ANALYSIS OF CHIRP AND MULTI-COMPONENT SIGNALS USING SIGNAL PROCESSING METHODS

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By

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DECLARATION BY THE SCHOLAR

I hereby declare that the work reported in the M-Tech thesis entitled **"Analysis of chirp and multi-component signals using signal processing methods"** submitted at **Jaypee University of Information Technology, Waknaghat, India,** is an authentic record of my work carried out under the supervision of **Prof. Sunil Datt Sharma.** I have not submitted this work elsewhere for any other degree or diploma.

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SUPERVISOR'S CERTIFICATE

This is to certify that the work reported in the M-Tech. thesis entitled "**Analysis of chirp and multi-component signals using time frequency tools**", submitted by **Jitain Sharma** at **Jaypee University of Information Technology, Waknaghat, India,** is a bonafide record of his / her original work carried out under my supervision. This work has not been submitted elsewhere for any other degree or diploma.

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Date Jitain Sharma

ABSTRACT

Signals are represented by function in time or space. These are classified as stationary and non stationary signals. Stationary signals are those signals in which frequency, amplitude and phase do not change with time, whereas these spectral components change with time in non stationary signals. Chirp and multi-component signals are the examples of non stationary signal. Chirp signals contain frequency which either increases or decreases with time. On the hand, multi-component signals have more than one frequency component. The importance of extracting information out of chirp or multi-component signals lies in the fact that in several research fields, for example speech recognition, medical fields, radar signals, micro seismic signals, micro doppler signal detection, instantaneous frequency estimation etc., the signals are often multi-component, chirp or both. So to extract the information at a particular time instant corresponding to a particular frequency, therefore various time frequency tools have been developed for the analysis of chirp and multi-component signals such as Short time fourier transform (STFT), Wavelet transform (WT), Chirplet Transform (CT), Polynomial Chirplet Transform (PCT) and S-transform (ST). As it is known that STFT has a limitation of fixed window length, therefore to overcome this problem of STFT, WT has been reported. WT does not have the direct relationship with frequency. Therefore to provide direct relationship with frequency, S transform has been introduced and it also provides phase information. The S-transform can give information about the phase of each frequency, but it degrades the time resolution at lower frequencies and degrades frequency resolution at higher frequencies. So, modified S-transform has been proposed to improvise the performance of S-transform. In this dissertation, the performance of the proposed method has been compared with STFT, ST, CT and PCT for multi-component signals. The implementation of these signal processing tools have been done using MATLAB software.

LIST OF ACRONYMS AND ABBREVIATIONS

CT	Chirplet Transform
DFT	Discrete Fourier Transform
FFT	Fast Fourier Transform
FT	Fourier Transform
IF	Instantaneous Frequency
PCT	Polynomial Chirplet Transform
STFT	Short Time Fourier Transform
ST	Stockwell Transform
TFA	Time frequency Analysis
WT	Wavelet Transform

LIST OF FIGURES

CHAPTER 1
INTRODUCTION

1.1 OVERVIEW

In our daily lives, most of the things that we deal with are images, velocity of a fluid, speech etc. These all are represented by functions in space or time called signals. Signals are classified into either stationary or non stationary categories. Stationary signals are constant in their statistical behaviour over time. It means a signal is said to be stationary if its frequency or spectral contents are not changing with respect to time. After a period of time these signals look the same as before. Its overall level will be same and its amplitude distribution and standard deviation would be same. Rotating machinery generally produces stationary signals. Non stationary signals are varying in their statistical parameters over time [2]. It means these signals will have many frequency contents and these components change continuously with time. Under non stationary signal there exists a signal called chirp. Chirp is a signal in which the frequency increases or decreases with time. These signals provide importance in the applications like RADAR, SONAR, Ultra short laser, Linear Frequency Modulation etc. That is why chirp signals have become our region of interest. There are several time frequency analysis tools to get information out of these chirp signals. For e.g. Chirplet transform, Polynomial chirplet transform etc.

On the basis of components signals are also classified as mono-component and multi-component signals. Mono-component signals contain single frequency component and multi-component signals have more than one frequency component [1]. In multi-component signals, component separation is very important to analyse a signal. As in chirp signals, Time frequency analysis plays an important role in this case also. STFT and S Transform are common time frequency tools used to analyse multi-component signals [3].

In many cases, it is more meaningful to look at frequencies of the functions. For example, sound is given by air pressure as a function of time. But for de-noising the high frequency noises, it is necessary for engineers to read the information in frequency domain for picking up a suitable filter.

The time-frequency analysis was first introduced by the French mathematician Joseph Fourier in the year 1822 which is known as Fourier transform. With the development of the time-frequency analysis technique, Dennis Gabor came up with the idea of Short-time fourier transform (STFT) in 1946. And in the last 20 years, there are several tools developed in the stream of time-frequency analysis [11].

Time-frequency tools provide efficient methods to characterize the time-frequency pattern of non stationary multi-component chirp signals [8]. Time-frequency representations obtained by using TF tools map a one dimensional signal in time as a function of two-dimensional signal of time and frequency. Therefore give a powerful insight into the complex structure of the signal consisting of several components.

1.2 MOTIVATION

The mathematical motivation for studying the time frequency tools come from the fact that functions and their transform representation are so much related to each other such that they can be understood better by studying them jointly, as a two-dimensional object, rather than separately.

The practical motivation for time–frequency analysis is that fourier analysis considers that the length of signals is infinite in time and are periodic. But in practice, many signals are of short duration and also change considerably with time. This is poorly illustrated by traditional methods, which motivates for the origin of time frequency analysis.

1.3 DIFFERENT METHODS

In the history of time-frequency analysis, Fourier transform has the great significance [6]. Since the origin of Fourier transform, a new chapter of the time-frequency analysis started. In theory, Fourier transform is used to be a tool to convert a signal expressed from time domain to frequency domain. It helps to represent any periodic functions as an infinite sum of periodic complex exponential functions. A time signal is decomposed into its different frequency components by calculating the fourier integral. Mathematically, Fourier transform of a function $x(t)$ is as given below

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \tag{1}$$

In above equation $x(t)$ is a time domain signal, $X(f)$ is the Fourier transform of an integrable function, f is the value of the angular frequency, j is the imaginary number.

To compute the $X(f)$, it is needed to integrate $x(t)$ over all time. Mathematically, due to both sine waves and cosine waves are significant in the whole time domain, so fourier transform is available at any given time. This means that during the whole intervals, Fourier transform cannot provide simultaneous time, frequency localization and the fourier coefficients (amplitude) which are depended on the behaviour of the function.

Inverse fourier transform is given by

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(f)e^{j2\pi f t}df \tag{2}$$

Motivation for the Fourier transform comes from the study of fourier series. In the study of fourier series, complicated but periodic functions are written as the sum of simple waves mathematically represented by sines and cosines. The Fourier transform is an extension of the fourier series that results when the period of the represented function is lengthened and allowed to approach infinity. Due to the properties of sine and cosine, it is possible to recover the amplitude of each wave in a fourier series using an integral. There is a close connection between the definition of fourier series and the fourier transform for functions f that are zero outside an interval. For such a function, we can calculate its fourier series on any interval that includes the points where f is not identically zero. The fourier transform is also defined for such a function. As we increase the length of the interval on which we calculate the fourier series, then the fourier series coefficients begin to look like the fourier transform and the sum of the fourier series of f begins to look like the inverse fourier transform.

Here is an example for two signals and their spectral analysis (FFT), Figure 1.1 is the presentation of two signal that have both sine waves with frequency 25Hz and 50Hz happen at the same time. Figure 1.2 is a step-change signal that changes its frequency from 25Hz to 50Hz at the middle of the time.

Looking at both the examples, it can be seen that two different signal obtain the same spectral analysis. In Figure 1.1 and Figure 1.2, it can be seen that peaks happen on the same frequencies which mean they have the same frequency contents, but obviously, fourier transform cannot recognize the difference between two signals that many different contents happened at the same time or at the different times. We can’t estimate the original signals through the results of Fourier transform in non-stationary signals (Stationary signals are signals which frequency contents are stable and don’t depend on time, non-stationary signals are the opposite of them).

