NOISE ESTIMATION AND IMAGE COMPRESSION USING SINGULAR VALUE DECOMPOSITION

Project report submitted in partial fulfillment of the requirement for the degree of

BACHELOR OF TECHNOLOGY

IN

ELECTRONICS AND COMMUNICATION ENGINEERING

By

Manikya Sabharwal (151088)
Divyansh Kaistha (151106)
Paras Gupta (151112)

UNDER THE GUIDANCE OF

Dr. Nafis Uddin Khan
(Assistant Professor-Senior Grade)

To

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT
MAY- 2019

TABLE OF CONTENTS

CHAPTER-3
SVD OVERVIEW

CHAPTER-4
MATLAB IMPLEMENTATION

DECLARATION BY THE SCHOLAR

I hereby declare that the work reported in the B-Tech thesis entitled **"NOISE ESTIMATION AND IMAGE COMPRESSION USING SINGULAR VALUE DECOMPOSITION"** submitted at **Jaypee University of Information Technology,Waknaghat India,** is an authentic record of my work carried out under the supervision of **Dr. Nafis Uddin Khan** . I have not submitted this work elsewhere for any other degree or diploma.

(Manikya Sabharwal   Divyansh Kaistha   Paras Gupta)

(151088   151106   151112)

Department of Electronics and Communication Engineering

Jaypee University of Information Technology, Waknaghat , India

13th May , 2019

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY

(Established by H.P. State Legislative vide Act No. 14 of 2002)

P.O. Waknaghat, Teh. Kandaghat, Distt. Solan - 173234 (H.P.) INDIA

Website: www.juit.ac.in

Phone No. (91) 01792-257999

Fax: +91-01792-245362

CERTIFICATE

This is to certify that the work reported in the B.Tech project report entitled **"NOISE ESTIMATION AND IMAGE COMPRESSION USING SINGULAR VALUE DECOMPOSITION"** which is being submitted by **Manikya Sabharwal(151088), Divyansh Kaistha(151106) and Paras Gupta(151112)** in fulfillment for the award of Bachelor of Technology in Electronics and Communication Engineering by the Jaypee University of Information Technology, is the record of candidate's own work carried out by him/her under my supervision. This work is original and has not been submitted partially or fully anywhere else for any other degree or diploma.

Dr. Nafis Uddin Khan

Assistant Professor (Grade - Senior)

Department of Electronics & Communication Engineering

Jaypee University of Information Technology, Waknaghat.

ACKNOWLEDGEMENT

This Project was successful largely due to guidance of many wonderful people who have given valuable advice to make this project a success. I would like to place my best regards to Dr. M. J. Nigam, Head of Department, Electronics and Communication Engineering, for his precious advice throughout the duration of project. I might want to thanks our undertaking guide Dr. Nafis Uddin Khan, Assistant Professor, Department of Electronics and Communication Engineering for giving us the correct course and honest to goodness print with about the undertaking. Without his direction and bolster this would not have been conceivable.

I likewise want to express gratitude toward Mr. Mohan, Lab specialist, Department of Electronics and Communication Engineering – JUIT .

Last however not the base, I generously welcome each and every one of those individuals who have helped me co-ordinate in making this undertaking a win. In this amazing condition, I should need to thank the diverse staff individuals, both instructing and non-preparing, which have developed their favorable help and energized my undertaking.

LIST OF ACRONYMS AND ABBREVIATIONS

ACRONYM	MEANING
SVD	Singular Value Decomposition
DCT	Discrete Cosine Transform
DWT	Discrete Wavelet Transform
PSNR	Peak Signal to Noise Ratio
MSE	Mean Squared error
SV	Singular Value
SC	Singular Vector
LSC	Left Singular Vector
RSC	Right Singular Vector
CR	Compression Ratio
MATLAB	Matrix laboratory

LIST OF FIGURES

ABSTRACT

We require to transmit and store many images in our day to day life through different applications .Less is the size of image , lower is the price required for transmission and storage. So we require to go for various data compression techniques to minimize the amount of space required by the image . The possible way to tackle this kind of problem is by using (SVD) where an whole image is considered as a matrix and then various kind of operations are performed on that matrix. One good approach to do this is to apply Singular Value Decomposition (SVD) on the matrix obtained from that image. It works as, digital image is given to SVD. The image can be refactored into three metrices by using SVD. The process of refactoration is carried out using singular values, the orignal image is represented by smaller values, which results in reducing of the storage space required by the image. Our goal here is to get a low size image while getting the important features which explain the original image properly but in reduced size. SVD can be used to any arbitrary, square, reversible and non-reversible matrix of m × n size.

