CHAPTER

A NEW MORGENSTERN TYPE BIVARIATE EXPONENTIAL DISTRIBUTION WITH KNOWN COEFFICIENT OF VARIATION BY RANKED SET SAMPLING

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Vishal Mehta

Department of Mathematics, Jaypee University of Information Technology, Waknaghat, Himachal Pradesh, India

10.1 INTRODUCTION

Cost-effective sampling methods are of major concern in some experiments, especially when the measurement of the characteristics is costly, painful, or time-consuming. The concept of ranked set sampling (RSS) was first introduced by McIntyre (1952) as a process of increasing the precision of sample mean as an unbiased estimator of population mean. The method of RSS provides an effective way to achieve observational economy or to achieve relatively more precision per unit of sampling. RSS as described by McIntyre (1952) is applicable whenever ranking of a set of sampling units can be done easily by judgment method. For a detailed discussion on theory and application of RSS, see Chen et al. (2004). In certain situations one may prefer exact measurements of some easily measurable variable X associated with the study variable Y to rank the units of samples rather than ranking them by a crude judgment method. Suppose the variable of interest Y, is difficult or much more expensive to measure, but an auxiliary variable X correlated with Y is readily measureable and can be ordered exactly. In this case as an alternative to McIntyre's (1952) method of ranked set sampling, Stokes (1977) used an auxiliary variable for the ranking of sampling units.

If $X_{(r)r}$ is the observation measured on the auxiliary variable X from the unit chosen from the rth set then we write $Y_{[r]r}$ to denote the corresponding measurement made on the study variable Y on this unit, then $Y_{[r]r}, r = 1, 2, \ldots, n$ from the ranked set sample. Clearly $Y_{[r]r}$ is the concomitant of the rth order statistic arising from the rth sample.

In many areas, especially in physical science, it is common to find the population standard deviation is proportional to the population mean, that is, the coefficient of variation (CV) is constant (e.g., Sen, 1978; Ebrahimi, 1984, 1985; Singh, 1986). In such cases it is possible to find a more efficient estimator of the mean assuming that the coefficient of variation (CV) is known than by using the sample mean.

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Let X be a random variable having the two-parameter exponential distribution as

$$f_X(x) = \frac{1}{\sigma}\exp\left(-\frac{x-\theta}{\sigma}\right); x \geq \theta > 0, \sigma > 0. \tag{10.1}$$

Here θ is the location parameter (guarantee period) and σ is the scale parameter (measuring the mean life). Since $E(X) = \theta + \sigma$ and $Var(X) = \sigma^2$, therefore the $CV = \frac{\sigma}{\theta+\sigma}$. Using the fact that the CV is some known constant we get that $\sigma = a_1\theta$, where $a_1(>0)$ is known (see, Samanta, 1984, 1985; Joshi and Nabar, 1991) and therefore Eq. (10.1) reduces to

$$f_X(x) = \frac{1}{a_1\theta}\exp\left(-\frac{x-\theta}{a_1\theta}\right); x \geq \theta > 0, a_1 > 0, \tag{10.2}$$

which has mean $\theta(a_1 + 1)$ and variance $\theta^2 a_1^2$, therefore the $CV = \frac{a_1}{(a_1+1)}$ is the same for all $\theta(>0)$.

The cumulative density function (cdf) of Eq. (10.2) is given by

$$F_X(x) = 1 - \exp\left(-\frac{x-\theta}{a_1\theta}\right); x \geq \theta > 0, a_1 > 0. \tag{10.3}$$

Ali and Woo (2002) considered parametric estimation of a special case of the two-parameter exponential distribution in which both the threshold (location) and the scale parameters are equal. For $a_1 = 1$ the probability density function (pdf) $f_X(x)$ in Eq. (10.2) reduces to:

$$f_X(x) = \frac{1}{\theta}\exp\left(-\frac{x-\theta}{\theta}\right); x \geq \theta, \tag{10.4}$$

which is due to Ali and Woo (2002).

A general family of bivariate distributions is proposed by Morgenstern (1956) with specified marginal distributions $F_X(x)$ and $F_Y(y)$ as

$$F_{X,Y}(x,y) = F_X(x)F_Y(y)[1 + \alpha(1 - F_X(x))(1 - F_Y(y))]; \ -1 \leq \alpha \leq 1, \tag{10.5}$$

where α is the association parameter between X and Y and $F_{X,Y}(x, y)$ is the joint distribution function (df) and $F_X(x)$ and $F_Y(y)$ are the marginal distribution function (df) of X and Y respectively (see Johnson and Kotz, 1972).

Also, the probability density function (pdf) of the Morgenstern family of distribution can be given as

$$f_{X,Y}(x,y) = f_X(x)f_Y(y)[1 + \alpha(1 - 2F_X(x))(1 - 2F_Y(y))]; \ -1 \leq \alpha \leq 1. \tag{10.6}$$

The pdf of the concomitants of order statistics $Y_{[r]r}$ arising from MTBED is obtained as (see Scaria and Nair, 1999)

$$f_{Y_{[r]r}}(y) = f_Y(y)\left[1 + \alpha\left(\frac{n-2r+1}{n+1}\right)(1 - 2F_Y(y))\right]; \ -1 \leq \alpha \leq 1. \tag{10.7}$$