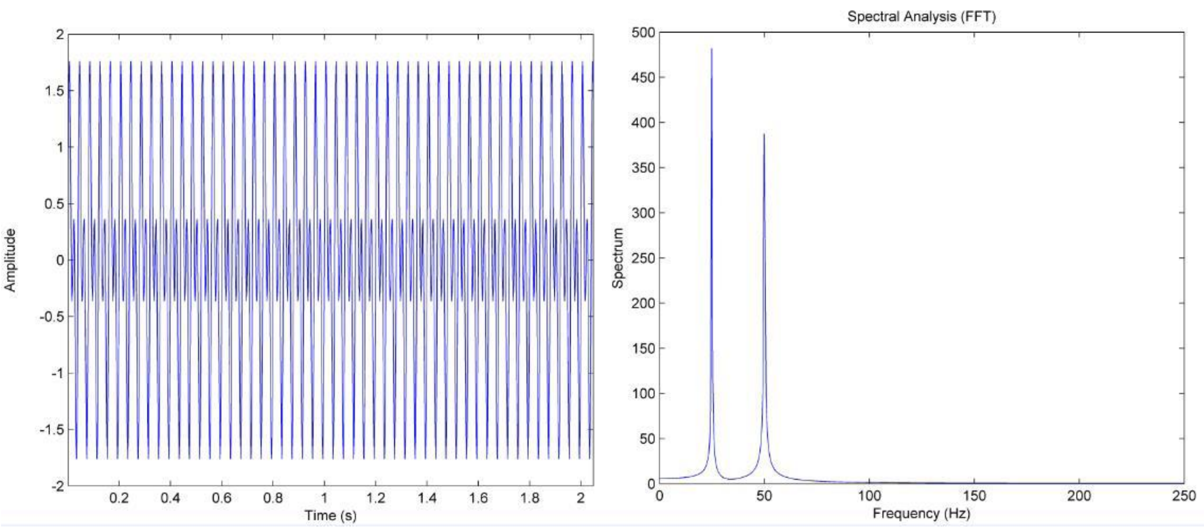


Figure 1.1: Superimposed sine wave signal and its Fourier transform

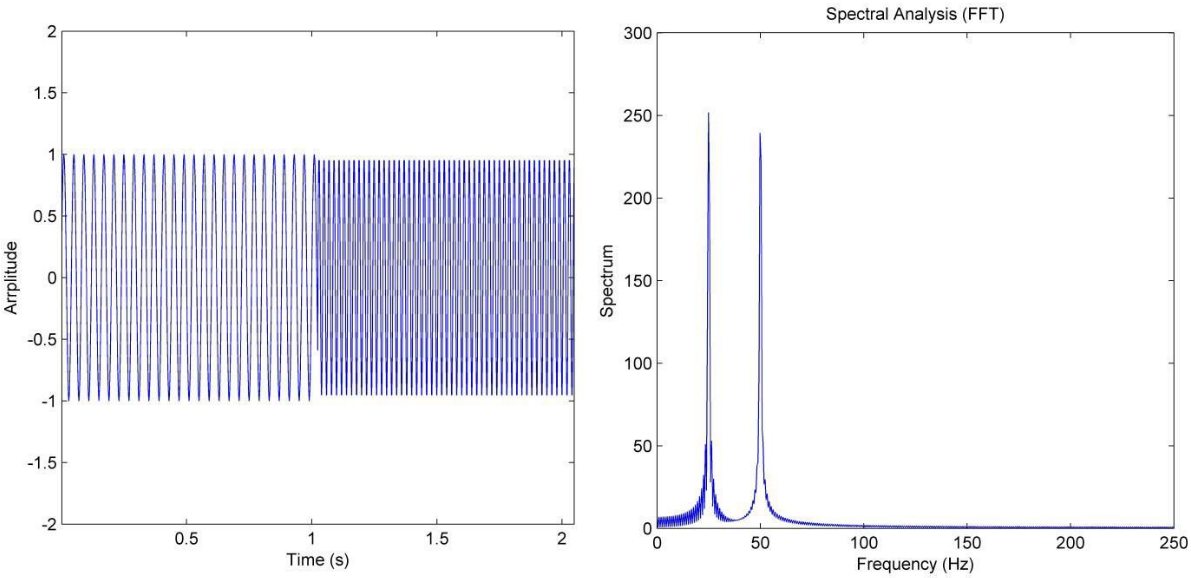


Figure 1.2: Step sine wave signal and its Fourier transform

Also these two things can be notice from the examples: first, in those diagrams, the sizes of the peaks (values of amplitude) related to how long the frequency contents exist on the time domain, and that's why the value of amplitude in the Figure 1.1 is double than it in Figure 1.2 The second thing is that the little ripples in the fourier transform in Figure 1.2 is caused by the sudden changes of frequency in the original signal, when the frequency change its values to another step, it causes the changes of the average frequencies in the short time intervals, therefore some of the spectral analysis values is around the actual value.

Fourier transform is a powerful tool for analysing stationary signals, but to track down the changes in non stationary it is inadequate [49], [50]. The property of concentrating the energy of a signal at and around the instantaneous frequency in the time-frequency plane shows how capable the TF representation based method is. Two approaches, namely, the short time Fourier transform and the continuous wavelet transform are commonly used to produce the TFDs for signals. We can see the difference between Fourier transform and STFT from the Figure 1.3. The Fourier transform can represent the frequency localization, but lost all information in time domain. The Short-time fourier transform, as an evolved scheme of fourier transform, has equal-length intervals in the time domain. And also, as shown in the figure, the segments of time intervals are not affected by the value of frequencies.

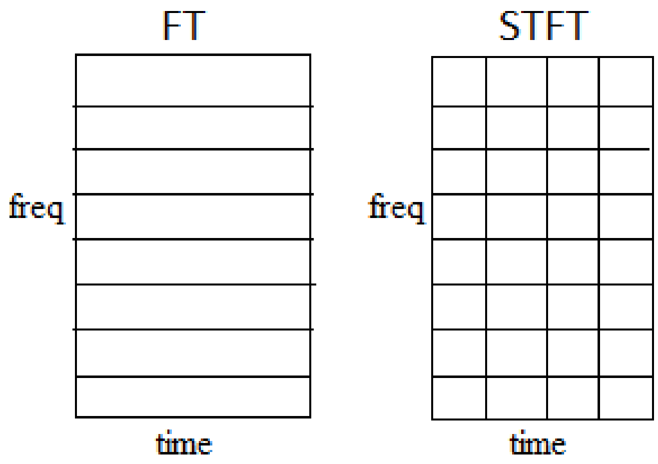


Figure 1.3: Time-frequency resolution diagrams of Fourier transform and STFT

To show that time frequency analysis is better than FT. We have taken an example for two signals and their STFT. From time frequency analysis we have chosen STFT because it is a simplest one. Figure 1.4 (a) is the presentation of two signal that have both sine waves with frequency 25 Hz and 50 Hz happen at the same time multiplied to each other. Figure 1.5 (a) is a step-change signal that changes its frequency from 25 Hz to 50 Hz at the middle of the time. Along with the signals the short time fourier of the two signals is also presented.

Looking at both the examples, it can be seen that two different signal obtain the different STFT results. In Figure 1.4 (a), it can be seen that frequency contents of 25 Hz and 50 Hz exist at the same time and they are well separated from each other. In Figure 1.5 (a), it can be seen that there are two frequency contents but they are at different time, obviously separated at middle.

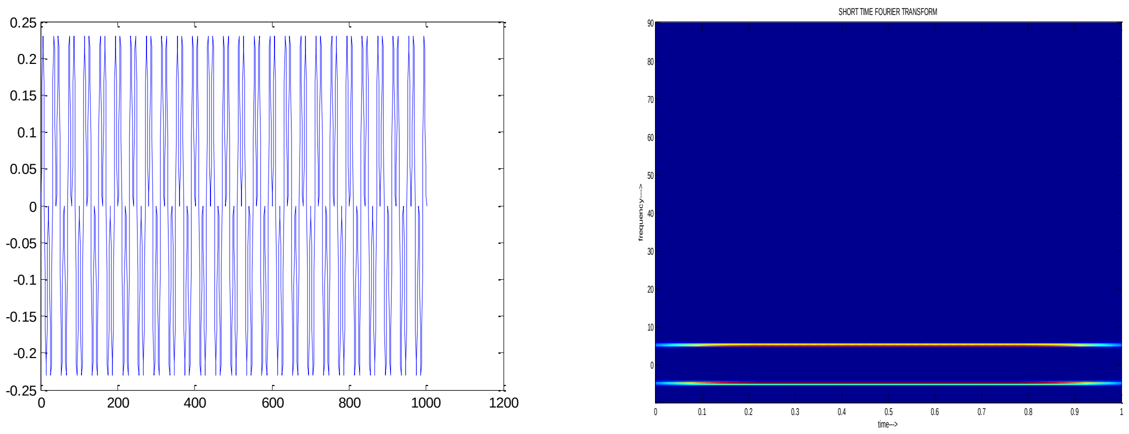


Figure 1.4: (a) Sine waves multiplied to each other (b) STFT of sine waves multiplied to each other

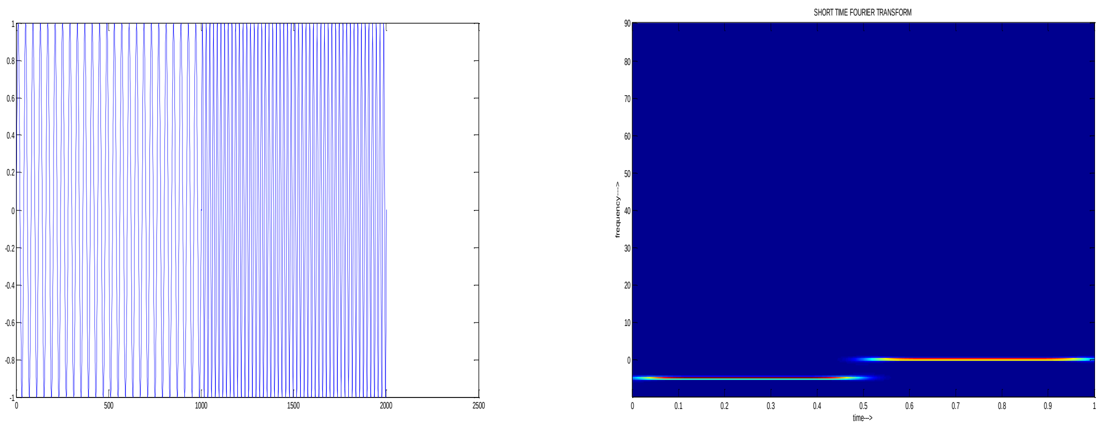


Figure 1.5: (a) Step sine wave signal (b) STFT of step sine wave signal

So that is why STFT is better than fourier transform because it can recognize the difference between two signals that have many different contents happened at the same time or at the different times. It is also a main reason due to which need for STFT arises. The STFT and the CWT are essentially a kind of linear transforms characterized by a static resolution in the time–frequency plane [28], [29]. However, due to the restriction of the Heisenberg–Gabor inequality, neither the STFT nor the CWT is able to achieve a fine resolution in both the time and frequency domains, a good time resolution definitely implying a poor frequency resolution and vice versa. When a signal is noise free and has a linear frequency law and constant amplitude, STFT and WT are the transforms that can achieve a highly accurate estimation. However, in the cases of noisy signals or nonlinear IF law, these two would be not effective. Efforts have been made to improve the capability of the STFT to achieve a better estimation for the nonlinear IF, mainly through adjusting the window length. Aside from the TFD analysis methods mentioned earlier, the Chirplet transform (CT) is another kind of TFA method which is particularly designed for the analysis of chirp like signals with linear IF law [32]. CT is not able to provide better resolution for the signals which follow non linear IF law, but PCT is. It was developed on the basis of polynomial kernel following the IF trajectory of the signal [34]. Basically it was an extension to CT. Concentration produced by PCT in time frequency domain is better than all other methods used in case of chirp signals, but it is highly vulnerable to noise.