CHAPTER 1
INTRODUCTION

1.1 Objectives

The main aim of the project is to demonstrate the usage of Singular Value Decomposition (SVD) in Image Compression applications. This report consist of several sections that discuss about different aspects of the SVD-based image processing scheme. First, a straight –forward compression and decompression scheme is used to show the basic idea of reducing storage requirement with SVD. Second, same image is compressed using Discrete Cosine Transform (DCT) a Lossy Compression Technique and the same image is compressed using Discrete Wavelet Transform (DWT) a Lossless Compression Technique.

1.2 Motivation

In daily applications we can notice that we usually transmit and store many images.If the size of the images is small ,directly the cost related to transmission and storage of that image will be less.So there is a way to reduce the storage and transmission cost i.e Data Compression Techniques. SVD provides ideal energy compaction as compared to DCT and it perform better for images having high pixel value.

1.3 MATLAB

MATLAB is the platform used for writing the various codes. MATLAB known as the Matrix Laboratory widely used as a multi-paradigm numerical computing platform and it is a fourth-generation programming language. It allows matrix calculations, plotting of functions and data, implementation of algorithm, develop user interfaces and interface with programs written in different languages (i.e C, C++, Java, Fortran and Python).

1.4 Layout

I. This project report is consist of four main chapters:

II. Chapter One: It provides an overview of the project. Contains the background about the project, objectives, motivation, scope of the project and finally the Project layout.

III. Chapter Two: Provides brief knowledge about the SVD. It contains Literature Review comprising of Introduction, SVD, SVD image properties, Basic concepts of image compression.

IV. Chapter Three: SVD overview.

V. Chapter Four: Provides us with practical implementation of codes of MATLAB and their respective outputs.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

The best matrix decomposition in a least square sense is SVD . It helps to combine the large amount of signal energy into as few coefficients as possible. One of the most stable and effecient method to divide the system into a set of linearly independent components is Singular value decomposition (SVD, each part is having their own energy contribution. SVD is a numerical technique of image compression used to diagonalize image matrices. SVD on other hand has vast number of advantages, some of the advantages like maximum energy packing which is used in image compression, also it has the ability to manipulate the image on two distinctive subspaces (data and noise subspaces), which have application like noise filtering and watermarking . Each of these applications shows us key properties of the SVD and give a lot of information about them. It is also used in solving problems like least squares problem, computing pseudo- inverse of a matrix and multivariate analysis. SVD is strong and reliable orthogonal matrix decomposition method, due to its speculative and stability reasons, SVD is becoming one of the key techniques to be used in signal processing area . SVD has the capability to be multivariate in local statistics of an image . Many SVD properties are attractive and are still not fully utilized in day to day applications. Under SVD image processing techniques were focused for compression of image, watermarking and quality measure of images.

2.2 Singular Value Decomposition (SVD)

SVD is a factorisation of a rectangular matrix that can be real or complex ,comparable to Hermitian square matrices using eigenvectors as basis. SVD is a firm and an effectual technique that is used to divide the system into sets of linearly independent components, each of the component is having its own energy contribution . A general digital Image X of size M X N, with $M \geq N$, is represented by SVD as;

$$[X]^{N}_{M} = {}_{M}[U]^{M}\,[S]^{N}_{M}\,[V]^{N\,T}_{N}$$

$$U = [u_1, u_2, \ldots u_m], \qquad V = [v_1, v_2, \ldots v_n]$$

$$S = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & O & \\ & & & \sigma_n \end{bmatrix}$$

Where U is an M X M orthogonal matrix, V is an N x N orthogonal matrix, and S is an M X N matrix with all the diagonal elements representing the singular values, s_i of X. We use subscript T to describe the transpose of the matrix. The columns of the orthogonal matrix U gives us left singular vectors, and the columns of the matrix V gives right singular vectors. The left particular vectors (LSCs) of X are eigenvectors of XXT and the correct solitary vectors (RSCs) of X are eigenvectors of XTX. Every particular esteem (SV) determines the brightness of a picture layer while the comparing pair of singular vectors (SCs) indicates the geometry of the picture. U and V are unitary symmetrical matrices i.e(the aggregate of squares of every section is unity and every one of the segments are uncorrelated) and S is a corner to corner grid (just the main slanting has non-zero qualities) of diminishing singular values. Since SVD is utilized to expand the biggest particular values, the primary eigen picture gives us the example that represents the best values of the difference covariance structure.