Now using Eqs. (10.2) and (10.3) in Eq. (10.6) we get a member of this family is Morgenstern type bivariate exponential distribution (MTBED) with the probability density function (pdf) as

$$f_{X,Y}(x,y) = \frac{\exp\left\{\left(-\frac{x-\theta_1}{a_1\theta_1}\right) + \left(-\frac{y-\theta_2}{a_2\theta_2}\right)\right\}\left[1 + \alpha\left(1 - 2\exp\left(-\frac{x-\theta_1}{a_1\theta_1}\right)\right)\left(1 - 2\exp\left(-\frac{y-\theta_2}{a_2\theta_2}\right)\right)\right]}{a_1a_2\theta_1\theta_2};$$

$$x \geq \theta_1, y \geq \theta_2, a_1, a_2 > 0, \ -1 \leq \alpha \leq 1. \tag{10.8}$$

Now the pdf of $Y_{[r]r}$ for $1 \le r \le n$ is given as (see Scaria and Nair, 1999)

$$f_{Y_{[r]r}}(y) = \frac{1}{a_2\theta_2}\exp\left(-\frac{y-\theta_2}{a_2\theta_2}\right)\left[1-\alpha\left(\frac{n-2r+1}{n+1}\right)\left(1-2\exp\left(-\frac{y-\theta_2}{a_2\theta_2}\right)\right)\right]; \tag{10.9}$$

$$y \ge \theta_2, a_2 > 0, -1 \le \alpha \le 1.$$

The mean and variance of $Y_{[r]r}$ for $1 \le r \le n$ are respectively given by

$$E\left(Y_{[r]r}\right) = \theta_2\xi_r \quad \text{and} \quad Var\left(Y_{[r]r}\right) = \theta_2^2\delta_r, \tag{10.10}$$

where

$$\xi_r = \left[1 + a_2\left(1 - \frac{\alpha}{2}\left(\frac{n-2r+1}{n+1}\right)\right)\right]$$

and

$$\delta_r = a_2^2\left[1 - \frac{\alpha}{2}\left(\frac{n-2r+1}{n+1}\right) - \frac{\alpha^2}{4}\left(\frac{n-2r+1}{n+1}\right)^2\right].$$

Stokes (1995) has considered the estimation of parameters of location-scale family of distributions using RSS. Lam et al. (1994, 1995) have obtained the BLUEs of location and scale parameters of exponential distribution and logistic distribution. Stokes (1980) has considered the method of estimation of correlation coefficient of bivariate normal distribution using RSS. Modarres and Zheng (2004) have considered the problem of estimation of the dependence parameter using RSS. A robust estimate of correlation coefficient for bivariate normal distribution has been developed by Zheng and Modarres (2006). Stokes (1977) has suggested the ranked set sample mean as an estimator for the mean of the study variate Y, when an auxiliary variable X is used for ranking the sample units, under the assumption that (X, Y) follows a bivariate normal distribution. Estimation of a parameter of Morgenstern type bivariate exponential distribution by using RSS was considered by Chacko and Thomas (2008). Barnett and Moore (1997) have improved the estimator of Stokes (1977) by deriving the best linear unbiased estimator (BLUE) of the mean of the study variate Y, based on ranked set sample obtained on the study variate Y. Lesitha et al. (2010) have considered application of RSS in estimating parameters of Morgenstern type bivariate logistic distribution. Tahmasebi and Jafari (2012) have considered upper RSS. For current references in this context the reader is referred to Sharma et al. (2016), Bouza (2001, 2002, 2005), Samawi and Muttlak (1996), Demir and Singh (2000); Singh and Mehta (2013, 2014a,b, 2015, 2016a,b,c, 2017), Mehta and Singh (2014, 2015), and Mehta (2017).

The remaining part of the chapter is organized as follows: Section 10.2.1 proposes an unbiased estimator $\hat{\theta}_2$ of the parameter θ_2 involved in Eq. (10.8) using ranked set sample mean along with its variance. In Section 10.2.2, we have derived BLUE θ_2^* of θ_2, when the association parameter α is known. We have also given the variance of BLUE θ_2^*. Section 10.2.3 deals with the problem of estimating the parameter θ_2 based on unbalanced multistage RSS. We have derived BLUE $\hat{\theta}_2^{n(r)}$ of θ_2 and obtained its variance. In Sections 10.2.4 and 10.2.5, we have discussed the problem of estimating the parameter θ_2 based on unbalanced single-stage and steady-state RSS, respectively, which are particular cases of the studies presented in Section 10.3.1. Section 10.3.2 compares the performance of the different estimators proposed in the chapter through a numerical illustration. In Section 10.4 we conclude the chapter with final remarks.

10.2 EXPERIMENTAL METHODS AND MATERIALS

10.2.1 RANKED SET SAMPLE MEAN AS AN ESTIMATOR OF θ_2

Let (X, Y) be a bivariate random variable which follows an MTBED with pdf defined by Eq. (10.8). Suppose RSS in the sense of Stokes (1977) has been carried out. Let $X_{(r)r}$ be the observation measured on the auxiliary variate X in the rth unit of the RSS and let $Y_{[r]r}$ be the measurement made on the Y variate of the same unit $r = 1, 2, \ldots, n$. Then clearly $Y_{[r]r}$ is distributed as the concomitant of rth order statistics of a random sample of n arising from Eq. (10.8). By using the expressions for mean and variances of concomitants of order statistics arising from MTBED obtained in Eq. (10.10), we propose an estimator $\hat{\theta}_2$ of θ_2 involved in Eq. (10.8) and proved that it is an unbiased estimator of θ_2.