For multi-component signals, not only the concentration of TFD matters but also the separation of components should be good. Most common tools developed for analysis of multi-component signals are STFT and S transform. S transform obviously shows better results than STFT because the window used in S transform depends on the frequency of the input signal which does not happen in case of STFT [36]. The window used in case of STFT is fixed. Thus it cannot provide good time and frequency resolution simultaneously.

1.4 PROBLEM STATEMENT

We have developed a time frequency analysis method which can work well on both chirp and multi-component signals. The motivation lies in fact that both chirp and multi-component signals are useful in many applications and there are existing methods which can either separate out components or follow the chirp behaviour of signals. So there is a need of a method which can fully track the chirping nature of signals and also at the same time separate the components of signals effectively. It leads to study and analyze the signal in a better way.

1.5 OBJECTIVE OF DISSERTATION

This dissertation provides the basics of time frequency analysis methods for chirp and multi-component signals. For chirp signals, the topics covered are Short time fourier transform, Chirplet transform and Polynomial Chirplet transform. Also the comparison of Short time fourier transform, Chirplet transform and Polynomial Chirplet transform is done. For multi-component signals, the methods of STFT and S transform are studied. The performance of both the methods is also analysed. The study of basics of time frequency analysis tools is also done to enhance knowledge about the project. Moreover, a MATLAB implementation of all the methods is presented on both the signals. We have also proposed a modified s transform. The dissertation concludes with a performance analysis of the results obtained from MATLAB implementations of new method. Our main goals are:

1) Analysis of chirp signals using STFT, CT and PCT.

2) Analysis of multi-component signals using STFT and ST.

3) Proposed signal processing tool for chirp and multi-component signals.

1.6 ORGANIZATION OF DISSERTATION

The objective of this work is to study various time frequency analysis tools for multi-component and chirp signals and develop a new time frequency tool for the analysis of both chirp and multi-component signals. The dissertation is organized as follows:

Chapter 2 gives an literature review of the TFA tools. It includes details of existing work in the field of spectral analysis.

Chapter 3 presents the analysis of chirp signal using TFA methods. It includes the basics of STFT, CT and PCT. Comparison is made between all three methods by MATLAB simulations and results are discussed as per observations.

In chapter 4, the analysis of multi-component signals is done using TFA tools. It contains the detail about STFT and S transform. Results are obtained by implementing both transforms on MATLAB software and observations are established accordingly.

In chapter 5, the modified S transform is proposed. It covers the details about the proposed method and how it is better than other existing methods. The results and discussions are also made by implementing the proposed method on both chirp and multi-component signals.

Conclusions are drawn in chapter 6. Future work has also been discussed in this chapter.

CHAPTER 2

LITERATURE REVIEW

The study about the stationary and non stationary is done first of all [2]. The information about the tools that are already present for the time frequency analysis are studied [5]. The basics of multi-component signals are known to get interest in project [1]. The analyzing methods of multi-components are studied [3], [4]. The fundamentals of spectral analysis provide a clear view of the basics [9]. A study is made on Fourier transform to get started with the origin of TFA methods [6].

STFT is the first time frequency tool developed in 1946. L. Durak presents two properties and implementation of STFT [20]. The investigation is made on shift and rotation properties of time frequency distributions. Uncertainty principle is studied to know about the limitations of STFT [9].Then there is a modified form of STFT for mono-component signals proposed by H. Guven [23]. Idea of minimum time bandwidth product is used in this. It has an adaptive window to get highest possible resolution for mono-component signals. Growth in time frequency analysis approach is quite evident from fact that another method called fast recursive STFT algorithm was proposed by S. Tomazic [22]. To make STFT adaptive was always the region of interest for researchers. Another method was introduced by Kwok and Jones [27]. In this IF estimation is also done.

After a period of time of introduction of time frequency tools, a boom was observed in the field of Wavelet Transform. A full study on implementation and interpretation of WT is given by Bentley and Mcdonnel [28]. This paper also includes the applications of Wavelet Transform. Most of the time is elapsed in computation in WT. So a new and fast algorithm is developed by Rioul and Duhamel [28]. The computational complexity is decreased to log L form L for large filter lengths.

Chirplet transform is considered to work on non stationary signals like chirp signals [32]. Quadratic chirp functions are used. Thus overtake the WT and STFT. There are options of not only obtaining time, frequency and scale resolutions, but also getting shear in time and shear in frequency. The chirplets used are related to each other in time frequency domain unlike the wavelets related in one dimensional space. A novel is done to update the Chirplet Transform by using Spline Kernel [33]. It was developed because of the fact that conventional TFA methods are less capable of dealing with non stationary signals. In this case, the kernel simply matches with the IF. The frequency rotate and frequency shift operators are used for developing this powerful tool for non linear frequency modulated signals.

PCT was proposed due to the limitations of CT to track non linear chirp signals [34]. In this modified form polynomial function is used unlike CT which uses linear chirp kernel. But it has a disadvantage of contamination by Gaussian noise. In 2013, same pair of researchers implemented PCT on multi-component signals [35]. Time fusion technique is used for this purpose. It is able to concentrate the energy along the IF of the signal. Filtration is used to remove the noise and preserve the component of interest.

To separate out components from multi-component signals, ST is considered. S transform is an extended version of CWT and have moving Gaussian window [36]. It has overcome the limitations of WT and STFT. It provides desired characteristics for better resolution. This paper is aimed to show the strength and limitations of S transform and its inverses [37]. The level of approximation for inverse of S Transform is tested. During the recovery of signal approximation is of very much importance. In this dissertation, the application of S Transform i.e. time frequency filtering is a also discussed. The evaluation of effects using S Transform and its inverses discussed in detail. A wavelet view of S Transform was proposed in 2008 [41]. S transform works well for continuous signal, but it does not provide accurate results in case of discrete signal. So ideas were taken from Wavelet Transform to make it a better tool in all cases. An improved S Transform was proposed in 2009 [39]. S Transform has an advantage of allowing multi-resolution analysis while retaining the phase information. But it has an disadvantage of poor energy concentration in time frequency domain. So to improve the energy concentration, the window length is improved in efficient manner. In 2011, non stationary signal analysis is done by generalized S Transform. Seismic signal processing is also done to compare S Transform and generalized S Transform. S Transform is deduced from STFT and WT [38]. S Transform achieves better flexibility due to its dependency on progressive frequency for good resolution. It has also better noise free performance as compared to other methods for non-stationary signals. In 2013, B. Han also discussed S Transform for signal analysis [40]. The result shows ST has better utility and variability in non stationary signal processing. It shows better performance than other existing methods under noise conditions. The applications of S Transform can give us an idea about the future scope of the topic. Its application is found in biomedical field [13], [14], microseismic systems [18]. It can be used for IF estimation purpose [27], [47]. The whole study is done to implement all these methods on MATLAB and also develop a new method with good energy concentration and localization both in time and frequency domain.

CHAPTER 3

ANALYSIS OF CHIRP SIGNAL USING SIGNAL PROCESSING TOOLS

3.1 INTRODUCTION

Chirp signals are non stationary signals which have continuously increasing or decreasing frequency. If the frequency of signal increases from lowest to the highest values then it is called an upchirp and if it is from highest to the lowest values then the chirp is called a downchirp. Importance of chirp signals lies in the fact that its features make it useful in the field of communication. For example, all the three modulation techniques namely, frequency modulation, amplitude modulation and phase modulation, can be applied to the chirp signals at the same time. Thus makes the transmission more optimal. In RADAR system, chirp signal can provide high resolution which contributes for the better ranging. . There are several tools which can be used to extract information out of chirp signals, namely STFT, CT, and PCT. Figure 3.1 shows an example of upchirp.