2.3 SVD Image Properties

SVD is a strong orthogonal matrix decomposition method. Due to SVD theoretical and security reasons, it turns out to be increasingly more famous in signal handling territory. SVD is an appealing mathematical change for image processing. SVD has unmistakable properties in image processing. SVD properties are completely used in picture preparing, regardless others needs more examination and added to. A few SVD properties are very blessed for pictures, for example, its most extreme energy packing, taking care of least squares issue, registering pseudoinverse of a network and multivariate examination. A key property of SVD is its connection to the position of a framework and its capacity to inexact matrices of a given rank. All Digital pictures are frequently spoken to by low position frameworks and, thusly, it can be portrayed by a whole of a moderately little arrangement of eigenimages. This idea rises the controlling of the signal as two distinct subspaces. A few theories will be given and checked in the accompanying segments.

SVD Subspaces:

SVD is made up of two orthogonal dominant and subdominant subspaces. This compares to parcel the M-dimensional vector space into predominant and subdominant sub spaces .This function of SVD is used in apllications like noise separating and watermarking.

SVD architecture:

For SVD decay of a picture, singular value (SV) indicates the luminance of a picture layer while the relating pair solitary vectors (SCs) determine the geometry of the picture layer. The biggest object segments in a picture discovered utilizing the SVD for the most part connect to eigen pictures connected with the biggest solitary qualities, while image noise relates to eigen pictures connected with the SVs.

SVD Multiresolution:

SVD has the most extreme energy packing among the various changes utilized in pressure. In numerous applications, it is important to acquire a factual characterization of a image at different goals. SVD breaks down a network into symmetrical segments with which best sub rank approximations may be acquired. With the multiresolution SVD, the accompanying significant attributes of a picture might be estimated, at every one of the various dimension of goals: isotropy, spercity of primary parts, self-similitude under scaling, and goals of the mean squared blunder into various important segments.

2.4 Basic Concepts Of Image Compression

The principle point of the picture compression is to reduce the unimportant and redundant piece of the picture data so as to store and transmit information in a proper form. This can be easily accomplished through the strategy for limiting the quantity of bits required to speak to every pixel of a picture. This easily decreases the space required to store pictures and encourages quick transmittion of images.

2.4.1 Data Redundancy

The field for studying various techniques for minimising the total number of bits required to represent a picture is digital image compression. This can be achieved by removing various types of redundancy that alreadtexist in the image. There are three basic type of redundancies.

- Coding Redundancy
- Inter-pixel Redundancy
- Psycho-visual Redundancy

(i) Coding Redundancy: The uncompressed image is coded with each pixel by giving a fixed length code word. By utilizing some factor length code plans, for example, Huffman coding and number-crunching coding, compression can be achieved.

(ii) Inter-pixel Redundancy: It is a repetition identified with measurable conditions among pixels, particularly between neighboring pixels. Data is unnecessarily imitates in the related pixels.

(iii) Psycho-visual Redundancy: It is an excess relating to various sensitivities of human eyes to all picture signals. In this way, taking out some less senstive data from the picture dependent on our visual preparing might be adequate.

2.4.2 Types Of Image Compression

Image compression techniques are divided into two categories:

a) Lossless image compression: In lossless compression the first information is completely recreated from the packed information. It utilize all the data in the first picture while pressure, so when the picture is decompressed, it will be actually same to the first picture. Pictures in which geometric shapes are moderately basic can be considered for lossless picture pressure.

b) Lossy Compression Techniques: In lossy pressure the remaking of a image is just an estimation of the first information. Certain amount of information is lost. It is commonly used to compact interactive media information. A preferred position of this system is that it considers higher compression proportion than the lossless method