Theorem 1.1: *Let $Y_{[r]r}, r = 1, 2, \ldots, n$ be the ranked set sample observations on a study variate Y obtained out of ranking made on an auxiliary variate X, when (X, Y) follows MTBED as defined in Eq. (10.8). Then the ranked set sample mean given by*

$$\hat{\theta}_2 = \frac{1}{n(a_2 + 1)} \sum_{r=1}^{n} Y_{[r]r}, \tag{10.11}$$

is an unbiased estimator of θ_2 and its variance is given by

$$Var\left(\hat{\theta}_2\right) = \frac{a_2^2 \theta_2^2}{n(a_2+1)^2} \left[1 - \frac{\alpha^2}{4n} \sum_{r=1}^{n} \left(\frac{n-2r+1}{n+1} \right)^2 \right]. \tag{10.12}$$

Proof: *Taking expectations of both sides of Eq. (10.11) we have*

$$E\left(\hat{\theta}_2\right) = \frac{1}{n(a_2 + 1)} \sum_{r=1}^{n} E\left(Y_{[r]r}\right) = \frac{\theta_2}{n(a_2 + 1)} \sum_{r=1}^{n} \left[1 + a_2 \left(\frac{n - 2r + 1}{n + 1} \right) \right]. \tag{10.13}$$

It is clear to note that

$$\sum_{r=1}^{n} (n - 2r + 1) = 0. \tag{10.14}$$

Using Eq. (10.14) in Eq. (10.13) we get

$$E\left(\hat{\theta}_2\right) = \theta_2.$$

Thus $\hat{\theta}_2$ is an unbiased estimator of θ_2.
The variance of $\hat{\theta}_2$ is given by

$$Var\left(\hat{\theta}_2\right) = \frac{1}{n^2(a_2+1)^2} \sum_{r=1}^{n} Var\left(Y_{[r]r}\right).$$

Now using Eq. (10.10) and Eq. (10.14) in the above sum we get,

$$\mathrm{Var}\left(\hat{\theta}_2\right) = \frac{a_2^2\theta_2^2}{n(a_2+1)^2}\left[1 - \frac{\alpha^2}{4n}\sum_{r=1}^{n}\left(\frac{n-2r+1}{n+1}\right)^2\right].$$

Thus the theorem is proved.♦

10.2.2 BEST LINEAR UNBIASED ESTIMATOR OF θ_2

In this section we provide a better estimator of θ_2 than that of $\hat{\theta}_2$ by deriving the BLUE θ_2^* of θ_2 provided the parameter α is known. Let $X_{(r)r}$ be the observation measured on the auxiliary variable X in the rth unit of ranked set samples and let $Y_{[r]r}$ be measurement made on the Y variable of the same unit, $r = 1, 2, \ldots, n$. Let $\mathbf{Y}_{[n]} = \left(Y_{[1]1}, Y_{[2]2}, \ldots, Y_{[n]n}\right)'$ and if the parameter α involved in ξ_r and δ_r is known, then proceeding as in David and Nagaraja (2003, p.185) the BLUE θ_2^* of θ_2 is obtained as

$$\theta_2^* = \left(\xi' G^{-1}\xi\right)^{-1}\xi' G^{-1}\mathbf{Y}_{[n]} \tag{10.15}$$

and

$$\mathrm{Var}\left(\theta_2^*\right) = \left(\xi' G^{-1}\xi\right)^{-1}\theta_2^2, \tag{10.16}$$

where $\xi = \left(\xi_1, \xi_2, \ldots, \xi_n\right)'$ and $G = diag(\delta_1, \delta_2, \ldots, \delta_n)$. On substituting the values of ξ and G in Eqs. (10.15) and (10.16) and simplifying we have

$$\theta_2^* = \frac{\sum_{r=1}^{n}\left(\xi_r/\delta_r\right)Y_{[r]r}}{\sum_{r=1}^{n}\left(\xi_r^2/\delta_r\right)} \tag{10.17}$$

and

$$\mathrm{Var}\left(\theta_2^*\right) = \frac{\theta_2^2}{\sum_{r=1}^{n}\left(\xi_r^2/\delta_r\right)}. \tag{10.18}$$

10.2.3 ESTIMATION OF θ_2 BASED ON UNBALANCED MULTISTAGE RANKED SET SAMPLING

Al-Saleh and Al-Kadiri (2000) have extended first the usual concept of RSS to double-stage ranked set sampling (DSRSS) with the objective of increasing the precision of certain estimators of the population when compared with those obtained based on usual RSS or using random sampling. Al-Saleh and Al-Omari (2002) have further extended DSRSS to multistage ranked set sampling (MSRSS) and shown that there is an increase in the precision of estimators obtained based on MSRSS when compared with those based on usual RSS and DSRSS. The MSRSS (in r stages) procedure is described below:

(1) Randomly select n^{r+1} sample units from the target population, where r is the number of stages of MSRSS.

(2) Allocate the n^{r+1} selected units randomly into n^{r-1} sets, each of size n^2.

(3) For each set in step (2), apply the procedure of RSS method to obtain a (judgment) ranked set, of size n; this step yields n^{r-1} (judgment) ranked sets, of size n each.
(4) Arrange n^{r-1} ranked sets of size n each, into n^{r-2} sets of n^2 units each and without doing any actual quantification, apply ranked set sampling method on each set to yield n^{r-2} second stage ranked sets of size n each.
(5) This process is continued, without any actual quantification, until we end up with the rth stage (judgment) ranked set of size n.
(6) Finally, the n identified elements in step (5) are now quantified for the variable of interest.