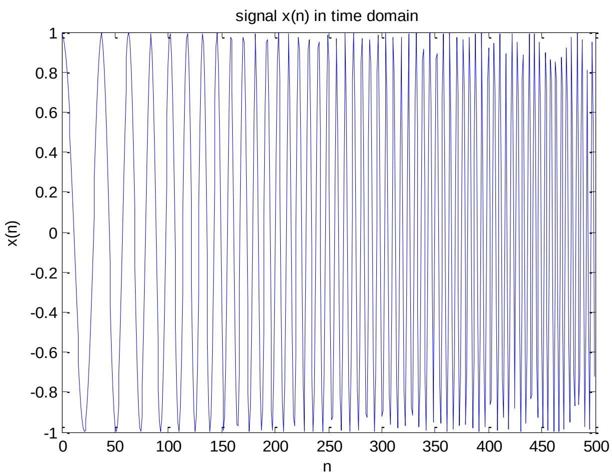


Figure 3.1: Chirp signal

3.2 SIGNAL PROCESSING TOOLS FOR CHIRP SIGNALS

3.2.1 SHORT TIME FOURIER TRANSFORM

STFT is a commonly used time-frequency representation [20], [21]. It is the modification of Fourier transform. The general formulation in time domain is given by

$$Z(t,f) = \int_{-\infty}^{\infty} w(t-\tau)z(t)e^{-j2\pi ft}\,dt \tag{3.1}$$

In this equation the $w(t)$ is the window function.

$$Z(t, f) = F\{w(t-\tau)z(t)\} \tag{3.2}$$

From the above equation it can be easily understood that $Z(t, f)$ is essentially the Fourier transform of $w(t-\tau)z(t)$ over an interval of the original signal which is segmented by window function. Compare equation (3.1) with the equation (3.2), we can find out that the STFT can get the time localization by first windowing the signal, so as to cut off only a well-localized slice of signal, and doing the fourier transform of each segments. To segment signals into narrow intervals, we use a appropriate window function $\mathrm{w(t)}$ as a slicing tool. From equation (3.1), we can get that no matter which bands of frequency they have, the window function $\mathrm{w(t)}$ always has the same window length τ. So during the whole procedure of STFT, the length of window function doesn't change. Because of that, choosing a suitable window functions length is very important in STFT.

Requirements of the sliding window function

1) $w(t)$ is an even function. i.e. $w(t) = w(-t)$.

2) Maximum value of window function is at $t = 0$.i.e.max$(w(t)) = w(0), w(t1) \geq w(t2)$ if $|t_1|<|t_2|$.

3) $w(t) \approx 0$ when $|t|$ is large.

The energy density spectrum of the STFT is given by

$$E(t, f) = |Z(t, f)|^2$$

$$E(t, f) = \left| \int_{-\infty}^{\infty} w(t-\tau)z(\tau)e^{-j2\pi f\tau} d\tau \right|^2$$

which is also called the spectrogram. Simply, the spectrogram averages the frequency content of a specific time interval of the original signal using a window shifted in time and frequency.

The resolution of time-frequency representations in short time fourier transform, however, is limited by the uncertainty principle. Let us consider a short duration signal which is constructed by multiplying the signal with an appropriate window function $h(t)$.

$$y(t_o) = z(t_o)h(t-t_o)$$

The uncertainty principle then states that the product of the standard deviations in time and frequency for the signal in the above equation cannot be made arbitrarily small [10]. Instead, the following relation holds

$$\sigma_t \sigma_\omega \leq \frac{1}{2}$$

The uncertainty principle was first derived by Werner Heisenberg in 1927. For a signal of the form $y(t_o)$, high resolution in the time domain corresponds to low resolution in the frequency domain and vice versa.

Even through STFT can provide simultaneous time and frequency localization, it is still limited by using the sine and cosines to represent the signals. Because of sine and cosine functions have the same amplitude in the whole infinity time domains, they have infinity energy which distribute averagely with time. Imagine if we can use a group of waves which have concentrated energy around one time point to present signals, we might get better time resolutions. That’s where the idea of chirplet transform originated.

3.2.2 CHIRPLET TRANSFORM

Due to the limitations of Short time fourier transform, Chirplet transform is proposed [32], [33]. It is a different approach for obtaining a time-frequency representation of a signal $x(t)$ is based on the decomposition of $x(t)$ using basis functions. The Chirplet transform approach uses a representation of the signal components, which in this case are called chirplets.

The Chirplet transform is a multi-dimensional parameter. Compared to other linear Time-Frequency representations, such as STFT or the Wavelet Transform, the Chirplet Transform represents 1-D time-domain signals by multi-dimensional functions. Most often, the function is five-dimensional, where the five dimensions are:

1) Chirping in time,

2) Chirping in frequency,

3) Time translation,

4) Frequency translation,

5) Dilation in frequency/ Contraction in time

The mapping of a signal in the five-dimensional space helps to overcome the resolution problem from which other time-frequency representations suffer.

3.2.2.1 Chirplet Operators The Chirplet transform is capable of rotating each cell of the time-frequency plane as well as shearing it along the time and frequency axes. The Chirplet Transform's capability of shaping the time-frequency cells presents new opportunities of optimizing the time-frequency resolution according to the analyzed signal. For example, it is possible to rotate and shear each cell according to the local slope of the trajectory of the analyzed instantaneous frequency, thus giving the opportunity of better tracking the frequency's evolution versus time. These degrees of freedom are provided by modifying the chirp's parameters in order to perform basic chirp operations such as the ones described next:

1) Chirping in time

Chirping in time is obtained by multiplying the mother chirplet with a linear Frequency Modulated (FM) signal, chirp ($e^{j2\pi\frac{\alpha}{2}t^2}$) where α is the chirp-rate. It causes a rotation of each cell as well as its shear along the frequency axis. The rotation angle with respect to the time axis is α. The slope of the cell on the time-frequency plane is equal to the value of chirp-rate.

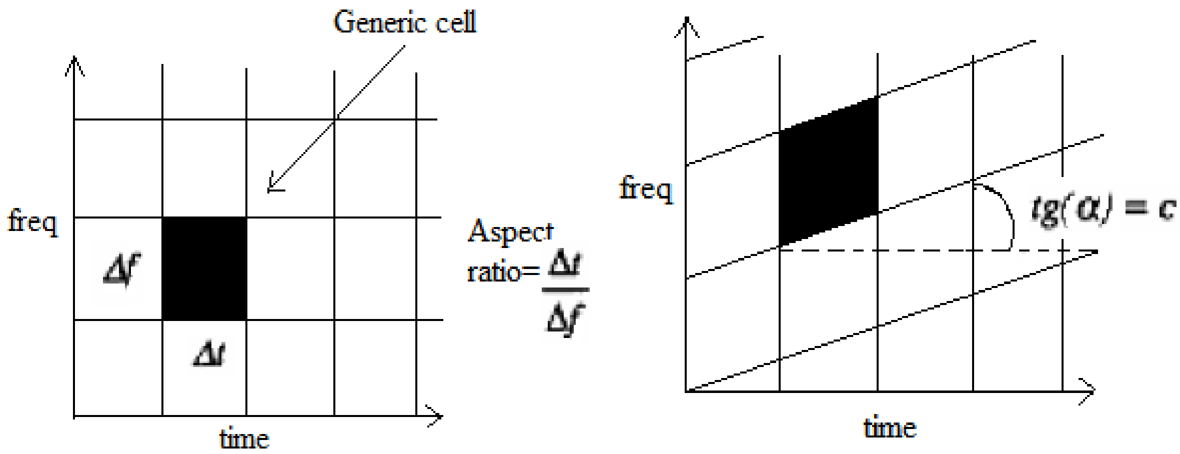


Figure 3.2: Time-frequency resolution diagrams of (a) STFT and (b) CT during chirping in time.

2) Chirping in frequency (time shearing)

Chirping in frequency (time shearing) is given by convolving, in the time domain, the mother chirplet with chirp $(-jd)^{\frac{-1}{2}}e^{j2\pi\frac{1}{2\pi}t^2}$ where the parameter d accounts for the shear amount along the time axis imposed on the cell. In the frequency domain, this is accomplished by multiplying the fourier transform of the mother chirplet and the function $e^{-j2\pi\frac{d}{2}f^2}$ which can be considered a chirp in the frequency domain with the chirp-rate equal to d. The rotational angle with respect to the frequency axis is β which depends on the chirp-rate d.As with chirping in time, the slope of the cell with respect to the frequency axis is equal to d.

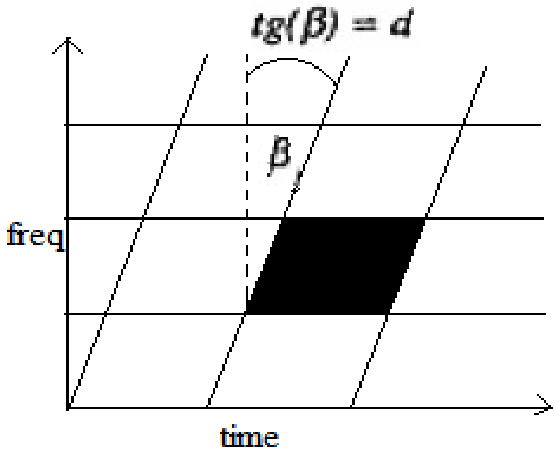


Figure 3.3: Time-frequency resolution diagrams of CT during chirping in frequency

3) Time translation

Time translation is done by shifting the mother chirplet by t_c in time where t_c is the time offset.

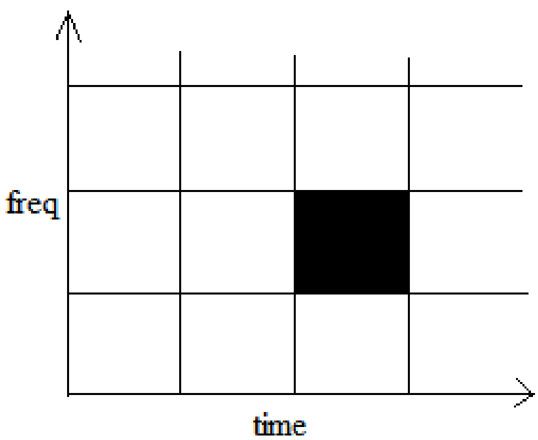


Figure 3.4: Time-frequency resolution diagrams of CT during time translation

4) Frequency Translation

Frequency translation is done by multiply the mother chirplet by $e^{j2\pi f_c t}$ where f_c is the amount of shift in frequency. The Figure 3.5 shows the TFD diagram during the frequency translation. Basically it shows the effect on the resolution in time and frequency domain due to the frequency translation. We can compare Figure 3.4 and Figure 3.5 to see the effects due to both the functions of CT.

