scheme for 2 × 2 system is suggested by Alamouti [3]. Let H_{11}, H_{21}, H_{12} and H_{22} be the channel coefficients between transmitter and receiver pair. The combined signals are given by

$$\hat{S}0 = H_{11}^* y_{11} + H_{12}^* y_{12} + H_{21}^* y_{21} + H_{22}^* y_{22} \tag{6}$$

$$\hat{S}1 = H_{12}^* y_{11} - H_{12} y_{12}^* + H_{22}^* y_{22} - H_{22} y_{22}^* \tag{7}$$

In (7), $y_{a,b}$ represents the received signals at receiver a from the transmitter b. These signals are given by

$$\begin{aligned} y_{11} &= H_{11}S0 + H_{12}S1 + n_0 \\ y_{12} &= -H_{11}S1^* + H_{12}S0^* + n_1 \\ y_{21} &= H_{21}S0 + H_{22}S1 + n_2 \\ y_{22} &= -H_{21}S1^* + H_{22}S0^* + n_3 \end{aligned} \tag{8}$$

Here S0 and S1 are the simultaneously transmitted symbols from antenna 1 and 2. The combined signal is then equalized by applying different equalizers like decision feedback equalization (DFE), maximum likelihood (ML) detector and SD.

3 ST/SF/STF Coding of MIMO-OFDM

In ST coding scheme [2], the coding is combined with transmit diversity to achieve high diversity performance in wireless systems. It can be implemented in two ways one is ST Trellis and other is ST Block coding. In ST trellis scheme, data symbols are encoded via M_T convolutional encoders to get M_T streams of symbols. In this scheme, the decoding complexity would increase exponentially with reference to diversity level i.e. by increasing number of transmit and receive antennas and transmission rate [28]. Alamouti [3] proposes orthogonal ST block code (OSTBC) design for 2 × 1 and 2 × 2 systems to alleviate above problem. Alamouti suggested that at a particular time instant two symbols can be simultaneously transmitted from the two antennas. Let the symbols transmitted from antenna 1 and 2 are S0 and S1 during time instant t and $-S1^*$ is transmitted symbol from antenna 1 and $S0^*$

Table 2 Space time scheme for 2 × 1 system

	Transmitter antenna 1	Transmitter antenna 2
Time (t)	S0	S1
Time (t + T)	−S1*	S0*

Table 3 Received signals at two receivers

	Receiver antenna 1	Receiver antenna 2
Time (t)	y_{11}	y_{21}
Time (t + T)	y_{12}	y_{22}

Table 4 SF coding for two transmit antenna

	OFDM-Subcarrier	
	K	L
Transmitting antenna 1	S0	−S1*
Transmitting antenna 2	S1	S0*

Table 5 STF coding for two transmit antenna

	OFDM-Subcarrier		
	K	L	
Transmitting antenna 1	S0	−S1*	OFDM Block (n)
Transmitting antenna 2	S1	S0*	OFDM Block (n+1)

from antenna 2 during next time instant t + T, where * is the complex conjugate operation. This scheme is illustrated in Table 2.

Diversity order can be increased for more reliable communication by employing two receiver antennas on receiver side as in Table 3.

Alamouti code can provide full diversity of 2 with rate 1 along with simple single symbol detection due to its orthogonal nature. It can further be generalized for higher number of transmit antennas case based upon theory of orthogonal design. In such cases OSTBC cannot provide rate more than 3/4 [17]. In ST coding full multipath diversity can not be achieved, to exploit it coding is done across antennas and OFDM subcarriers called SF coding [17]. Alamouti based SF coding can be realized by spreading Alamouti code across two subcarriers in one OFDM block as shown in Table 4.

Table 4 shows that two symbols S0 and −S1* are sent from subcarriers K and L of the same OFDM block through transmitting antenna 1. Similarly symbols S1 and S0* are sent from subcarriers K and L of the same OFDM block but through transmitting antenna 2. This code still can't achieve full diversity especially in frequency selective channels. Performance can be further enhanced by spreading Alamouti coding across space, time and frequency called STF codes. Table 5 shows that two symbols S0 and −S1* are sent from subcarriers K and L of OFDM block n through transmitting antenna 1. Similarly symbols S1 and S0* are sent from subcarriers K and L of OFDM block (n + 1) through transmitting antenna 2.

4 Performance Design Criteria

In this section ST, SF and STF coding approaches for MIMO-OFDM are analyzed and their performance criteria are derived [14]. These coding schemes can be compared in terms of coding rate, diversity gain and decoding complexity. The code rate generally defined as the ratio of total number of information symbols sent per channel and mathematically, it is approximately equal to $N_g/N_C T$ symbols per channel use (PCU), which means N_g information symbols are sent over $N_C T$ channels where N_C channels are used in T times. Diversity gain is the number of faded replicas of same information symbol that can be provided to the receiver in the form of redundancy in various domains like space, time and frequency. Since the probability that all the signal replicas fade equally and simultaneously is extremely small, thus receiver performance is enhanced significantly. In flat MIMO channels, full diversity gain is $M_T M_R$ whereas in frequency-selective MIMO channels it is $M_T M_R L$. ST coded MIMO-OFDM has a simple implementation with minimal decoding complexity, but it cannot achieve multipath diversity nor high rate. SF coding can achieve maximum diversity and full rate over multipath fading channels, but the decoding complexity is increased. A joint ML decoding method is needed for such cases. STF coded MIMO-OFDM can achieve full diversity and full rate. However, its decoding complexity is higher than ST and SF coding.