Instead of the judgment method of ranking at each stage if there exists an auxiliary variate on which one can make measurement very easily and exactly and if the auxiliary variate is highly correlated with the variable under study, then we can apply ranking based on these measurements to obtain the ranked set units at each stage of MSRSS. Then, on the finally selected units, one can make measurement on the study variable.

In this section we deal with the MSRSS by assuming that the random variable (X, Y) has an MTBED as defined in Eq. (10.8), where Y is the study variable and X is an auxiliary variable. In Section 10.2.2, we have considered a method for estimating θ_2 using the $Y_{[r]r}$ measured on the study variate Y on the unit having rth smallest value observed on the auxiliary variable X, of the rth sample $r = 1, 2, \ldots, n$, and hence the RSS considered there was balanced.

Abo-Eleneen and Nagaraja (2002) have shown that, in a bivariate sample of size n arising from MTBED, the concomitant of largest-order statistic possesses the maximum Fisher information on θ_2 whenever $\alpha > 0$ and the concomitant of smallest order statistic possesses the maximum Fisher information on θ_2 whenever $\alpha < 0$. Hence, in this section, first we considered $\alpha > 0$ and carry out an unbalanced MSRSS with the help of measurements made on an auxiliary variate to choose the ranked set and then estimate θ_2 involved in MTBED based on the measurements made on the study variable. At each stage and from each set we choose a unit of a sample with the largest value on the auxiliary variable as the units of ranked sets with an objective of exploiting the maximum Fisher information on the ultimately chosen ranked set sample.

Let $U_i^{(r)}, i = 1, 2, \ldots, n$ be the units chosen by the (r stage) MSRSS. Since the measurement of an auxiliary variable on each unit $U_i^{(r)}, i = 1, 2, \ldots, n$ has the largest value, we may write $Y_{[n]i}^{(r)}, i = 1, 2, \ldots, n$ to denote the value measured on the variable of primary interest on $U_i^{(r)}, i = 1, 2, \ldots, n$. Then it is easy to see that each $Y_{[n]i}^{(r)}$ is the concomitant of the largest-order statistic of n^r independently and identically distributed bivariate random variables with MTBED. Moreover $Y_{[n]i}^{(r)}, i = 1, 2, \ldots, n$ are also independently distributed with *pdf* given by (see Scaria and Nair, 1999)

$$f_{[n]i}^{(r)}(y) = \frac{1}{a_2\theta_2}\exp\left(-\frac{y-\theta_2}{a_2\theta_2}\right)\left[1+\alpha\left(\frac{n^r-1}{n^r+1}\right)\left(1-2\exp\left(-\frac{y-\theta_2}{a_2\theta_2}\right)\right)\right]; \tag{10.19}$$

$$y \geq \theta_2, a_2 > 0, \ -1 \leq \alpha \leq 1.$$

Thus the mean and variance of $Y_{[n]i}^{(r)}, i = 1, 2, \ldots, n$ are given below

$$E\left(Y_{[n]i}^{(r)}\right) = \theta_2\left[1 + a_2\left\{1 + \frac{\alpha}{2}\left(\frac{n^r-1}{n^r+1}\right)\right\}\right] = \theta_2\xi_{n^r}, \tag{10.20}$$

$$Var\left(Y_{[n]i}^{(r)}\right) = \theta_2^2 a_2^2 \left[1 + \frac{\alpha}{2}\left(\frac{n^r - 1}{n^r + 1}\right) - \frac{\alpha^2}{4}\left(\frac{n^r - 1}{n^r + 1}\right)^2\right] = \theta_2^2 \delta_{n^r}, \tag{10.21}$$

where

$$\xi_{n^r} = \left[1 + a_2\left\{1 + \frac{\alpha}{2}\left(\frac{n^r - 1}{n^r + 1}\right)\right\}\right] \tag{10.22}$$

and

$$\delta_{n^r} = a_2^2 \left[1 + \frac{\alpha}{2}\left(\frac{n^r - 1}{n^r + 1}\right) - \frac{\alpha^2}{4}\left(\frac{n^r - 1}{n^r + 1}\right)^2\right]. \tag{10.23}$$

Let $\mathbf{Y}_{[n]}^{(r)} = \left(Y_{[n]1}^{(r)}, Y_{[n]2}^{(r)}, \ldots, Y_{[n]n}^{(r)}\right)'$, then by using Eqs. (10.20) and (10.21) we get the mean vector and dispersion matrix of $\mathbf{Y}_{[n]}^{(r)}$ as

$$E\left(\mathbf{Y}_{[n]}^{(r)}\right) = \theta_2 \xi_{n^r} \mathbf{1} \tag{10.24}$$

and

$$D\left(\mathbf{Y}_{[n]}^{(r)}\right) = \theta_2^2 \delta_{n^r} \mathbf{I}, \tag{10.25}$$

where $\mathbf{1}$ is the column vector of n ones and $\mathbf{I}$ is a unit matrix of order n.