Chapter 3
SVD Overview

3.1 Singular Value Decomposition

SVD is a tool used to compress image .It can be used to compress both coloured and black and white images. SVD converts the image matrix into three vectors S , V and D where S contains Singular value and is a diagonal matrix of size n * m . The image matrix is represented by using lower singular or information containing values .Therefore only the important information is retained rest all the unimportant information or reduntant information is removed leading to compression of the image .So the SVD processed image requires lesser size compared to the original image which contains redundant information. SVD is based on selecting the desired values of K , if a very low k value is selected then all the image data that might be important is lost and very high value of k results in the no compression of the image .Therefore, an adequate value of k has to be choosen , k values are actually the eigen values used for compressing and de-compression of the image .It is a lossy compression technique some data is lost and could not be retrieved making the choice of k values an important factor. The value of k depends from application to applications ,some applications might require very high commpression (i.e information is not very important) where as in some cases no information could be lost (i.e compression ratio and k has to be choosen very carefully).therefore if k value is equal to the rank of the image same original image could be formed where as with decrease in k value results in decrease of image quality the image, the reconstructed image is closer to the original image. As the value of k decreases from the rank . Another factor is compression ratio , if the compression ratio is high i.e image much more compressed leading to lower image quality whereas if compression ratio is small , image quality is high and image is not too much compressed.

There are many other compression techniques too such as JPEG i.e Joint Photographic Expert Group which usses DCT i.e Discrete Cosine transform , as the name tells it is based on frequency domain and takes the cosine transform for compressing the image , it is one of the most widely used technique for image compression. In this report we will be using SVD mainly but its comparison is done with some other widely use techniques as well such as DCT which is known for giving high energy compaction but our propossed SVD performs better than DCT for images of high standard deviation i.e with high pixel quality.

3.2 Mathematical steps to calculate SVD of a matrix

1. Convert the image into a matrix A

2.Then calculate A*AT and AT * A where AT is A Transpose

3. Find the eigen values using these two matix ,i.e form A * AT will be used to find eigen values for U

$(A * AT – ƛ*I) * \ddot{x} = 0.$

4. Form AT * A will be used to find eigen values for V

$(AT * A – ƛ * I) * \ddot{x} =0.$

5 .Eigen vector by its value (i .e magnitude) is used to form the data of U and V.

6. Square of the eigen values is used to form singular values which are arranged in decesending order to form a diagonal matrix S, such that : $\sigma 1 \geq \sigma 2 \geq \ldots \geq \sigma r \geq 0$

3.3 PROPERTIES OF SVD

Properties of S , V and U are as follows:

- S, U and V are multiplied in such a manner that they equate to A .

  As $A=U * S * V^T$.

2 U is an orthogonal matrix of dimensions n * k and follows the orthogonal properties of matices .

2 V is also is an orthogonal matrix of dimensions k * k and follows the orthogonal properties of matices .

3 Whereas S is an orthogonal matrix of dimensions k * k .It is also a diagonal matrix with eigen values values arranged diagonally in a descending manner and with no negative values having singular values, s1, s2, ... s_K, on the diagonal.

4 Singular values of matrix S are arranged in descenging orger such that

$s1 \geq s2 \geq . . . \geq s_K \geq 0$.

3.4 PERFORMANCE PARAMETERS

Results of various compression techniques are compared with the help of some parameters and these parameters define that which technique is better than the other technique and how well they perform under different conditions .It is also used to measure that to what degree an image is compressed. Some of these parameters can be stated as follow:

1.Compression Ratio: Ratio of original image size with respect to the size required to store the compressed image is known as Compression Ratio is and is given by the formula:

Compression Ratio = m1* n1 / (k1 * (m1 + n1 + 1))

It measures the degree to which the image can and is compressed.

2.Mean Square Error(MSE): MSE is used to measure the increase or decrease in image quality after compression with respect to the quality of the image before compression. Mathematically , it is the sq. of pixel value difference beteween the original image and the image that has been compressed with an average over the entire image ..

Mathematically,

$$MSE = [\sum \sum g(x , y) – g'(x , y)] / (m \times n)$$

3.Peak Signal to Noise Ratio (PSNR): Ratio between the max signal power to the power of that of the noise that has been added and results in the distortion of the image is known as Peak signal to Noise Ratio (PSNR) . The Image has the maximum power when it is not compressed i.e the original image wherase noise is the extra signal or values added to it during the compression .