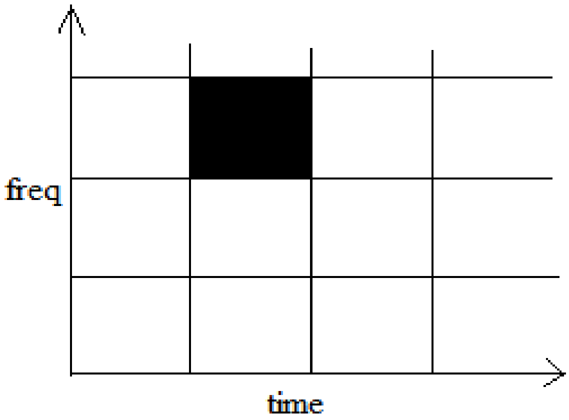


Figure 3.5: Time-frequency resolution diagrams of CT during frequency translation

5) Dilation in Frequency/ Contraction in Time:

Changing the sampling rate or resolution of the signal in time cause the cell to be either dilated or contracted in time. When that happens, the reverse effect is caused in the frequency scale.

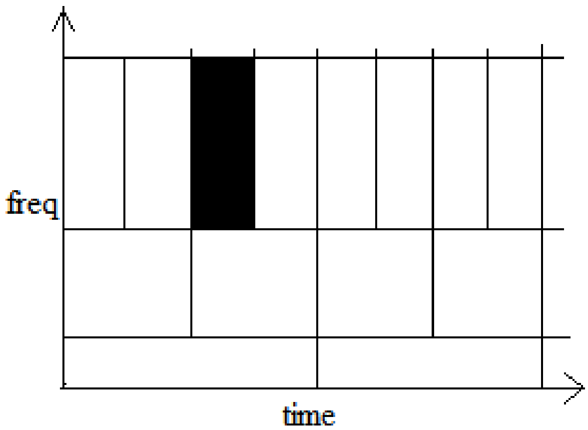


Figure 3.6: Time-frequency resolution diagrams of CT during frequency dilation in contraction in time

All of the above operations can be combined to a complete analytic expression of the chirplet. The CT of a signal $s(t)$ is defined as

$$CT_S(T_0, \omega, \alpha; \sigma) = \int_{-\infty}^{\infty} z(t)\, \Psi^*_{(t_{0,\alpha,\sigma})}(t) \exp(-j\omega t)\, dt \qquad (3.3)$$

where $z(t)$ is the analytical signal of $s(t)$, generated by the Hilbert transform H, i.e., $z(t) = s(t) + jH[s(t)]$, and $\Psi^*_{(t_o,\alpha,\sigma)}(t)$ is a complex window given by

$$\Psi_{(t_{o,\alpha,\sigma})}(t) = w_{(\sigma)}(t - t_o)\exp(-j\frac{\alpha}{2}(t - t_o)^2)$$

Here, the parameters t_o, α stand for the time and chirp rate.

From the definition of the CT given as (3.3), the CT may be seen as the STFT of the analytical signal multiplied by the complex window $\Psi^*_{(t_{0,\alpha,\sigma})}(t)$. The definition of the CT is :

$$CT_S(T_0, \omega, \alpha; \sigma) = A(t_o) \int_{-\infty}^{\infty} \bar{z}\, w_{(\sigma)}(t - t_o) \exp(-j\omega t)\, dt$$

where

$$\bar{z}(t) = z(t)\Phi^R_\alpha(t)\Phi^M_\alpha(t, t_o)$$

$$\Phi^R_\alpha(t) = \exp(-j\alpha \frac{t^2}{2})$$

$$\Phi^M_\alpha(t, t_o) = \exp(j\alpha t_o t)$$

$$A(t_o) = \exp(-jt_o^2 \frac{\alpha}{2})$$

Clearly, $\Phi^R_\alpha(t)$ is a frequency rotating operator which rotates the analytical signal $z(t)$ by an angle θ with $tg(\theta) = -\alpha$,in the time–frequency plane; $\Phi^M_\alpha(t, t_o)$ is the frequency shift operator that relocates a frequency component at ω to $\omega + \alpha t_o$ and $A(t_o)$ is a complex number with modulus $|A(t_o)| = 1$.

In the time–frequency analysis, it is the modulus of the TFD $|CT_S(T_0, \omega, \alpha; \sigma)|$ that is usually of interest and have high value, and therefore, the definition of the CT can be simplified as

$$CT_S(T_0, \omega, \alpha; \sigma) = \int_{-\infty}^{\infty} \bar{z}\, w_{(\sigma)}(t - t_o) \quad (3.4)$$

From this definition, it can be seen that the CT can be decomposed into a series of operators: 1) rotating the signal under consideration by a degree $\arctan(-\alpha)$ in the time–frequency plane; 2) shifting the signal by a frequency increment of αt_o; and 3) doing STFT with window $w_{(\sigma)}$.

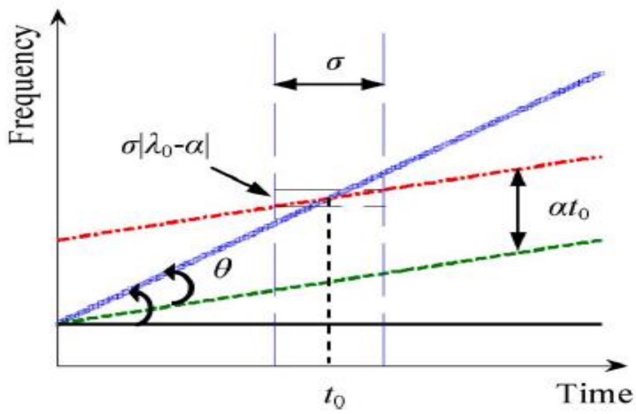


Figure 3.7: Interpretation of CT as θ shows the rotation and αt_oshows the frequency shift

3.2.3 POLYNOMIAL CHIRPLET TRANSFORM

PCT is an extension to Chirplet transform [34], [35]. For non linear IF trajectory signals CT does not provide good time frequency distribution. It spreads out at times. So when the chirp rate is selected wisely than PCT gives good concentration in time frequency domain. PCT is:

$$PCT_S(t_0, \omega, \alpha_1, \alpha_2 \dots \dots \dots \alpha_n; \sigma)$$

$$= \int_{-\infty}^{\infty} z(t)\varphi^R_{\alpha_1 \dots\dots\dots \alpha_n}(t) \times \varphi^M_{\alpha_1 \dots\dots\dots \alpha_n}(t, t_0) w_{(\sigma)}(t - t_0) \exp(-j\omega t)\, dt \quad (3.5)$$

in which

$$\varphi^R_{\alpha_1 \dots\dots\dots \alpha_n}(t) = exp\left(-j \sum_{k=2}^{n+1} \frac{1}{k} \alpha_{k-1}\, t^k\right)$$

and

$$\varphi^M_{\alpha_1 \dots\dots\dots \alpha_n}(t, t_0) = exp\left(j \sum_{k=2}^{n+1} \alpha_{k-1} t_0^{k-1}\, t\right)$$

where $\varphi^R_{\alpha_1 \dots\dots\dots \alpha_n}(t)$ is a non linear frequency rotating operator, $\varphi^M_{\alpha_1 \dots\dots\dots \alpha_n}(t, t_0)$ is the frequency shift operator and $\alpha_1 \dots \dots .. \alpha_n$ are the polynomial kernel characteristic parameters. PCT gives better concentration than CT. But provide noise in the TFD generation.

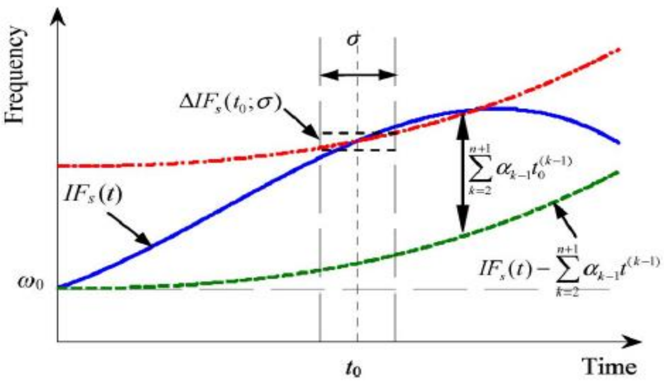


Figure 3.8: Interpretation of PCT

3.3 SIMULATION RESULTS AND DISCUSSIONS

The performance of time frequency distribution is assessed by applying STFT, CT and PCT on synthetic signals.

3.3.1 EXAMPLE 1

The first test signal is a linear chirp signal. The instantaneous frequency of a linear chirp signal varies with respect to time.

$$z(t) = \sin(2\pi(12 + 2.5t)t) \quad (3.6)$$

$$(0 \leq t \leq 15s).$$

The signal is sampled at 200 Hz. The generated signal is as shown in Figure 3.7 as below

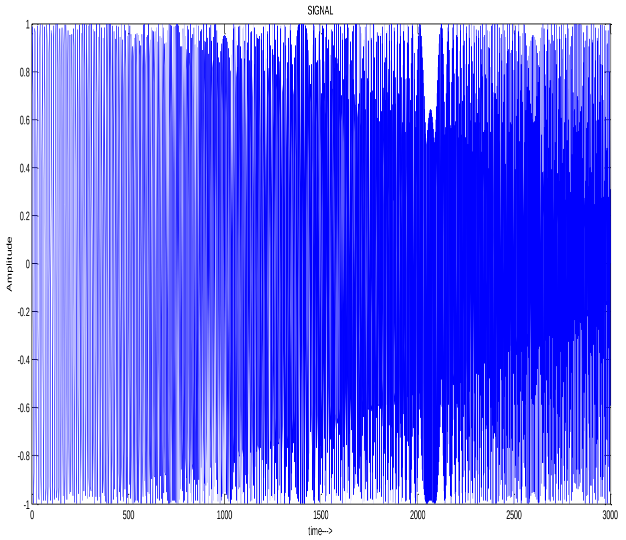


Figure 3.9: Signal with linear IF law

The frequency of signal in Figure 3.7 is changing very fast, which shows chirping behaviour.