If $\alpha > 0$ involved in ξ_{n^r} and δ_{n^r} is known then Eqs. (10.24) and (10.25) together define a generalized Gass–Markov setup and hence the BLUE of θ_2 is obtained as

$$\hat{\theta}_2^{n(r)} = \frac{1}{n\xi_{n^r}} \sum_{i=1}^{n} Y_{[n]i}^{(r)} \tag{10.26}$$

with variance given by

$$Var\left(\hat{\theta}_2^{n(r)}\right) = \frac{\theta_2^2 \delta_{n^r}}{n\left(\xi_{n^r}\right)^2}. \tag{10.27}$$

10.2.4 ESTIMATION OF θ_2 BASED ON UNBALANCED SINGLE-STAGE RANKED SET SAMPLING

If we take $r = 1$ in the MSRSS method described above, then we get the usual single-stage unbalanced RSS. By putting $r = 1$ in Eqs. (10.26) and (10.27) we get the BLUE $\hat{\theta}_2^{n(1)}$ of θ_2 based on single-stage unbalanced ranked set sampling as

$$\hat{\theta}_2^{n(1)} = \frac{1}{n\xi_n} \sum_{i=1}^{n} Y_{[n]i} \tag{10.28}$$

with variance

$$\mathrm{Var}\left(\hat{\theta}_2^{n(1)}\right) = \frac{\theta_2^2 \delta_n}{n\left(\xi_n\right)^2}, \tag{10.29}$$

where, we write $Y_{[n]i}$ instead of $Y_{[n]i}^{(1)}$ and it represents the measurement on the study variable of the unit selected in the RSS. Also ξ_n and δ_n are obtained by putting $r = 1$ in Eqs. (10.22) and (10.23), respectively i.e.,

$$\xi_n = \left[1 + a_2\left\{1 + \frac{\alpha}{2}\left(\frac{n-1}{n+1}\right)\right\}\right] \tag{10.30}$$

and

$$\delta_n = a_2^2\left[1 + \frac{\alpha}{2}\left(\frac{n-1}{n+1}\right) - \frac{\alpha^2}{4}\left(\frac{n-1}{n+1}\right)^2\right]. \tag{10.31}$$

10.2.5 ESTIMATION OF θ_2 BASED ON UNBALANCED STEADY-STATE RANKED SET SAMPLING

Al-Saleh (2004) has considered the steady-state RSS by letting r go to $+\infty$. For the steady-state RSS the problem considered in having the asymptotic distribution of $Y_{[n]i}^{(r)}$ is given by

$$f_{[n]i}^{(\infty)}(y) = \frac{1}{a_2\theta_2}\exp\left(-\frac{y-\theta_2}{a_2\theta_2}\right)\left[1 + \alpha\left(1 - 2\exp\left(-\frac{y-\theta_2}{a_2\theta_2}\right)\right)\right]; \tag{10.32}$$

$$y \geq \theta_2, a_2 > 0, \ -1 \leq \alpha \leq 1.$$

From the definition of unbalanced MSRSS it follows that $Y_{[n]i}^{(\infty)}, i = 1, 2, \ldots, n$ are independent and identically distributed random variables each with *pdf* as defined in Eq. (10.32). Then $Y_{[n]i}^{(\infty)}, i = 1, 2, \ldots, n$ may be regarded as an unbalanced steady-state ranked set sample of size n. The mean and variance of $Y_{[n]i}^{(\infty)}, i = 1, 2, \ldots, n$ are given below

$$E\left(Y_{[n]i}^{(r)}\right) = \theta_2\left[1 + a_2\left\{1 + \frac{\alpha}{2}\right\}\right], \tag{10.33}$$

$$\mathrm{Var}\left(Y_{[n]i}^{(r)}\right) = \theta_2^2 a_2^2\left[1 + \frac{\alpha}{2} - \frac{\alpha^2}{4}\right]. \tag{10.34}$$

Let $\mathbf{Y}_{[n]}^{(\infty)} = \left(Y_{[n]1}^{(\infty)}, Y_{[n]2}^{(\infty)}, \ldots, Y_{[n]n}^{(\infty)}\right)'$. Then the BLUE $\hat{\theta}_2^{n(\infty)}$ based on $\mathbf{Y}_{[n]}^{(\infty)}$ and the variance of $\hat{\theta}_2^{n(\infty)}$ is obtained by taking the limits as $r \to \infty$ in Eqs. (10.26) and (10.27), respectively, and are given by

$$\hat{\theta}_2^{n(\infty)} = \frac{1}{n\left[1 + a_2\left(1 + \frac{\alpha}{2}\right)\right]}\sum_{i=1}^{n} Y_{[n]i}^{(\infty)} \tag{10.35}$$

and

$$\mathrm{Var}\left(\hat{\theta}_2^{n(\infty)}\right) = \theta_2^2\frac{a_2^2\left(1 + \frac{\alpha}{2} - \frac{\alpha^2}{4}\right)}{n\left[1 + a_2\left(1 + \frac{\alpha}{2}\right)\right]}. \tag{10.36}$$

Remark 1: *As mentioned earlier for MTBED the concomitant of smallest-order statistic possesses the maximum Fisher information on* θ_2 *whenever* $\alpha < 0$. *Therefore when* $\alpha < 0$ *we consider an* *unbalanced MSRSS in which at each stage and from each set we choose a unit of a sample with the smallest value on the auxiliary variable as the units of ranked sets with an objective of exploiting the maximum Fisher information on the ultimately chosen ranked set sample.*

Let $Y_{[1]i}^{(r)}, i = 1, 2, \ldots, n$, *be the value measured on the variable of primary interest on the units selected at the rth stage of the unbalanced MSRSS. Then it is easily to see that each* $Y_{[1]i}^{(r)}, i = 1, 2, \ldots, n$ *is the concomitant of the smallest-order statistic of* n^r *independently and identically distributed bivariate random variables with MTBED. Moreover* $Y_{[1]i}^{(r)}, i = 1, 2, \ldots, n$ *are also independently distributed with pdf given by*

$$f_{[1]i}^{(r)}(y) = \frac{1}{a_2\theta_2}\exp\left(-\frac{y-\theta_2}{a_2\theta_2}\right)\left[1-\alpha\left(\frac{n^r-1}{n^r+1}\right)\left(1-2\exp\left(-\frac{y-\theta_2}{a_2\theta_2}\right)\right)\right]; \quad y \ge \theta_2, a_2 > 0, -1 \le \alpha \le 1. \tag{10.37}$$