Mathematically, PSNR is given by

$$PSNR = 10 * \log10 [255 / \sqrt{MSE}]$$

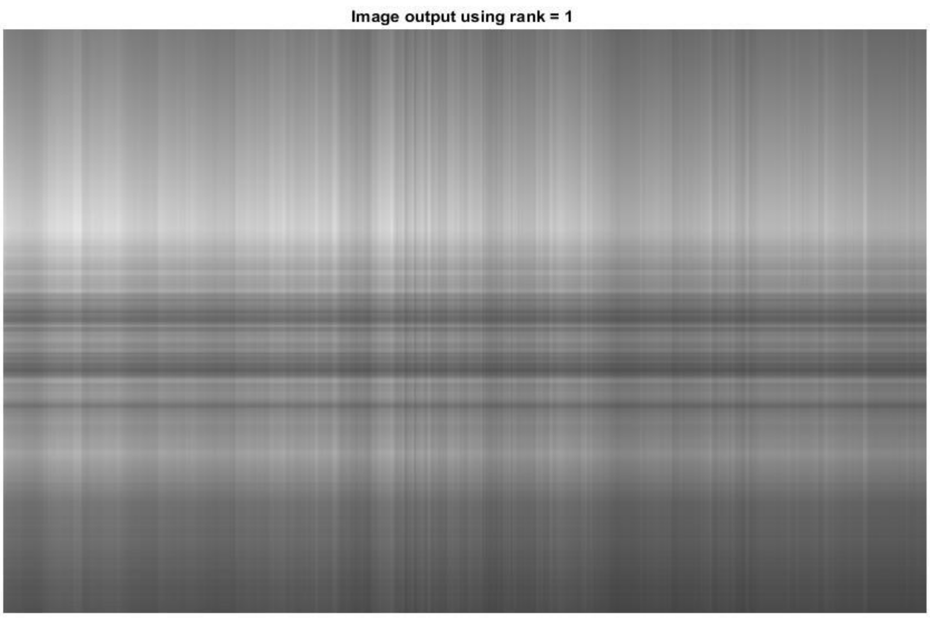
CHAPTER 4
MATLAB IMPLEMENTATION

4.1 Image Compression using different techniques

4.1.1 SVD(Lossy Compression) using different rank values

```
close all
clear all
clc

inImage=imread('C:\Users\Paras Gupta\Desktop\JUIT3.jpg');
inImage=rgb2gray(inImage);
inImageD=double(inImage);

[U,S,V]=svd(inImageD);

dispEr = [];
numSVals = [];
for N=5:70:300

    C = S;

      C(N+1:end,:)=0;
    C(:,N+1:end)=0;

    D=U*C*V';
```

```
figure;
buffer = sprintf('Image output using rank = %d ', N)
imshow(uint8(D));
title(buffer);
error=sum(sum((inImageD-D).^2));

   dispEr = [dispEr; error];
   numSVals = [numSVals; N];
end

figure;
title('Error in compression');
plot(numSVals, dispEr);
grid on
xlabel('for different values of K i.e rank');
ylabel('Error between compress and original image');
```