4.1 Pairwise Error Probability (PEP) Criteria

The design criteria for performance evaluation of STF coded MIMO-OFDM is derived [14], which serve us a formulation to evaluate any coding scheme. The received signal in (4) can be rewritten in vector form as

$$Y = \sqrt{\frac{\rho}{M_T}} DH + Z \tag{9}$$

In (9), D is a $KN_C M_T \times KN_C M_T M_R$ matrix constructed from STF codeword in (3) is given by

$$D = I_{M_R} \otimes \left[D_1 \ D_2 \ D_3 \ \ldots \ D_{M_T} \right] \tag{10}$$

where $\otimes$ denotes Kronecker product, I_{M_R} is the identity matrix of size $M_R \times M_R$ and

$$D_i = \text{diag}\{C_i(0), C_i(1), \ldots C_i(KN_C - 1)\}. \tag{11}$$

For any $i = 1, 2, \ldots, M_T$. Each D_i in (11) is related to ith column of the STF codeword in (3). The channel vector H of size $KN_C M_T M_R \times 1$ can be combined as

$$H = \begin{bmatrix} H_{1,1}^T \ldots H_{M_T,1}^T & H_{1,2}^T \ldots H_{M_T,2}^T \\ \ldots H_{1,M_R}^T & \ldots H_{M_T,M_R}^T \end{bmatrix}^T \tag{12}$$

where

$$H_{i,j} = \left[H_{i,j}(0) \quad H_{i,j}(1) \quad \ldots \quad \ldots \quad H_{i,j}(KN_C - 1) \right]^T \tag{13}$$

The received signal vector Y of size $KN_C M_R \times 1$ is given by

$$Y = \begin{bmatrix} y_1(0) & \ldots & y_1(KN_c - 1) & y_2(0) & \ldots \\ \ldots & y_{M_R}(0) & \ldots & \ldots & y_{M_R}(KN_C - 1) \end{bmatrix}^T \tag{14}$$

And noise vector Z is same as of Y, i.e.

$$Z = \begin{bmatrix} z_1(0) & \dots & z_1(KN_c - 1) & z_2(0) & \dots \\ \dots & z_{M_R}(0) & \dots & \dots & z_{M_R}(KN_C - 1) \end{bmatrix}^T \tag{15}$$

Suppose that D and $\tilde{D}$ are two different matrices related to two different STF codewords C and $\tilde{C}$ respectively. Then, the pairwise error probability (PEP) between D and $\tilde{D}$ can be upper bounded as

$$P(D \to \tilde{D}) \le \binom{2r-1}{r} \left(\prod_{i=1}^{r} \lambda_i \right)^{-1} \left(\frac{\rho}{M_T} \right)^{-r} \tag{16}$$

where r is the rank of $(D \to \tilde{D})\Re(D \to \tilde{D})^H$, $\lambda_1, \lambda_2, \lambda_3 \dots \lambda_r$ are the non-zero eigenvalues of $(D \to \tilde{D})\Re(D \to \tilde{D})^H$, and $\Re = E\{HH^H\}$ is the correlation matrix of H. The superscript ^{H} stands for Hermitian operator, which means complex conjugate and transpose of a matrix. E stands for expectation operator. Based on the upper bound on the PEP in (16), two general STF performance criteria are derived as follows

Diversity Criteria: It is also called rank criteria, which proposes that minimum rank of $(D \to \tilde{D})\Re(D \to \tilde{D})^H$ over all pairs of different codewords C and $\tilde{C}$ should be as large as possible.

Product Criteria: It proposes that minimum value of the product $\prod_{i=1}^{r} \lambda_i$ over all pairs of different codewords C and $\tilde{C}$ should be maximized.

4.2 Diversity Analysis Criteria

In spatially uncorrelated MIMO channels [14], the channel taps $\alpha_{i,j}^k$ (l) between each pair of transmit antenna i and receive antenna j are independent of each other. Thus, correlation matrix R of size $KN_CM_TM_R \times KN_CM_TM_R$ can be combined as

$$\Re = \text{diag} \begin{pmatrix} \Re_{1,1}, \dots \Re_{M_T,1}, \Re_{1,2}, \dots \\ \dots \Re_{M_T,2}, \dots \Re_{1,M_R}, \dots \Re_{M_T,M_R} \end{pmatrix} \tag{17}$$

where

$$R_{i,j} = E\left[H_{i,j}H_{i,j}^H\right] \tag{18}$$

is the correlation matrix of the channel frequency response from transmit antenna i to receive antenna j. Using notation $w = e^{-j2\pi\Delta f}$, $H_{i,j}$ can be decomposed as

$$H_{i,j} = (I_K \otimes W)A_{i,j} \tag{19}$$

where

$$W = \begin{bmatrix} 1 & 1 & \dots & 1 \\ w^{\tau_0} & w^{\tau_1} & \dots & w^{\tau_{L-1}} \\ \dots & \dots & \dots & \dots \\ w^{(N_c-1)\tau_0} & w^{(N_C-1)\tau_1} & \dots & w^{(N_C-1)\tau_{L-1}} \end{bmatrix}_{N_C \times L} \tag{20}$$

which is related to delay distribution, and

$$A_{i,j} = \begin{bmatrix} \alpha_{i,j}^1(0), \alpha_{i,j}^1(1), \dots \alpha_{i,j}^1(L-1) \dots \\ \dots \alpha_{i,j}^k(0), \alpha_{i,j}^k(1), \dots \alpha_{i,j}^k(L-1) \end{bmatrix}^T \tag{21}$$

This is related to the power distribution of the channel impulse response. In general, W is not a unitary matrix. If all of the L delay paths fall at the sampling instances of the receiver, then W is a part of the DFT matrix which is unitary. The correlation matrix of the channel frequency response vector between transmit antenna i and receive antenna j can be calculated as

$$\Re_{i,j} = E\left\{(I_k \otimes W)\, A_{i.j}A_{i,j}^H\, (I_k \otimes W)^H\right\} \tag{22}$$

$$\Re_{i,j} = (I_k \otimes W)\, E\left\{A_{i,j}A_{i,j}^H\right\} \left((I_K \otimes W)^H\right) \tag{23}$$