Clearly from Eqs. (10.19) and (10.37) we have

$$f_{[1]i}^{(r)}(y;\alpha) = f_{[n]i}^{(r)}(y;-\alpha) \tag{10.38}$$

and hence $E\left(Y_{[n]i}^{(r)}\right)$ *for* $\alpha > 0$ *and* $E\left(Y_{[1]i}^{(r)}\right)$ *for* $\alpha < 0$ *are identically equal. Similarly,* $Var\left(Y_{[n]i}^{(r)}\right)$ *for* $\alpha > 0$ *and* $Var\left(Y_{[1]i}^{(r)}\right)$ *for* $\alpha < 0$ *are identically equal. Consequently, if* $\hat{\theta}_2^{1(1)}$ *is the BLUE of* θ_2, *involved in MTBED for* $\alpha < 0$, *based on the unbalanced MSRSS observations* $Y_{[1]i}^{(r)}, i = 1, 2, \ldots, n$ *then the coefficients of* $Y_{[1]i}^{(r)}, i = 1, 2, \ldots, n$ *in the BLUE* $\hat{\theta}_2^{1(1)}$ *for* $\alpha < 0$ *is the same as the coefficients of* $Y_{[n]i}^{(r)}, i = 1, 2, \ldots, n$ *in the BLUE* $\hat{\theta}_2^{n(r)}$ *for* $\alpha > 0$. *Further we have* $Var\left(\hat{\theta}_2^{1(1)}\right) = Var\left(\hat{\theta}_2^{n(r)}\right)$ *and hence* $Var\left(\hat{\theta}_2^{1(1)}\right) = Var\left(\hat{\theta}_2^{n(1)}\right)$ *and* $Var\left(\hat{\theta}_2^{1(\infty)}\right) = Var\left(\hat{\theta}_2^{n(\infty)}\right)$, *where* $\hat{\theta}_2^{1(1)}$ *are the BLUE of* θ_2 *for* $\alpha < 0$ *based on the usual unbalanced single stage RSS observations* $Y_{[n]i}, i = 1, 2, \ldots, n$ *and* $\hat{\theta}_2^{1(\infty)}$ *are the BLUE of* θ_2 *for* $\alpha < 0$ *based on the unbalanced steady-state RSS observations* $Y_{[n]i}^{(\infty)}, i = 1, 2, \ldots, n$.

Remark 2: *If we have a situation with* α *unknown, we introduce an estimator (moment type) for* α *as follows. For MTBED the correlation coefficient between the two variables is given by* $\rho = \frac{\alpha}{4}$. *If* q *is the sample correlation coefficient between* $X_{(i)i}$ *and* $Y_{[i]i}, i = 1, 2, \ldots, n$ *then the moment type estimator for* α *is obtained by equating with the population correlation coefficient* ρ *and is obtained as (see Chacko and Thomas, 2008):*

$$\hat{\alpha} = \begin{cases} -1 & \text{if } q < -1/4 \\ 4q & \text{if } -\dfrac{1}{4} \le q \le \dfrac{1}{4}. \\ 1 & \text{if } q > 1/4 \end{cases} \tag{10.39}$$

10.3 OBSERVATIONS, RESULTS, AND DISCUSSION

10.3.1 RELATIVE EFFICIENCY

We have obtained the relative efficiencies $e_1 = RE\left(\theta_2^*, \hat{\theta}_2\right) = \frac{\mathrm{Var}(\hat{\theta}_2)}{\mathrm{Var}(\theta_2^*)}, e_2 = RE\left(\hat{\theta}_2^{n(1)}, \hat{\theta}_2\right) = \frac{\mathrm{Var}(\hat{\theta}_2)}{\mathrm{Var}\left(\hat{\theta}_2^{n(1)}\right)}$ and $e_3 = RE\left(\hat{\theta}_2^{n(\infty)}, \hat{\theta}_2\right) = \frac{Var(\hat{\theta}_2)}{Var\left(\hat{\theta}_2^{n(\infty)}\right)}$ of $\theta_2^*, \hat{\theta}_2^{n(1)}$ and $\hat{\theta}_2^{n(\infty)}$ relative to $\hat{\theta}_2$ respectively, for $n = 2(2)20, \alpha = 0.25(0.25)1.00$ and $a_2 = 1(1)5$ and these are presented in Table 10.1.