Output_1

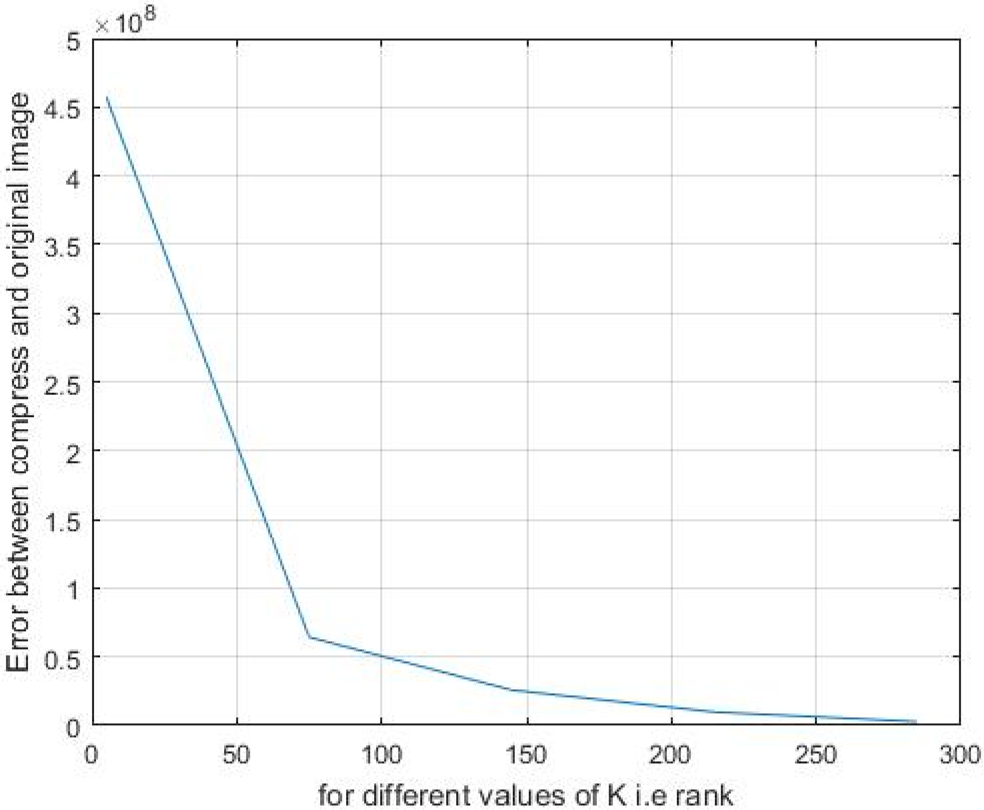
Output_2

Output_3

Output_4

Output_5

Output_6



Output_7

4.1.2 DCT (Lossy Compression) using different Coefficient values

```
clc
close all
im=imread('C:\Users\Paras Gupta\Desktop\JUIT3.jpg');
im = double(im)/255;
im = rgb2gray(im);
subplot(211)


imshow(im)
title('Original image');
img_dct=dct2(im);
img_pow=(img_dct).^2;
img_pow=img_pow(:);
[B,index]=sort(img_pow);
B=flipud(B);
index=flipud(index);
compressed_dct=zeros(size(im));
coeff = 20000;
for k=1:coeff
compressed_dct(index(k))=img_dct(index(k));
end
im_dct=idct2(compressed_dct);
subplot(212)
imshow(im_dct)
title('DCT Compress Image');
```