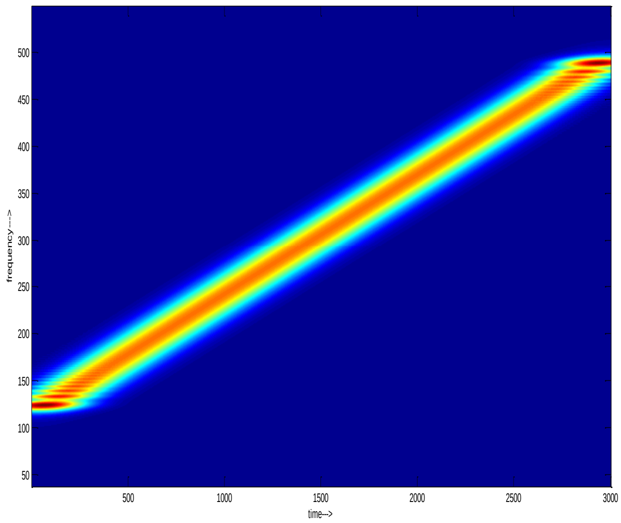


Figure 3.10: TFD generated by STFT

The Figure 3.8 shows that STFT results broadens in shape and concentration is also not up to desired levels. The time frequency resolution is also not good.

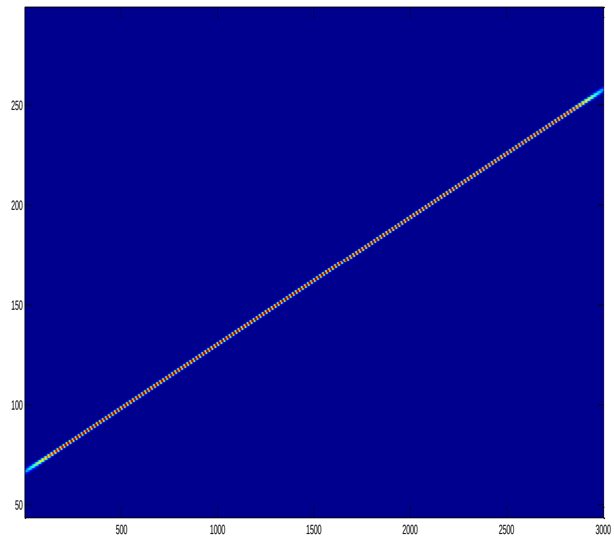


Figure 3.11: TFD generated by CT

Figure 3.9 shows that TFD generated by CT provides better concentration than STFT. Also the CT kernel follows the linear increase in frequency very well and without expanding. CT provides good results when IF trajectory of chirp signal is linear. The performance analysis of CT on non linear signals is done ion following example.

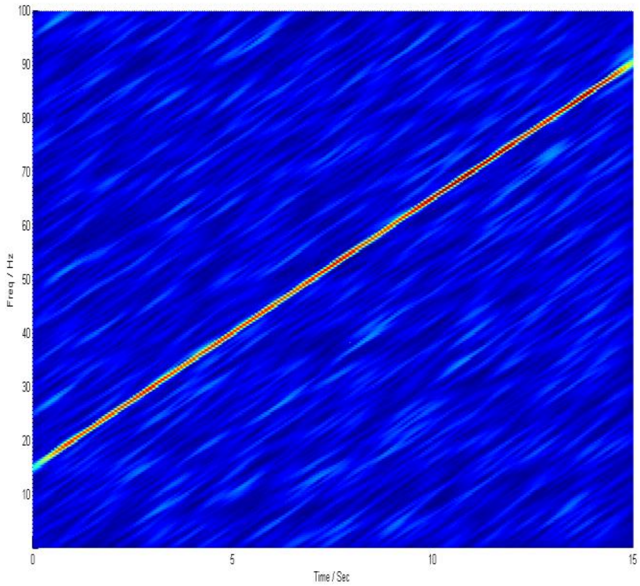


Figure 3.12: TFD generated by PCT

PCT provides very good concentration in time frequency domain. The IF trajectory of signal is followed but there is a disadvantage of addition of unwanted signal in TFD. The contamination is due to white Gaussian noise.

3.3.2 EXAMPLE 2

In last example we have illustrated results on s signal following linear IF law. So in this case, we have taken an example of a signal in which frequency increases non linearly. Mathematically given as follows:

$$z(t) = sin\left(2\pi(10t + \frac{5}{4}t^2 + \frac{1}{9}t^3 - \frac{1}{160}t^4)\right) \tag{3.7}$$

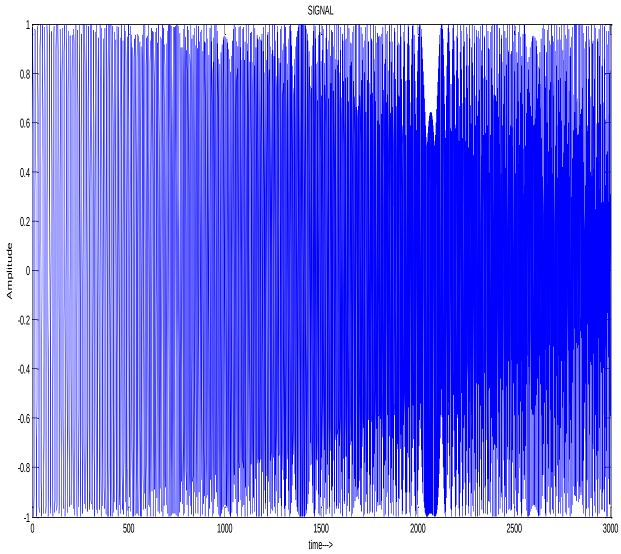


Figure 3.13: Signal with non linear IF

This signal contains chirp signal in which the frequency increase and in latter part decreases with time.

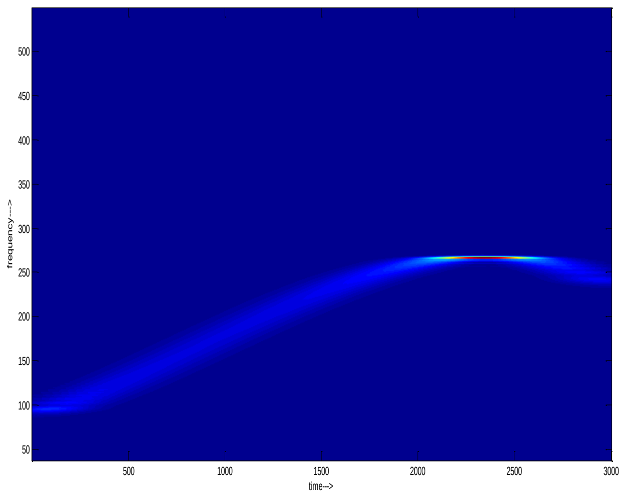


Figure 3.14: TFD generated by STFT

Figure 3.12 shows the broadening of TFD in earlier and latter part. In the middle part, it shows good concentration. STFT is unable to provide localization in both time and frequency.

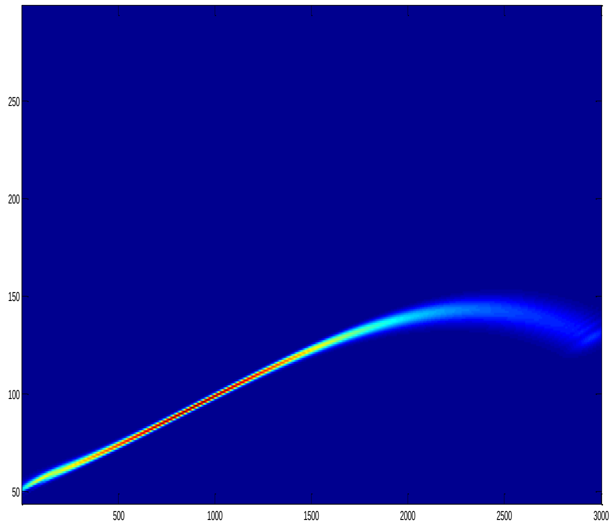


Figure 3.15: TFD generated by CT

In Figure 3.12, we can see that in latter part the TFD broadens and CT kernel is unable to follow the trajectory of signal. So PCT was proposed to remove this limitation of CT to track the non linear nature of the signal.