It is assumed that the path gains are independent for different paths and different pairs of transmit and receive antennas. The second order statistics of the time correlation is same for all transmit and receive antenna pairs for all paths. Thus the correlation matrix $E\{A_{I,J}A_{I,J}^H\}$ can be expressed as

$$E\left\{A_{i,j}A_{i,j}^H\right\} = \Re \otimes \mathrm{diag}\left(\delta_0^2, \delta_1^2 \ldots \delta_{L-1}^2\right) \tag{24}$$

where $\Re_T$ is temporal correlation matrix of size $k \times k$, thus frequency correlation matrix $\Re_F$ can also be expressed as

$$\Re_F = E\left\{H_{i,j}^K H_{i,j}^{K^T}\right\} \tag{25}$$

where, $H_{i,j}^K = \lfloor H_{i,j}^K(0) \ldots H_{i,j}^K(N_C - 1)^T \rfloor$ then, $\Re_F = W\, \mathrm{diag}\left(\delta_0^2,\ \delta_1^2 \ldots \delta_{L-1}^2\right) W^H$ As a result

$$\Re_{I,J} = (I_K \otimes W)\left\{\Re_T \otimes \mathrm{diag}\left(\delta_0^2,\ \delta_1^2 \ldots \delta_{L-1}^2\right)\right\}\left(I_K \times W^H\right) \tag{26}$$

$$\Re_{I,J} = \left\{\Re_T \otimes W\mathrm{diag}\left(\delta_0^2, \delta_1^2 \ldots \delta_{L-1}^2\right)\right\}\left(W^H\right) = \Re_T \otimes \Re_F \tag{27}$$

Finally, the expression for r in Eq. (16) can be rewritten as

$$\left(D - \tilde{D}\right) R \left(D - \tilde{D}\right)^H = I_{MR} \otimes \left[\sum_{i=1}^{M_T}\left(D_i - \tilde{D}_i\right)(\Re_T \otimes \Re_F)\left(D_i - \tilde{D}_i\right)^H\right] \tag{28}$$

$$= I_{MR} \otimes \left\{\left[\left(\left(C_i - \tilde{C}_i\right)\left(C_i - \tilde{C}_i\right)^H\right)\right] O\,(R_T \otimes R_F)\right\} \tag{29}$$

In (29), symbol o denotes the Hadamard product. Let $\Delta \cong \left(C_i - \tilde{C}_i\right)\left(C_i - \tilde{C}_i\right)$ and $\Re \cong \Re_T \otimes R_F$. Substitute (29) into (16) it becomes

$$P\left(C \to \tilde{C}\right) \leq \binom{2\nu M_R - 1}{\nu M_R}\left(\prod_{i=1}^{\nu}\lambda_i\right)^{-M_R}\left(\frac{\rho}{M_T}\right)^{-VM_R} \tag{30}$$

where ν is rank of $\Delta o \mathcal{R}$ and $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_\nu$ are the non-zero eigenvalues of $\Delta o \mathcal{R}$. As result Diversity and product criteria can be modified as follows.

Diversity (rank) criterion: The minimum rank of $\Delta o \mathcal{R}$ over all pairs of distinct codewords C and $\tilde{C}$ should be as large as possible.

Product criterion: The minimum value of the product $\prod_{i=1}^{\nu}\lambda_i$ over all pairs of distinct codewords C and $\tilde{C}$ should also be maximized.

If the minimum rank of $\Delta o \ \mathcal{R}$ is ν for any pair of distinct STF codewords C and $\tilde{C}$ Then STF code achieves a diversity order of vM_R for a fixed number of OFDM blocks, transmitting antennas and correlation matrices $\mathcal{R}_T$ and $\mathcal{R}_F$.

According to the rank inequalities on Hadamard product and Kronecker product we have

$$\text{rank}\ (\Delta\ \text{o}\mathcal{R})\ \leq\ \text{rank}\ (\Delta)\cdot\ \text{rank}\ (\mathcal{R}_{\text{T}})\cdot\ \text{rank}\ (\mathcal{R}_{\text{F}}) \tag{31}$$

Since the rank Δ of is at the most M_T, the rank of $\mathcal{R}_F$ at the most is L, and the rank of $o\quad\mathcal{R}$ is at most KN_C, So we get

$$\text{rank}\ (\Delta\text{o}\mathcal{R})\ \leq\ \min \text{LM}_\text{T}\ \ \text{rank}(\Re_\text{T}), \text{KN}_\text{C} \tag{32}$$

Thus, the full achievable diversity is at most min $[\text{LM}_\text{T}\text{M}_\text{R}\text{rank}(\Re_\text{T}),\ \text{KN}_\text{C}\text{M}_\text{R}]$. In rest of the section, we will show that the above mentioned diversity would be achieved. In case, if the channel remains constant over multiple OFDM blocks then rank of time correlation matrix would be close to 1. Thus, full diversity in such cases would be min $[\text{LM}_\text{T}\text{M}_\text{R}\text{rank}(\Re_\text{T}),\ \text{KN}_\text{C}\text{M}_\text{R}]$ same as that of SF coding.

4.3 STF Code Design Criteria

In this section, a full diversity STF code criterion is derived from SF coding. For this purpose, it is assume that the number of subcarriers N_C is not less than LM_T. Our objective is to show that the maximum achievable diversity order is $[\text{LM}_\text{T}\text{M}_\text{R}\text{rank}(\Re_\text{T})]$

Suppose C_SF is a full diversity code of size $\text{N}_\text{C} \times \text{M}_\text{T}$. We can construct a STF code by repeating C_SF codeword K times (over K OFDM blocks) as shown below

$$\text{C}_\text{STF} = 1_{\text{K}\times 1} \otimes \text{C}_\text{SF} \tag{33}$$

where $1_{\text{K}\times 1}$ is an all one matrix of size $\text{k}\times 1$, Let $\Delta_\text{STF} = \left(\text{C}_\text{STF} - \tilde{\text{C}}_\text{STF}\right)\left(\text{C}_\text{STF}-\tilde{\text{C}}_\text{STF}\right)^H$ and $\Delta_\text{SF}=\left(\text{C}_\text{SF}-\tilde{\text{C}}_\text{SF}\right)\left(\text{C}_\text{SF}-\tilde{\text{C}}_\text{SF}\right)^H$. Also we have

$$\begin{aligned}\Delta_\text{STF} &= \left[1_{\text{K}\times 1} \otimes \left(\text{C}_\text{SF}-\tilde{\text{C}}_\text{SF}\right)\right] \times \left[1_{\text{K}\times 1} \otimes \left(\text{C}_\text{SF}-\tilde{\text{C}}_\text{SF}\right)^H\right] \\ &1_{\text{K}\times 1} \otimes \Delta_\text{SF}\end{aligned} \tag{34}$$