Original image

DCT Compress Image with coeff 200

Output_9

Original image

DCT Compress Image with coeff 1000

Output_9

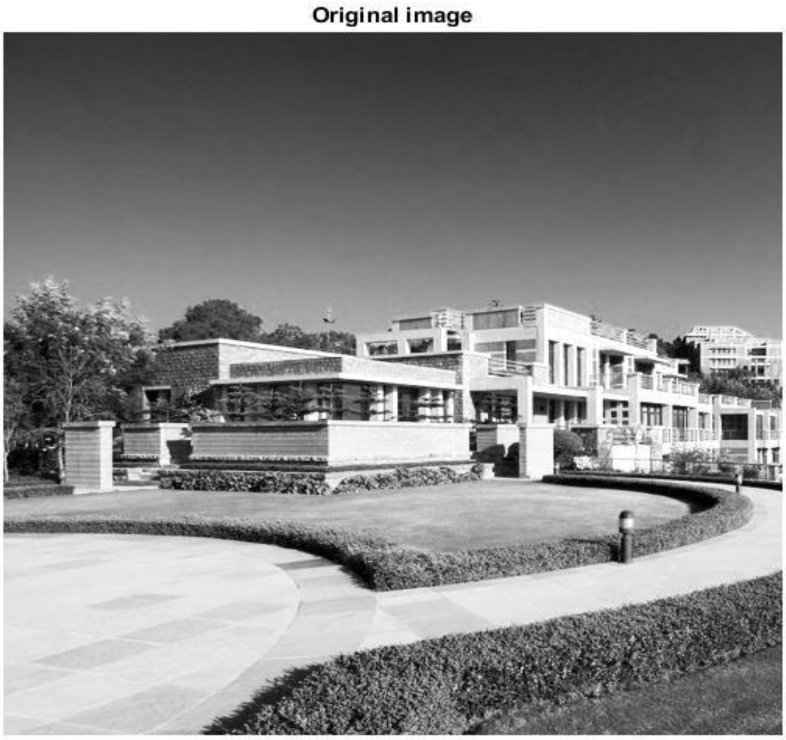
Output_10

Output_11

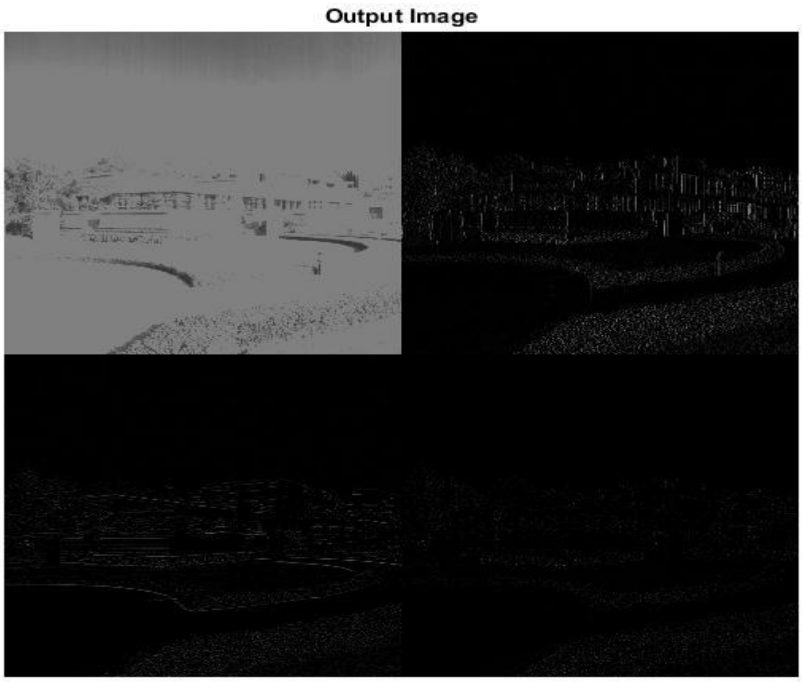
Output_12

4.1.3 DWT (Lossless Compression)

```
X=imread('C:\Users\Paras Gupta\Desktop\Jp.jpg');
X=X(1:500,1:500);
figure;
imshow(uint8(X));
f=X;
[m, n]=size(X);
k=n/2;
for i=1:1:m
for j=1:1:k
f(i,j) =X(i,2*j)+X(1,2*j-1);
f(i,j+k) = X(i,2*j) - X(i,2*j-1);
end
end

X = f;


 k=m/2;
 for j=1:1:n
 for i=1:1:k
 X(i,j) = uint8((f(2*i,j)+f(2*i-1,j))/2);
 X(i+k,j)= uint8((f(2*i,j)-f(2*i-1,j))/2);
 end
 end

 title('Original image');
 figure; imshow(X(1:500,1:500));
 title('Output Image');
 figure; imshow(Y(1:500,1:500));
```



Output_13



Output_14

4.2 Image Compression On Colored Images For Different Rank Values

```
close all
iptsetpref('ImshowBorder','tight')
clear Q
L=imread('C:\Users\Paras Gupta\Desktop\pic6.jpg');
imshow(L)
L1=L(:,:,1);
L2=L(:,:,2);
L3=L(:,:,3);
I1=im2double(L1); I2=im2double(L2); I3=im2double(L3);
figure
imshow(I1)
figure
imshow(I2)
figure
imshow(I3)

[u1,s1,v1]=svd(I1);
[u2,s2,v2]=svd(I2);
[u3,s3,v3]=svd(I3);

C1=zeros(size(I1));
C2=zeros(size(I2));
C3=zeros(size(I3));

k=290;
for j=1:k
   C1=C1+s1(j,j)*u1(:,j)*v1(:,j).';
end
for j=1:k
```

```
    C2=C2+s2(j,j)*u2(:,j)*v2(:,j).';
  end
  for j=1:k
    C3=C3+s3(j,j)*u3(:,j)*v3(:,j).';
  end
  figure
  imshow(C1)
  figure
  imshow(C2)
  figure
  imshow(C3)

  C1(k)=1;
  C2(k)=1;
  C3(k)=1;

  R1=im2uint8(C1);
  R2=im2uint8(C2);
  R3=im2uint8(C3);


Q(:,:,1)=R1;
Q(:,:,2)=R2;
Q(:,:,3)=R3;
figure
imshow(Q,[])
```

Original Image (Size : 21kb , Dimensions : 300*200)

Rank : 50

Output_15

Red Layer

Output_16

Green Layer

Output_17

Blue Layer

Output_18

SVD Red Layer

Output_19

SVD Green Layer

Output_20

SVD Blue Layer

Output_21

Compressed Image (Size : 13.7kb)