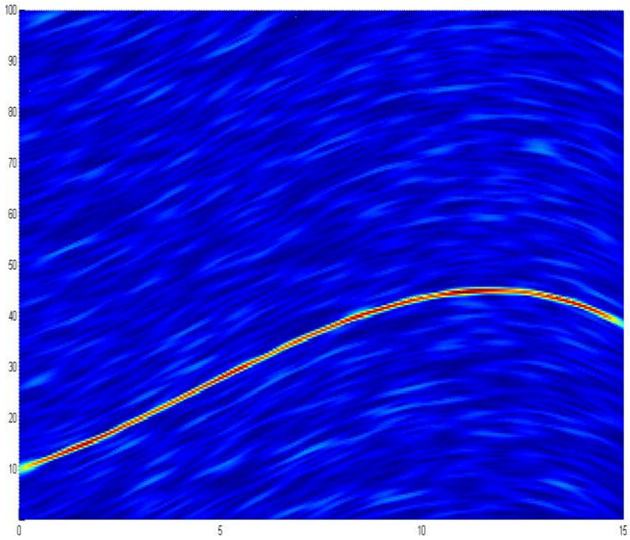


Figure 3.16: TFD generated by PCT

Figure 3.12 shows the TFD generation by PCT. On comparing it with Figure 3.11, it can be easily understood that concentration is very good from starting to end points in case of PCT. The shape of IF trajectory also shows that there is no expansion in the latter part of signal.

3.3.3 EXAMPLE 3

Now we have taken an example where the frequency of signal first increases and then decreases. The mathematical equation is as shown below:

$$z(t) = \sin(90\pi t + 13.5\pi t^2 - \frac{4}{3}\pi t^3 + \frac{3}{100}\pi t^4) \tag{3.8}$$

The signal in equation (3.8) is represented by

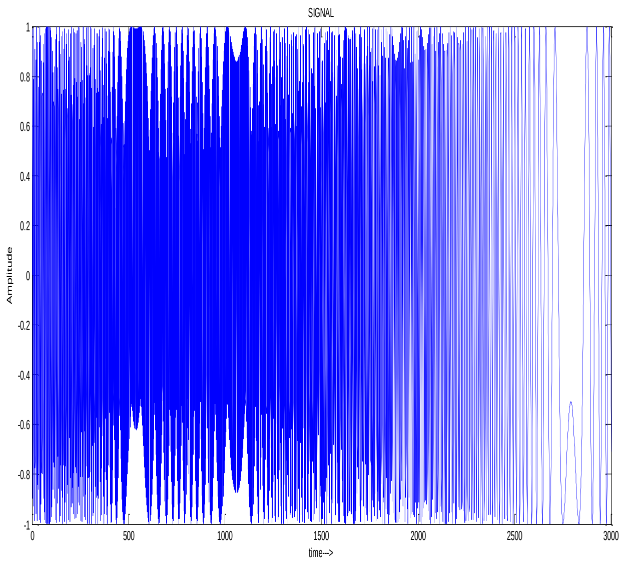


Figure 3.17: Signal generated

The frequency in Figure 3.17 is becoming lesser as signal expands in the latter part. Before ending there is again an abrupt increase in frequency. The TFDs generated are shown in Figure 3.18 Figure 3.19 and Figure 3.20 by STFT, CT and PCT respectively. This example is taken to show whether the TFD can track down small changes in frequencies or not. We have also implemented ST and Modified S transform in chapter 5. The observation need to be made is that whether these all transform are able to extract information at the last part of the signal or not. There are limitations to each of the transform. So it is also very necessary to have knowledge about the applications where we can apply these transforms. To clear this idea we have taken so many examples.

We can assess from the TFD generated by STFT that there is no good concentration. Also STFT is unable to give information about the signal due to broadening. In the latter part it does not give clear information about the signal whether its frequency is increasing or decreasing.

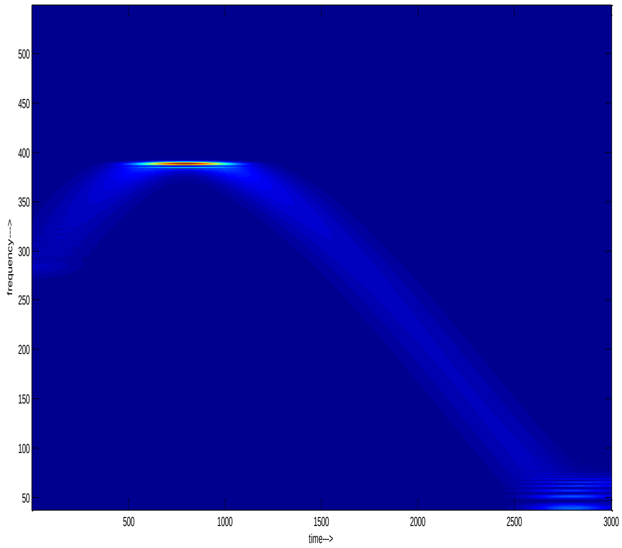


Figure 3.18: TFD generated by STFT

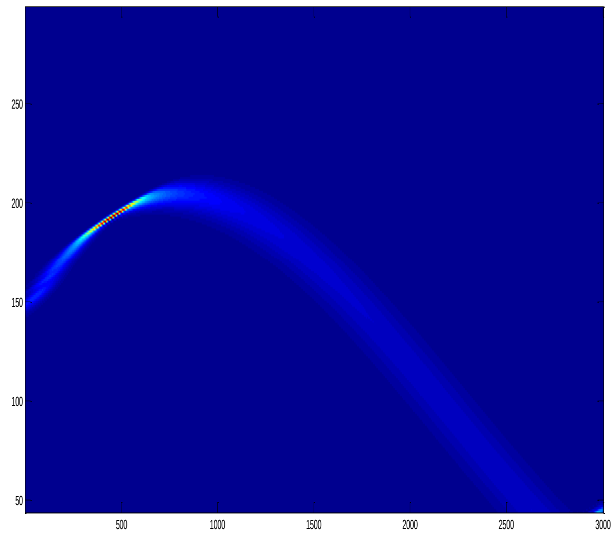


Figure 3.19: TFD generated by CT

The results of CT shown in Figure 3.19 confirms that for non linear IF law following signals, CT is not providing satisfactory TFD. Again its problem is expanding of TFD. In the end, there is no information about frequency with respect to time also.

TFD generated by PCT have much better concentration than in case of CT and STFT. It is because of the fact that PCT kernel tracks the IF of the signal completely. There is a problem of noise addition in case of PCT.

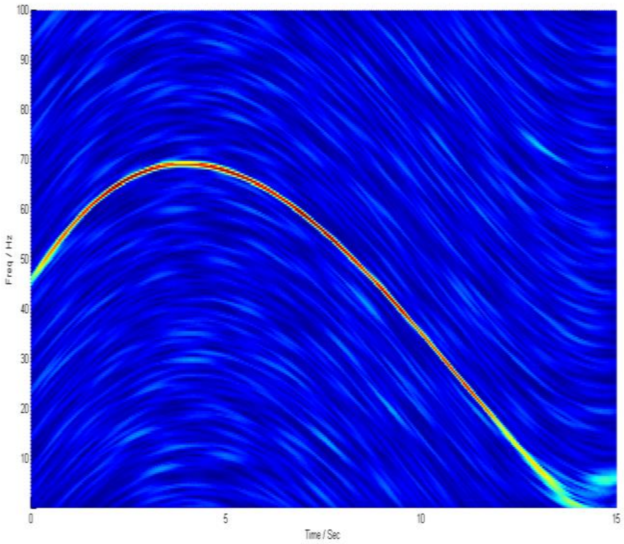


Figure 3.20: TFD generated by PCT

3.3.4 EXAMPLE 4

From the former examples it is clear that PCT is best for non linear chirp signals. Now before moving onwards to next chapter we should know that why these all methods, namely, STFT, CT and PCT are not suitable for multi-component signals. Thus in this example, we have taken a synthetic signal which is multi-component and contains chirp also.

$$z(t) = 0.3\cos(10\pi t_1) + 0.8\cos(30\,\pi t_1) + 0.7\cos(20\pi t_2 + \sin(\pi t_2)) + 0.4\cos((66\pi t_3) + \sin(4\pi t_3))$$

$$0 \leq t_1 \leq 6 \text{ sec}$$

$$6 \leq t_2 \leq 10 \text{ sec}$$

$$4.8 \leq t_3 \leq 7.6 \text{ sec} \tag{3.9}$$

This signal as seen in Figure 3.21 is made up of more than on components. All components have different magnitude as well as different frequencies. The time frequency distributions are given in Figure 3.22, Figure 3.23 and Figure 3.24. The discussions are done with TFDs. The emphasis is given on the fact that there are limitations of Chirp transforms when used on multi-component signals. That is why the need for S transform arises. It is important to know that separation of components of signal is quite of use in different applications. For example, image and speech processing.

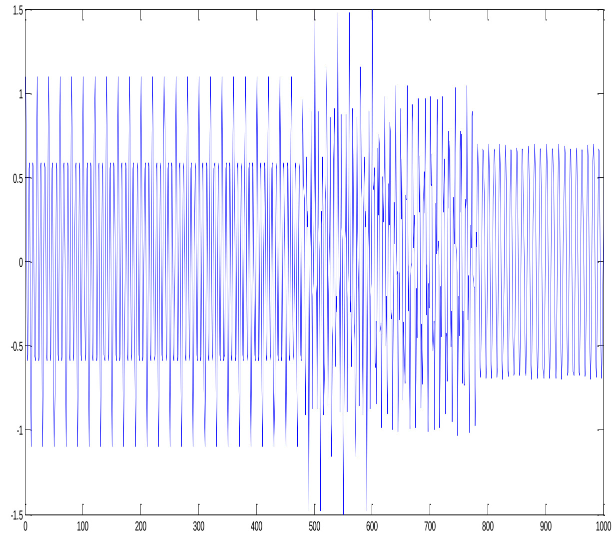


Figure 3.21: Signal generated

Figure 3.21 clearly shows that the signal is made up of different components and also contains chirp behaviour. Now the TFD generated by different transform should clearly divide each component segment and also need to follow the IF of all the components. The TFD generated in next part will lead us to result that the STFT, CT and PCT are not suitable for multi-component signals. Obviously CT and PCT are made for work on chirp signals, not on multi-component. Also choosing a suitable polynomial also necessary part of PCT performance.