Thus

$$\begin{aligned}\Delta_\text{STF}\text{o}\Re &= (1_{\text{K}\times\text{K}} \otimes \Delta_\text{SF})\ \text{o}\ (\Re_\text{T} \otimes \Re_\text{F}) \\ &= \Re_\text{T} \otimes (\Delta_\text{STF}\text{o}\Re)\end{aligned} \tag{35}$$

Since the SF code C_SF achieves full diversity in each OFDM block, the rank of $\Delta_{\text{STF}\circ}\text{o}\Re$ is LM_T. Therefore, the rank of $\Delta_{\text{STF}\circ}\text{o}\Re$ is LM_T rank $(\mathcal{R}_\text{T})$. It means C_STF achieves full diversity of $\text{LM}_\text{T}\text{M}_\text{R}\text{rank}\ (\mathcal{R}_\text{T})$.

It is observed that the maximum achievable diversity depends on the rank of the temporal correlation matrix $\mathcal{R}_\text{T}$. If the fading channels are constant during K OFDM blocks, i.e. rank $(\mathcal{R}_\text{T}) = 1$, the maximum achievable diversity order for STF codes (coding among several OFDM blocks) is the same as that for SF codes (coding within one OFDM block). Moreover, if the channel changes independently in time, i.e. $\mathcal{R}_\text{T} = \text{I}_\text{K}$, the repetition structure of STF code C_STF is sufficient, but not necessary to achieve the full diversity. We have

$$(\Delta\ \text{o}\mathcal{R})\ =\ \text{diag}\ (\Delta_1\text{o}\mathcal{R}_\text{F},\ \ \Delta_2\text{o}\mathcal{R}_\text{F}\ldots\Delta_\text{k}\text{o}\mathcal{R}_\text{F}) \tag{36}$$

where $\Delta_\text{K} = \left(\text{C}_\text{K}-\tilde{\text{C}}_\text{K}\right)\left(\text{C}_\text{K}-\tilde{\text{C}}_\text{K}\right)^H$ for $1 \leq \text{k} \leq$ for $1 \leq \text{k} \leq \text{K}$. Thus, the necessary and sufficient condition to achieve full diversity $\text{KLM}_\text{T}\text{M}_\text{R}$ is to make $\Delta_\text{K}\text{o}\mathcal{R}_F$ of rank LM_T over all pairs of distinct codewords for $1 \leq \text{k} \leq \text{K}$.

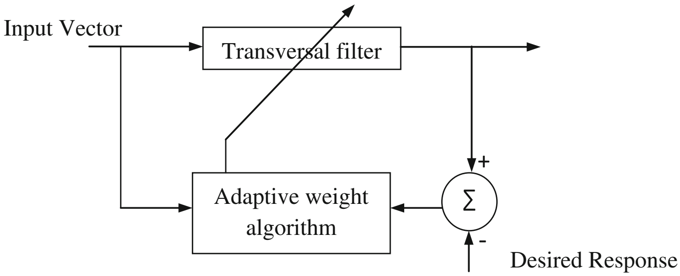

Fig. 2 Schematic of decision feedback equalizer (DFE)

5 Equalization and Decoding

The inter-symbol interference (ISI) caused by multipath MIMO channels distorts the MIMO-OFDM transmitted signal which causes bit errors at receiver. To minimize errors, equalization [29] is needed. Equalizer minimizes the error between actual output and desired output by continuous updating its filter coefficients. Equalization can be done in both time and frequency domain. Equalization in frequency domain is simpler to use as compared to time domain. The important parameter in equalization design is to choose number of taps. The number of taps is limited by maximum time delay spread offered by the channel. An equalizer can equalize for delay intervals less than or equal to the maximum delay within the filter structure. So it is important to know about the number of taps before selecting an equalizer structure and its algorithm. In this paper various equalizers like DFE, ML and SD are implemented and their performance evaluation is done in terms of BER.

5.1 Decision Feedback Equalizer

The basic idea behind DFE [30] is that once an input symbol has been detected and decided upon, the ISI that it produces on future symbols can be estimated and subtracted before detection of subsequent symbols. DFE can be realized in direct transversal form which consists of feed forward filter (FFF) and a feedback filter (FBF) as shown in Fig. 2. The FBF [31] is driven by decision on the output of the detector, and its coefficients can be adjusted to cancel the ISI on the current symbol from past detected symbols.

RLS (recursive least squares) algorithm is used to determine the coefficient of an adaptive filter. It uses information from all past input samples to estimate the autocorrelation matrix of the input vector. To decrease the influence of input samples from the past samples, a weighting factor is used for the influence of each sample. DFE equalizer comprises of filtering and adaptive process. In filtering process, algorithm computes the output of a linear filter in response to an input signal as given by

$$y(n) = \tilde{w}H\,(n)\,C(n) \tag{37}$$

where y(n) is output of a linear filter. All subscripts are omitted for simplification. Output y(n) is compared with desired response d (n) to generate estimation error e(n) which is given by

$$e(n) = d(n) - y(n) \tag{38}$$

In next phase tap-weight vector is updated by incrementing its old value by an amount equal to the complex conjugate of the estimation error as described in (38).

$$\tilde{w}(nr + 1) = \tilde{w}(n_r) + \mu C(n)e^*(n) \quad (39)$$

where n_R the number of iteration and μ is the step size, which controls the convergence rate and stability of algorithm.