Output_22

Original Image (Size : 21kb , Dimensions : 300*200)

Rank : 100

Output_23

Red Layer

Output_24

Green Layer

Output_25

Blue Layer

Output_26

SVD Red Layer

Output_27

SVD Green Layer

Output_28

SVD Blue Layer

Output_29

Compressed Image (Size : 15.4kb)

Output_30

Original Image (Size : 21kb , Dimensions : 300*200)

Rank : 200

Output_31

Red Layer

Output_32

Green Layer

Output_33

Blue Layer

Output_34

SVD Red Layer

Output_35

SVD Green Layer

Output_36

SVD Blue Layer

Output_37

Compressed Image (Size : 16kb)

Output_38

Original Image (Size : 80.9kb , Dimensions : 960*543)

Rank : 100

Output_39

Red Layer

Output_40

Green Layer

Output_41

Blue Layer

Output_42

SVD Red Layer

Output_43

SVD Green Layer

Output_44

SVD Blue Layer

Output_45

Compressed Image

Output_46

Original Image (Size : 80.9kb , Dimensions : 960*543)

Rank : 300

Output_47

Red Layer

Output_48

Green Layer

Output_49

Blue Layer

Output_50

SVD Red Layer

Output_51

SVD Green Layer

Output_52

SVD Blue Layer

Output_53

Compressed Image

Output_54

Original Image (Size : 80.9kb , Dimensions : 960*543)

Rank : 500

Output_55

Red Layer

Output_56

Green Layer

\

Output_57

Blue Layer

Output_58

SVD Red Layer

Output_59

SVD Green Layer

Output_60

SVD Blue Layer

Output_61

Compressed Image

Output_62

CONCLUSION

Singular Value Decomposition is a straightforward, robust and dependable procedure. This SVD procedure gives steady and compelling technique to part the picture framework into a lot of straightly autonomous networks. SVD gives great compression proportion and furthermore a useful answer for picture compression issue. The results shown above clearly displays the compressed outputs for different r values. Thus, selection of r value plays a crutial role in this SVD based image compression technique.

I. In the world of image and data compression SVD's applications arevery useful and resource-saving.

II. The portions of the matrix are arranged in order ofimportance with the help of SVD. The most important unit eigenvectors are produced by most important singular values

III. Large portion of the matrix can eliminated without losing quality

IV. We should choose a optimum vale of 'k' , which will convey the information contained in the original image, and has an file size required according to our demand.

TIME FRAME DISTRIBUTION

	P ROJECT SCHEDULE							
	SEMESTER 7				SEMESTER 8			
Months	Aug	Sep	Oct	Nov	Feb	Mar	Apr	May
Project								
Project Proposal								
Background Research								
Proposal Preparation								
Proposal presentation								
Implementation								
Denoising								
Design								
Coding								
Implementation								
Write up								
Submit Project								
Presentation								

REFERENCES

[1] H R Swathi, Shah Sohini, Surbhi and G Gopichand " Image compression using singular value decomposition", IOP conference series : Material Science and Engineering , vol.263(2017).

[2] Ruey-Feng Chang,Wen-Tsuen Chen,and Jia-Shung Wang,"A Fast Finite-State Algorithm for Vector Quantization Design", IEEE Transaction on Signal Processing, Vol.40, No.1, January 1992.

[3] Hong Wong, Ling LU, DA-Shun Que, Xun Luo,"Image Compression Based on Wavelet Transform and Vector Quantization" IEEE proceedings of the First international Conference on Machine Learning and Cybernetics, Beijing, November 2002.

[4] Y.W. Chen, Vector Quantization by principal component analysis, M.S. Thesis, National Tsing Hua University, June, 1998.

[5] M.F. Barnsley and L.P. Hurd, Fractal Image Compression, AK Peters, Ltd. Wellesley, Massachusetts, 1993.

[6] A.E. Jacquin, Imagecoding basedon a fractaltheory of iterated contractive image transformations. IEEE Trans. on Image Processing, vol. 1, 18-30, 1992.

[7] H.S. Chu, A very fast fractal compression algorithm, M.S. Thesis, National Tsing Hua University, June, 1997.

[8] Y. Fisher, Editor, Fractal Image Compression: Theory and Applications, Springer-Verlag, 1994.