Table 10.1 The Values of $e_i's, i = 1, 2, 3$

		$a_2 = 1$			$a_2 = 2$			$a_2 = 3$		
n	α	e_1	e_2	e_3	e_1	e_2	e_3	e_1	e_2	e_3
2	0.25	1.0008	1.0005	1.0160	1.0000	1.0138	1.0559	1.0004	1.0210	1.0766
	0.50	1.0016	1.0008	1.0581	1.0009	1.0280	1.1383	1.0004	1.0415	1.1793
	0.75	1.0041	1.0013	1.1241	1.0023	1.0416	1.2463	1.0014	1.0617	1.3093
	1.00	1.0075	1.0016	1.2150	1.0037	1.0537	1.3824	1.0022	1.0803	1.4703
4	0.25	1.0000	1.0034	1.0143	1.0009	1.0281	1.0549	1.0000	1.0401	1.0751
	0.50	1.0033	1.0118	1.0521	1.0018	1.0595	1.1316	1.0014	1.0842	1.1729
	0.75	1.0083	1.0235	1.1095	1.0047	1.0946	1.2304	1.0037	1.1307	1.2928
	1.00	1.0224	1.0388	1.1880	1.0125	1.1311	1.3517	1.0083	1.1782	1.4369
6	0.25	1.0000	1.0052	1.0135	1.0000	1.0343	1.0540	1.0000	1.0489	1.0743
	0.50	1.0024	1.0182	1.0487	1.0027	1.0762	1.1296	1.0022	1.1052	1.1704
	0.75	1.0126	1.0397	1.1049	1.0070	1.1225	1.2235	1.0044	1.1652	1.2852
	1.00	1.0316	1.0628	1.1760	1.0190	1.1730	1.3382	1.0138	1.2299	1.4230
8	0.25	1.0000	1.0060	1.0127	1.0000	1.0375	1.0530	1.0000	1.0536	1.0736
	0.50	1.0033	1.0225	1.0470	1.0037	1.0863	1.1285	1.0014	1.1172	1.1687
	0.75	1.0135	1.0481	1.1004	1.0075	1.1387	1.2190	1.0045	1.1853	1.2805
	1.00	1.0355	1.0771	1.1680	1.0236	1.1992	1.3312	1.0170	1.2615	1.4154
10	0.25	1.0000	1.0079	1.0135	1.0023	1.0417	1.0545	1.0000	1.0571	1.0736
	0.50	1.0082	1.0283	1.0487	1.0023	1.0920	1.1270	1.0018	1.1249	1.1674
	0.75	1.0127	1.0532	1.0967	1.0071	1.1495	1.2161	1.0056	1.2001	1.2791
	1.00	1.0402	1.0890	1.1650	1.0248	1.2151	1.3248	1.0195	1.2812	1.4090
12	0.25	1.0000	1.0063	1.0111	1.0000	1.0431	1.0540	1.0000	1.0603	1.0743
	0.50	1.0049	1.0312	1.0487	1.0028	1.0966	1.1265	1.0000	1.1291	1.1653
	0.75	1.0152	1.0593	1.0967	1.0114	1.1597	1.2167	1.0067	1.2093	1.2767
	1.00	1.0486	1.0987	1.1640	1.0269	1.2271	1.3210	1.0211	1.2974	1.4068
14	0.25	1.0000	1.0101	1.0143	1.0000	1.0436	1.0530	1.0000	1.0614	1.0736
	0.50	1.0057	1.0292	1.0445	1.0032	1.1005	1.1265	1.0025	1.1358	1.1674
	0.75	1.0179	1.0613	1.0940	1.0099	1.1663	1.2161	1.0052	1.2156	1.2743
	1.00	1.0506	1.1048	1.1620	1.0315	1.2394	1.3216	1.0219	1.3085	1.4041

Table 10.1 The Values of $e_i^{'}s, i = 1, 2, 3$ *Continued*

		$a_2 = 1$			$a_2 = 2$			$a_2 = 3$		
n	α	e_1	e_2	e_3	e_1	e_2	e_3	e_1	e_2	e_3
16	0.25	1.0065	1.0122	1.0160	1.0036	1.0465	1.0549	1.0000	1.0628	1.0736
	0.50	1.0066	1.0300	1.0436	1.0037	1.1034	1.1265	1.0029	1.1374	1.1653
	0.75	1.0204	1.0676	1.0967	1.0076	1.1681	1.2122	1.0060	1.2227	1.2748
	1.00	1.0507	1.1091	1.1600	1.0280	1.2430	1.3158	1.0252	1.3176	1.4025
18	0.25	1.0000	1.0077	1.0111	1.0000	1.0465	1.0540	1.0000	1.0635	1.0732
	0.50	1.0074	1.0314	1.0436	1.0000	1.1027	1.1234	1.0033	1.1415	1.1666
	0.75	1.0153	1.0678	1.0940	1.0128	1.1753	1.2150	1.0067	1.2257	1.2725
	1.00	1.0574	1.1151	1.1610	1.0362	1.2533	1.3190	1.0248	1.3225	1.3988
20	0.25	1.0000	1.0064	1.0095	1.0000	1.0453	1.0521	1.0000	1.0648	1.0736
	0.50	1.0082	1.0376	1.0487	1.0046	1.1056	1.1244	1.0036	1.1425	1.1653
	0.75	1.0169	1.0728	1.0967	1.0095	1.1772	1.2133	1.0075	1.2295	1.2720
	1.00	1.0545	1.1183	1.1600	1.0302	1.2525	1.3120	1.0236	1.3289	1.3982