5.2 Maximum Likelihood (ML) Detection

The MSE based linear equalizers are optimum [32] when channel does not introduce much amplitude distortion. In such situations we choose ML based equalizer which tests all possible data sequences and chooses the data sequence which has maximum probability at the output. These equalizers require knowledge of channel characteristics in order to compute the metrics for making decisions. They also require knowledge of statistical distribution of the noise, which determines the form of metric for optimum demodulation of the received signal. Maximum likelihood (ML) decoding [33] finds the codeword $\hat{C}$ that solves the following minimization problem

$$\hat{C} = \arg\min_{\hat{C}} \left[|y - CH|\right]_F^2 \quad (40)$$

where F is Frobenius norm. Equation (16) can be expanded using Frobenius norm as following

$$\hat{C} = \arg\min_{\hat{C}} \left\lfloor \mathrm{Tr} \left|(y - CH)^H y - CH\right| \right\rfloor \quad (41)$$

$$\hat{C} = \arg\min_{\hat{C}} \left[\mathrm{Tr}\left[y^H \cdot y + H^H C^H CH - H^H C^H y - y^H CH\right]\right] \quad (42)$$

If $y^H \cdot y$ is independent of the transmitted codeword (42) can be written as

$$\hat{C} = \arg\min_{\hat{C}} \left[\mathrm{Tr}\left[H^H C^H CH\right] - 2 \cdot \mathrm{Real}\left(\mathrm{Tr}\left[H^H C^H y\right]\right)\right] \quad (43)$$

Also (43) can be generalized for multiple receivers as given by

$$\mathrm{Tr}|H^H C^H CH| = \sum_{m=1}^{M_R} H_m^H C^H CH_m \quad (44)$$

$$\mathrm{Tr}|H^H C^H y| = \sum_{m=1}^{M_R} H_m^H C^H y_m \quad (45)$$

Applying (44) and (45) in (43) we get

$$\hat{C} = \arg\min_{\hat{C}} \left[\sum_{m=1}^{M_R} H_m^H C^H CH_m - 2 \cdot \mathrm{Real}\left(\sum_{m=1}^{M_R} H_m^H C^H y_m\right)\right] \quad (46)$$

In case of one receiving antenna, the minimization function reduce to

$$H_1^H C^H CH_1 - 2 \cdot \mathrm{Real}\left(H_1^H C y_1\right) \quad (47)$$

For multiple receivers, we can write the function for one receiver and add the correct summation in front of it to achieve the ML decoding formulas for general case of M_R receiving antennas. It is equivalent to maximum ratio combining (MRC). In case of ST coding with Alamouti structure, the above metric can be decomposed into two separate parts for detecting each individual symbol i.e. ML decoding becomes single symbol decodable ML (SML). In SF coding, single symbol ML decoder doesn't yield optimum results because channel orthoganlity is disturbed in case of frequency-selective channels. In such cases, joint ML decoder (JML) is preferred which detects two symbols jointly. ML decision metric, which detects S0 and S1 jointly is given by

$$D_m(\hat{S}0, \hat{S}1) = \left(\left[y - CH\right]\right)^2 \tag{48}$$

$$= \left[y_0 - H_{11}S0 - H_{12}S1\right]^2 + \left[y_1 + H_{21}S1^* - S0^*\right]^2 \tag{49}$$

By neglecting the terms that are independent of transmitted symbols S0 and S1, the above decision metric reduces to

$$D_m\left(\hat{S}0, \hat{S}1\right) = \left([H_{11}]^2 + [H_{22}]^2\right)[S0]^2 + \left([H_{12}]^2 + [H_{21}]^2\right)[S1]^2 - 2R\left[\left(H_{11}y_0^* + H_{22}^*y_1\right)S0 + (H_{21}y_0^* - H_{12}^*y_1)[S1] + (H_{12}H_{22}^* - H_{11}H_{21}^*)S0S1^*\right] \tag{50}$$

5.3 Sphere Decoder (SD)

SD is preferably used [34] when decoder complexity is high, it might be high due to increase in number of transmit and receive antenna or when coding is in three dimensions like space, time and frequency. The main idea behind SD is to limit the search space for finding the closest codeword to the particular received vector. The search space which includes optimal lattice point is given by a hyper-sphere of radius r centered on the received signal vector. Equation (4) can be rewritten after omitting all subscripts and superscripts as follows

$$Y = CH + Z \tag{51}$$

In ML decoding, we find the codeword C that minimizes the Frobenius norm $\left[|y\text{-}CH|\right]_F^2$. Using a full search for finding the optimal codeword is computationally very demanding. If the modulation utilizes a constellation with 2^b points to transmit b bits, the number of possibilities for C is $2^{bM}_{T.}$ It will become more impractical with higher constellation size and with more transmitting antennas. Thus in SD, instead of searching all possible vectors for finding C in the above optimization problem, we will search over a hyper-sphere of radius r_s centered on the received signal vector [35] i.e. we look for vectors C which satisfy $\hat{C} = \arg\min_{\hat{C}}[|y - CH|]_F^2 \leq r_s^2$. If the point is actually found in the sphere, the radius of the search sphere is lowered to the distance of this point to the center. The algorithm is repeated until no point is found inside the search sphere.

It comprises of two steps, (1) pre-processing (2) search step. In step 1, we consider the solution of optimization problem, i.e., to minimize $[|y - CH|]_F^2$ is $Z_S = H^{\downarrow}Y$. where $H^{\downarrow}$ pseudo inverse of matrix H. The optimization problem can equivalently written as

$$\min_{\Lambda \epsilon \Lambda} (C - z_S)^H H^H H(C - z_S) \tag{52}$$

where Λ is a complex lattice in the sense that each coordinates of C is chosen from the defined complex constellation. By performing Cholskey decomposition on the $H^H H$ matrix, we obtain the upper triangular matrix $U = \langle u_{k,1}|u_{kk} \in r_S > 0\rangle$ such that $H^H H = U^H U$. We now consider the problem of finding solution to $[|U(z_s - C)|]^2 \leq r_s^2$.

In the preprocessing step, the unconstrained solution z_s and upper triangular matrix U are calculated. Then the $M_T M_T$ matrix Q is formed according to

$$Q = \begin{vmatrix} q_{K_S,K_S} = u^2_{K_s K_S} \\ q_{K_S,1} = u_{K_S 1}/u_{k_S,K_S},\, k_s < 1 \end{vmatrix} \tag{53}$$

In step 2, the points inside the sphere are examined to locate the vector C which has the lowest distance from the center. Using the fact that the matrix Q is upper-triangular, we can re-write the problem as follows

$$\sum_{i=0}^{K_S} \left| q_{i,i}\left(C_i - z_{s_i}\right) + \sum_{j=i+1}^{K_S} q_{i,j}(C_j + z_{s_i}) \right|^2 \leq r_s^2 \tag{54}$$

The relation above can be used to find C_k's, $k_s = 1, \ldots, k_s$ in an iterative fashion. Specifically, we start with $k_s = k_s$ and find the distance between C_k and the center of the k_s-dimensional sphere as

$$d^2_{k_s} = \sum_{l=k}^{k_s} \left| \sum_{i=1}^{k_s} q_{l,i}\left(C_i - z_{s_i}\right) \right|^2 \tag{55}$$

If we define

$$S_{K_S} = z_{S_{K_S}} - \sum_{i=K_S+1}^{K_S} q_{K_S,i}\left(C_i - z_{s_i}\right) \tag{56}$$

For the other $k_s - k_s$ elements, the condition for being inside the search sphere can be written as

$$d^2_{K_S} = d^2_{K_S+1} + q_{K_S,K_S}\left|C_{K_S} - S_{K_S}\right|^2 \leq r_S^2 \tag{57}$$

Thus a search space for S_{K_S} can be specified as

$$\left|C_{K_S} - S_{K_S}\right|^2 \leq \frac{r_S^2 - d^2_{K_S}+1}{q_{K_S,K_S}} \tag{58}$$

When k_s become equal to 1, a valid vector C is found. If the distance from the center to the point is found to be less than the radius of the sphere [36], this distance is chosen as the new radius of the sphere. The procedure is then repeated starting again with $k_s = k_s$. If at any time, $d^2_{K_S}$ is greater than the radius of the sphere, the procedure is terminated.

The decoder complexity is measured and compared in terms of complex valued additions, subtractions and multiplications required to decode one block of transmitted symbols. A complex multiplication is equivalent to 4 real multiplications R_M and 2 real additions R_A, while a complex addition is equivalent to 2 real additions. Furthermore, the multiplication of a real valued quantity by a factor 2, like the term on right hand side of Eq. (50), is implemented by means of one real valued addition. In case of ML decoding, we have to compare single symbol decodable ML and jointly decodable ML. In first case, we need to compute 2^b metrics for each of the two transmitted symbols, where b is number of bits per modulated symbol. In joint ML, we require 2^{2b} metrics to determine symbols which jointly minimizes (50).The number of necessary complex valued additions/subtractions and multiplications are summarized in Table 6.

Table 6 Number of complex valued operations in SML and JML

Parameter	SML	JML
Number of additions/subtractions	$4 + 3 * 2^b$	$4 + 3.5 * 2^{2b}$
Number of complex multiplications	$8 + 2.5 * 2^b$	$10 + 4 * 2^{2b}$

Table 7 Number of complex valued operations in SML and JML

Parameter	BPSK(b = 1)		QPSK(b = 2)		16-QAM(b = 4)	
	SML	JML	SML	JML	SML	JML
Number of additions/subtractions	10	18	16	60	52	900
Number of complex multiplications	13	26	18	74	48	1034

Table 8 Simulation parameters

Parameter	Value
Total bandwidth	1 MHz
Number of transmit antenna	2
Number of receiving antenna	1 and 2
Maximum Doppler freq.(f_m)	200 Hz
Maximum Doppler shift	$2\pi f_m = 1.256 \times 10^{-3}$
Channel model used [37] Channel coefficients amplitudes Channel delay spreads (in μ s) Carrier modulation used	Six-ray urban TU channel model 0.2, 0.5, 0.4, 0.1, 0.06, 0.4 0, 0.2, 0.5, 1.6, 2.3, 5 BPSK
Spectral efficiency	1bit/sec/Hz
Number of data subcarriers	128
Number of pilot-subcarriers	None
IFFT size	128
Guard period type	Cyclic extension
Cyclic prefix length	32
Window type	No windowing used
Number of channel taps	12 and 34 for flat and selective channels
Channel fading	Rayleigh independent frequency flat and selective fading i.e. rank ($\mathcal{R}_T$) = 1,

The results of complexity analysis for different modulation techniques like BPSK, QPSK, and 16-QAM are shown in Table 7.

From Table 7, it is clear that complexity incases with increase in constellation size and among ML decoders, joint Ml decoder exhibits higher complexity. In SD, complexity is measured in terms of average floating point operations (FLOPS) which include all arithmetic operations. Average FLOPS per block in case of SD used in this paper is apprx. 10 for BPSK and increased up to 400 for 64-QAM; Thus, number of FLOPS are remarkably less than number of real multiplications and additions in SML and JML. Thus, SD is considerably less complex than SML and JML. However, system complexity needs to be put in relation to the complexity of other functional blocks for e.g. in FFT operation at the receiver, the number of complex multiplication required for an N-point FFT is $N * \log_2 N$, which equals to 896 complex multiplications for $N = 128$.

6 Simulation Results

6.1 Simulation Parameters

Simulation parameters used for simulation of MIMO-OFDM transceiver model mentioned in Fig. 1 are listed in Table 8. Simulations is done with 2 transmit antenna, 1 receiver antenna and then with 2 transmit and 1 receive antenna.