		$a_2 = 4$			$a_2 = 5$		
n	α	e_1	e_2	e_3	e_1	e_2	e_3
2	0.25	1.0000	1.0249	1.0887	1.0000	1.0277	1.0970
	0.50	1.0003	1.0498	1.2043	1.0003	1.0551	1.2210
	0.75	1.0006	1.0737	1.3477	1.0006	1.0818	1.3738
	1.00	1.0016	1.0964	1.5244	1.0012	1.1073	1.5611
4	0.25	1.0000	1.0474	1.0873	1.0000	1.0526	1.0957
	0.50	1.0006	1.0986	1.1975	1.0006	1.1082	1.2139
	0.75	1.0026	1.1523	1.3306	1.0018	1.1668	1.3561
	1.00	1.0066	1.2072	1.4896	1.0055	1.2265	1.5250
6	0.25	1.0000	1.0580	1.0870	1.0000	1.0635	1.0948
	0.50	1.0010	1.1223	1.1948	1.0009	1.1336	1.2110
	0.75	1.0039	1.1915	1.3233	1.0036	1.2094	1.3493
	1.00	1.0101	1.2642	1.4744	1.0093	1.2885	1.5107
8	0.25	1.0000	1.0639	1.0866	1.0012	1.0705	1.0951
	0.50	1.0013	1.1357	1.1929	1.0012	1.1486	1.2096
	0.75	1.0039	1.2150	1.3195	1.0024	1.2333	1.3440
	1.00	1.0136	1.2986	1.4661	1.0125	1.3251	1.5019
10	0.25	1.0000	1.0669	1.0856	1.0000	1.0733	1.0935
	0.50	1.0016	1.1446	1.1918	1.0015	1.1588	1.2093
	0.75	1.0033	1.2291	1.3156	1.0045	1.2507	1.3424
	1.00	1.0153	1.3212	1.4602	1.0141	1.3491	1.4959
12	0.25	1.0000	1.0700	1.0859	1.0000	1.0767	1.0938
	0.50	1.0019	1.1512	1.1914	1.0018	1.1660	1.2089
	0.75	1.0039	1.2405	1.3143	1.0054	1.2625	1.3408
	1.00	1.0185	1.3392	1.4582	1.0151	1.3671	1.4926

(*Continued*)

Table 10.1 The Values of $e_i^{\prime}s, i = 1, 2, 3$ *Continued*

		$a_2 = 4$			$a_2 = 5$		
n	α	e_1	e_2	e_3	e_1	e_2	e_3
14	0.25	1.0000	1.0718	1.0856	1.0000	1.0796	1.0945
	0.50	1.0022	1.1560	1.1910	1.0000	1.1698	1.2071
	0.75	1.0069	1.2503	1.3148	1.0042	1.2710	1.3392
	1.00	1.0168	1.3504	1.4543	1.0155	1.3793	1.4889
16	0.25	1.0000	1.0730	1.0853	1.0000	1.0807	1.0938
	0.50	1.0026	1.1604	1.1914	1.0000	1.1737	1.2068
	0.75	1.0052	1.2539	1.3109	1.0048	1.2771	1.3376
	1.00	1.0192	1.3620	1.4543	1.0177	1.3898	1.4871
18	0.25	1.0000	1.0750	1.0859	1.0000	1.0820	1.0938
	0.50	1.0029	1.1624	1.1903	1.0026	1.1782	1.2079
	0.75	1.0059	1.2617	1.3131	1.0054	1.2840	1.3384
	1.00	1.0186	1.3680	1.4509	1.0171	1.3982	1.4857
20	0.25	1.0000	1.0740	1.0839	1.0000	1.0844	1.0951
	0.50	1.0000	1.1647	1.1899	1.0029	1.1806	1.2075
	0.75	1.0033	1.2627	1.3092	1.0030	1.2852	1.3344
	1.00	1.0207	1.3750	1.4504	1.0190	1.4049	1.4843

It is observed from Table 10.1 that

- for fixed a_2, the values of $e_i{}'s, i = 1, 2, 3$ increase as n increase;
- for fixed n, the value of $e_i{}'s, i = 1, 2, 3$ increase as α increases;
- the values of $e_i{}'s, i = 1, 2, 3$ greater than "unity" for all values of (n, α, a_2), which follows that the estimators $\theta_2^*, \hat{\theta}_2^{n(1)}$ and $\hat{\theta}_2^{n(\infty)}$ are more efficient than unbiased estimator $\hat{\theta}_2$;
- when n is fixed, larger gain in efficiencies are observed for large values of α and all values of a_2;
- the values of $e_i{}'s, i = 2, 3$ increase as the value of a_2 increases. It follows that the larger gain in efficiency by using $\hat{\theta}_2^{n(1)}$ and $\hat{\theta}_2^{n(\infty)}$ over $\hat{\theta}_2$ can be obtained when the population is more heterogeneous. No trend is observed for e_1 as a_2 increases.

Therefore we conclude that the BLUE of steady-state RSS $\hat{\theta}_2^{n(\infty)}$ of θ_2 is a better estimator of $\hat{\theta}_2$, θ_2^* and $\hat{\theta}_2^{n(1)}$, respectively.

10.4 CONCLUSION

In this chapter, taking the motivation from Ebrahimi (1984, 1985), we have developed a new Morgenstern type bivariate exponential distribution (MTBED) with known coefficients of variation (CV) using the results due to Morgenstern (1956) and Scaria and Nair (1999). The mean and variance of newly developed MTBED with known CV have also been obtained. We have discussed the problem of estimating parameter θ_2 in MTBED in the presence of known CV. For estimating the parameter θ_2 of MTBED, we have derived an unbiased estimator $\hat{\theta}_2$ using ranked set sample mean and the BLUE θ_2^* based on RSS and their variances are given. We have further addressed the problem of estimating θ_2 using unbalanced RSS and its special cases known as unbalanced single-stage and steady-state RSS are also discussed. The reflective performance of the various proposed estimators of the parameter θ_2 are evaluated through numerical illustration and finally obtained that the BLUE of the steady-state RSS $\hat{\theta}_2^{n(\infty)}$ is more efficient among the estimators discussed in the chapter.

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