6.2 Results

To illustrate analytical results derived in Sect. 3, we plotted simulation results in terms of BER with variation in signal to noise (SNR) ratio. Simulations are done in two phases. In phase 1, results are plotted considering two transmit antennas and one receiving antenna and in 2, with two transmit antennas and two receiving antennas. In both cases the channel between transmit and receive antenna pair is assumed to be a quasi-static Rayleigh flat and selective channel with higher tap order. The channel is modeled by Jake's model which takes into account the effects of Doppler shift and Doppler spread existing due to relative motion between transmitter and receiver. Further, the channel experienced by each transmitting antenna is assumed to be independent of channel received by other transmitting antennas. In addition, the transmitting power of each transmitting antenna is considered same. Further, we assume that the receiver has perfect knowledge of the channel while transmitter doesn't know the channel.

Figures 3 and 4 shows BER performance of 2×1 MIMO-OFDM system with different equalizers in frequency flat and frequency selective channels. It shows STF coding scheme clearly dominates ST and SF coding, In other words, STF coding showing higher diversity order than other schemes. In Sect. 4, we conclude that maximum achievable diversity in case of STF coding is LM_RM_T, assuming independent channel fading i.e. temporal coefficient $\mathcal{R}_T = I_k$ (identity matrix).Therefore, maximum achievable diversity in flat channels with 2×1 configuration is $12 * 1 * 2 = 24$ (as shown in Fig. 3) and $34 * 1 * 2 = 68$ in selective channels as shown in Fig. 4. The symbols are chosen from BPSK constellation, therefore ignoring the cyclic prefix, we have a spectral efficiency of 1/bit/s/Hz. Moreover, SF and STF schemes have same spectral efficiency because full diversity STF coding is obtained from full diversity SF coding via repetition mapping. Among equalizers, STF with SD outperforms all other equalizers i.e. STF-ML and STF-DFE by almost 0.5 and 1 dB. Among coding schemes, STF-SD dominates SF-SD and ST-SD by approximately 2 and 3 dB. Thus, we conclude that STF coding with SD is the best combinations among all permutations. Comparing results of Figs. 3 and 4, it clearly shows that BER performance is better in flat channels with all combinations.

Figures 5 and 6 shows BER performance of 2×2 MIMO-OFDM system in both flat and selective channels. The achievable diversity is 48 and 136 in case of flat and selective channels with 2×2 configuration, higher than that of 24 and 68 with 2×1 configuration. The results also show that employing 2 antennas on receiver side greatly enhance the system

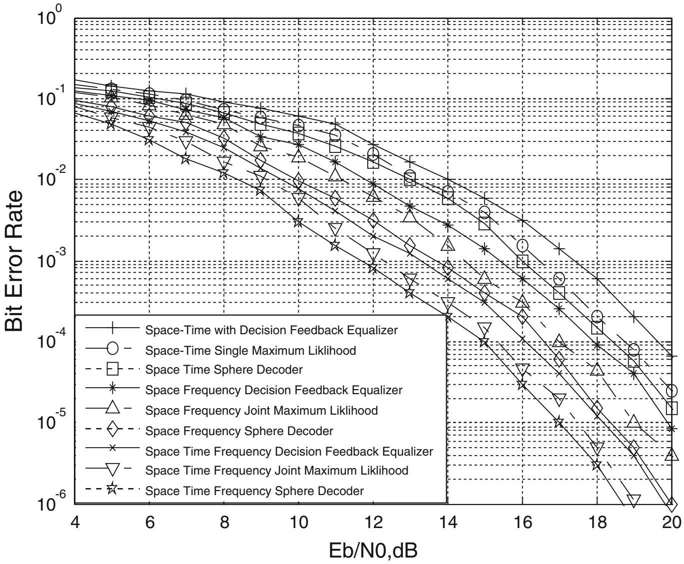

Fig. 3 Comparison of BER for BPSK using 2 × 1 MIMO-OFDM system in quasi-static Rayleigh frequency flat channel

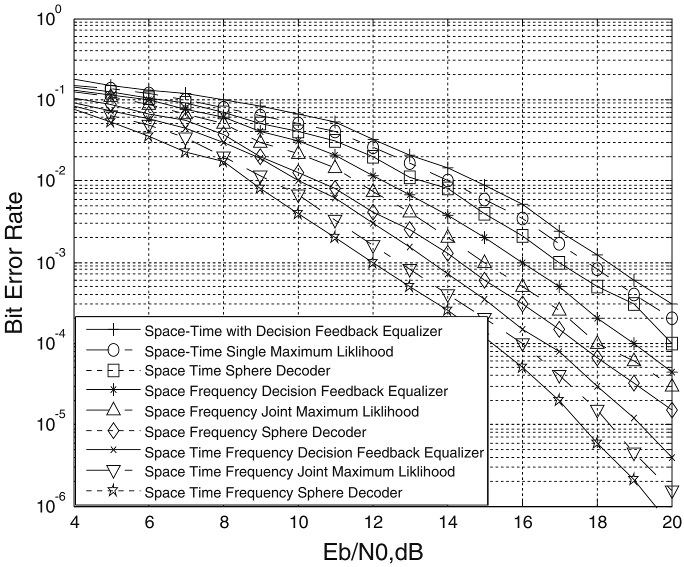

Fig. 4 Comparison of BER for BPSK using 2 × 1 MIMO-OFDM system in quasi-static Rayleigh frequency selective channel

performance as the diversity order is increased. Further, the BER performance is better in flat channels than selective channels. Therefore, it can be concluded that STF-SD is the best coding scheme in both cases.

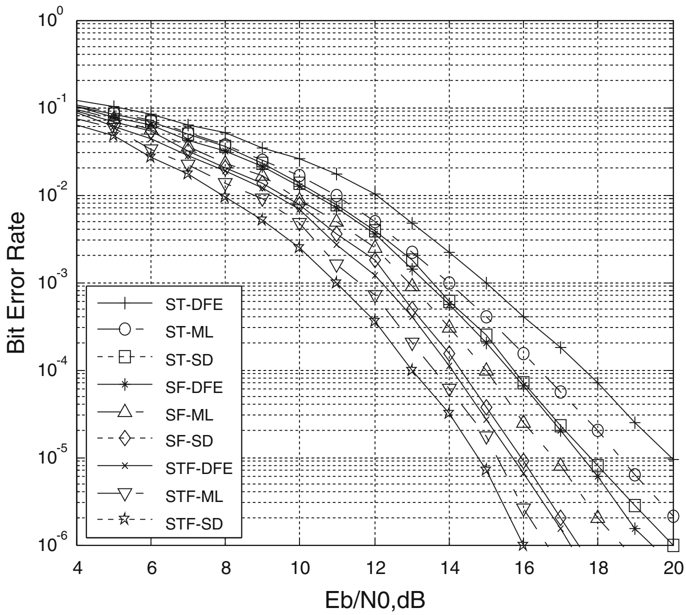

Fig. 5 Comparison of BER for BPSK using 2 × 2 MIMO-OFDM system in quasi-static Rayleigh frequency flat channel. ST-DFE *space time with decision feedback equalizer*, *ST-ML* space-time maximum likelihood, *ST-SD* space-time sphere decoder, *SF-DFE* space frequency with decision feedback equalizer, *SF-ML* space frequency maximum likelihood, *SF-SD* space frequency sphere decoder, *STF-DFE* space time frequency decision feedback equalizer, *STF-ML* space time frequency maximum likelihood, *STF-SD* space time frequency sphere decoder

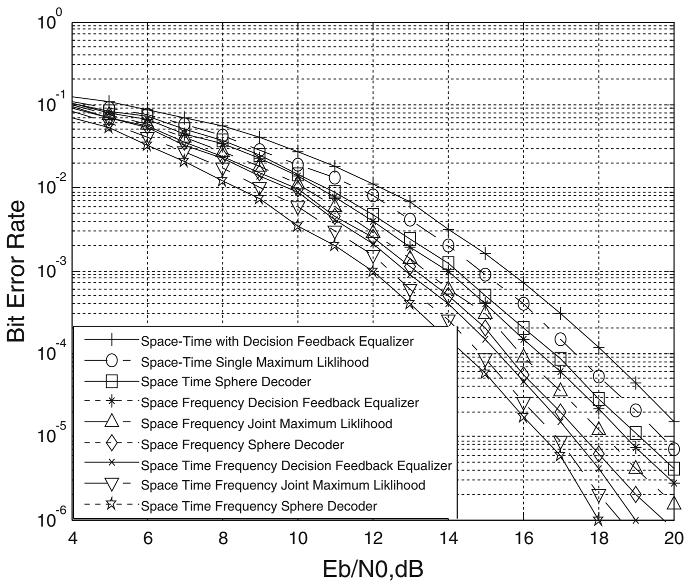

Fig. 6 Comparison of BER for BPSK using 2 × 2 MIMO-OFDM system in quasi-static Rayleigh frequency selective channel

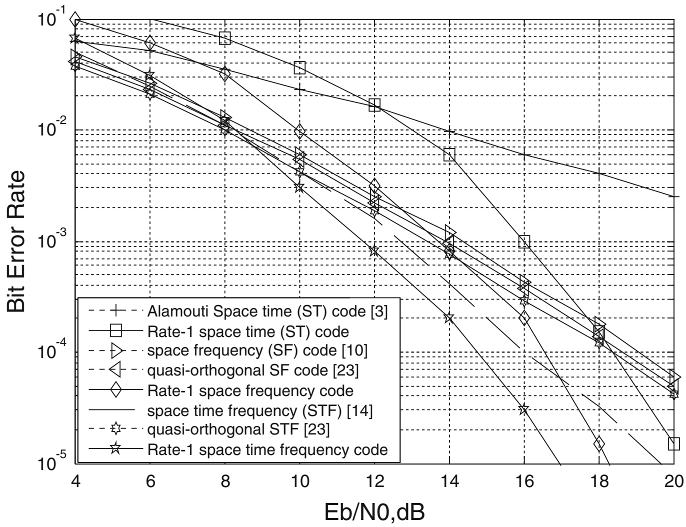

Fig. 7 BER comparison of STFC and SFC with rate-1 for 2 × 1 MIMO-OFDM system in quasi-static Rayleigh frequency flat channel

Figure 7 shows the BER comparison of our ST, SF and STF codes with existing codes. Results show that existing codes dominate the performance in lower SNR region as they use precoding matrix [10]. In higher SNR region, our ST, SF and STF codes outperforms their corresponding counterparts as the effect of higher diversity order is more dominating then using precoding matrix. The superiority of our STF code over that of [14] and [23] is evident from Fig. 7. In Fig. 7, at almost every point our STF code is better than code of [14] and [23] by almost 2–3 dB. Our SF codes also dominates SF codes in [10] and [23] by 1–1.5 dB.

7 Conclusion

In this paper, system performance of 2 × 1 and 2 × 2 ST/SF/STF coded MIMO-OFDM is investigated under broadband wireless channels. Maximum achievable diversity is computed and verified with simulation results. Simulation is done for both quasi static Rayleigh frequency flat and selective channels modeled by Jake's model which considers the effect of Doppler shift and spread along with more multiple paths. Choosing higher number of multiple paths helps in to achieve higher diversity order, which further helps to combat deep fades occur due to harsh wireless environment. It is concluded that effects of higher diversity is more dominating in higher SNR region. Decoder complexity which increases due to increase in diversity order is resolved by employing SD at receiver. The major limitations of MIMO technology are to choose antenna spacing which must be appropriate depending upon the type of channels. The limitation of the proposed design is increased complexity in transmitter and receiver designs. Work can be done to reduce this complexity. System performance can be improved further if we increase code rate and coding gain.

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