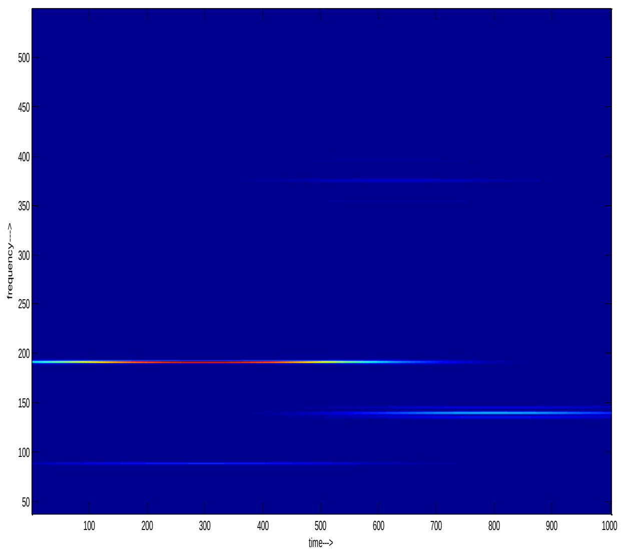


Figure 3.22: TFD generated by STFT

STFT provides results which are better than CT and PCT here, but they are far from ideal one. It could not locate the components properly.

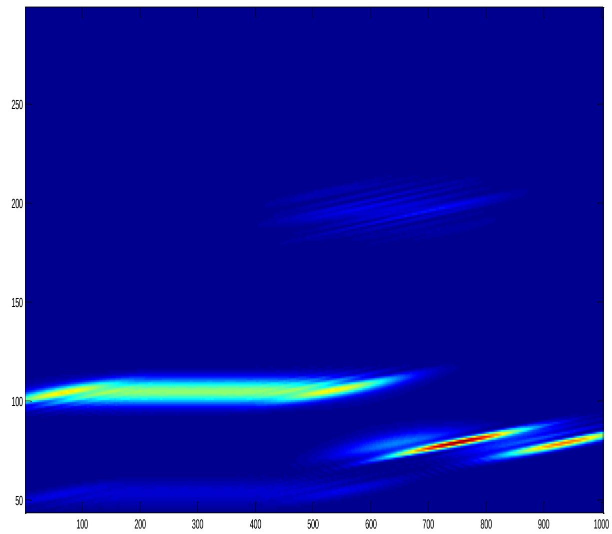


Figure 3.23: TFD generated by CT

CT can't even provide the information about the components in the signal. Also its TFD is flattened at many places. Thus unable to give knowledge of at what time what frequency is present?

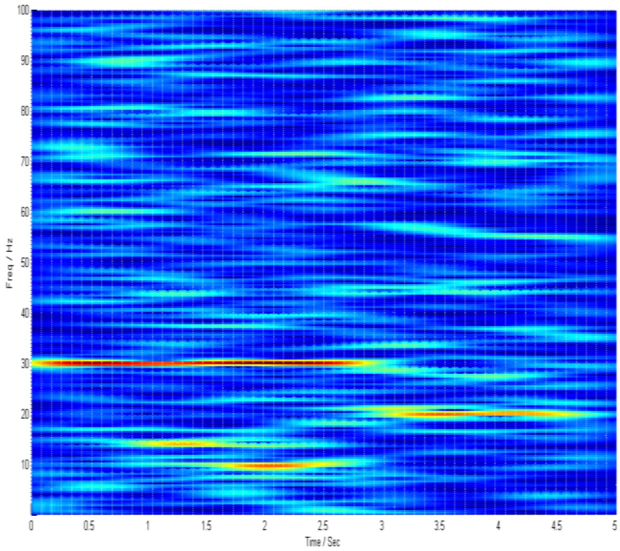


Figure 3.24: TFD generated by PCT

PCT perform even worse in case of multi-component signals. In addition, it contains traces of noise as well. This noise is difficult to remove and contaminates the whole TFD.

CHAPTER 4

ANALYSIS OF MULTI-COMPONENT SIGNAL USING SIGNAL PROCESSING TOOLS

4.1 INTRODUCTION

On basis of components, signal is classified into mono-component and multi-component. Mono-component signals contain single frequency component and multi-component signals are the one which have more than single frequency component. A prime example of multi-component signal is human speech. The importance of extracting information out of these multi-component signals lies in the fact that in several research fields, for example speech recognition, medical fields etc., the signals are often multi-component. The example of multi-component signal is given in following figure:

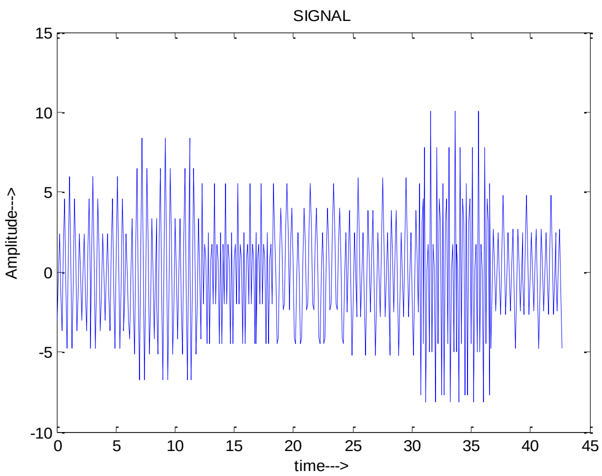


Figure 4.1: Multi-component signal

4.2 SIGNAL PROCESSING TOOLS FOR MULTI-COMPONENT SIGNALS

4.2.1 SHORT TIME FOURIER TRANSFORM

STFT is discussed in detail in section 3.2.1. The mathematical formula is given as:

$$X(t,f) = \int_{-\infty}^{\infty} w(t-\tau)x(t)e^{-j2\pi ft}\,dt \qquad (4.1)$$

In this chapter, we have used STFT for the analysis of multi-component signal and compared the results with S transform.

4.2.2 S TRANSFORM

S transform was proposed by Stockwell in 1996. S transform is an extension of STFT and also provides premium characteristics of wavelet transform [36], [40]. It is derived from the fixed resolution of STFT and the absence of phase information of WT. So before studying S transform, we should have some idea about wavelet transform also. The Wavelet transform is comprised of basis functions which are called as wavelets [2]. These wavelets are defined as

$$k_{a,b}(t) = \frac{1}{\sqrt{a}} h^* \left(\frac{t-b}{a}\right) \tag{4.2}$$

Where a is the scaling factor and b is the time shift variable. The Wavelet transform is given by

$$CWT(b,a) = \frac{1}{\sqrt{a}} \int h^* \left(\frac{t-b}{a}\right) s(t)dt \tag{4.3}$$

From equation (6), it can be easily understood that Wavelet transform does the decomposition of a signal $s(t)$ into a weighted set of scaled wavelet functions $h(t)$.

Now the definition of S Transform is given by:

$$S(\tau,f) = \int_{-\infty}^{\infty} z(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2 f^2}{2}} e^{-i2\pi f t} dt \tag{4.4}$$

where τ,t are time domain variables and f is a frequency domain variable. $h(t)$ is a one dimensional time domain signal and $S(\tau,f)$ is a two dimensional signal in time frequency domain. The feature which makes S transform unique is that its window in time domain changes inversely with respect to the frequency. So for analysis of high frequencies, shorter window in time domain is used and vice versa. S transform is also good because it provides exactly same signal after taking inverse of the transform. Inverse S transform is given by:

$$z(t) = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} S(\tau,f) d\tau \right\} e^{i2\pi f t} df \tag{4.5}$$

S transform has disadvantage of being computationally more complex and in this window has no parameter to allow its width in time or frequency to be adjusted.

The procedure to find out S transform is given as

1) Determine

$$Z(\alpha) \leftrightarrow z(t)$$

2) Calculate $H(\alpha) \times \delta(\alpha - f)$, by doing this $H(\alpha)$is translated to f.

3) Multiply $w(\sigma)$ to shifted $H(\alpha)$. $w(\alpha, \sigma)$ is the Gaussian window function given by:

$$w(\sigma) = \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2 f^2}{2}}$$

4) Take the inverse Fourier transform.

We can notice that the standard deviation of window is taken as

$$\sigma = \frac{1}{f}$$

The variation of Gaussian window provide the time localization and the $e^{-2\pi f t}$ provides the frequency localization. The exponential factor remains stationary and window is translated. S transform is also important because it gives time frequency plots unlike WT. It has applications in many fields which include Geophysics [12], Biomedical [13], Micro seism [18], Power transformer protection [15], [16], [17].

4.2.2.1 GENERALIZED S TRANSFORM

Generalization of S transform provides better control over the window function [42], [45]. The formula is given by:

$$S(\tau, f, \beta) = \int_{-\infty}^{\infty} z(t) w(\tau - t, f, \beta)\, e^{-j2\pi f t} dt \tag{4.6}$$

In equation 4.4, w is the window function and β is a set of parameters that decide the properties of Gaussian window. The width of window is controlled by changing the standard deviation of the window.

4.3 SIMULATION RESULTS AND DISCUSSIONS

We have started with a simplest of example, which is taken in last chapter. Then performance of S transform is evaluated on different types of multi-component signals. Simulations are done in MATLAB software.

4.3.1 EXAMPLE 1

Now in this case, we have taken example from section 3.3.4 to show the advantage of S transform to separate out the components of a signal. We have used equation 3.9 in this example. The TFD is produced using S transform. This example is only taken to make an observation that why ST is used in case of multi-component signals.

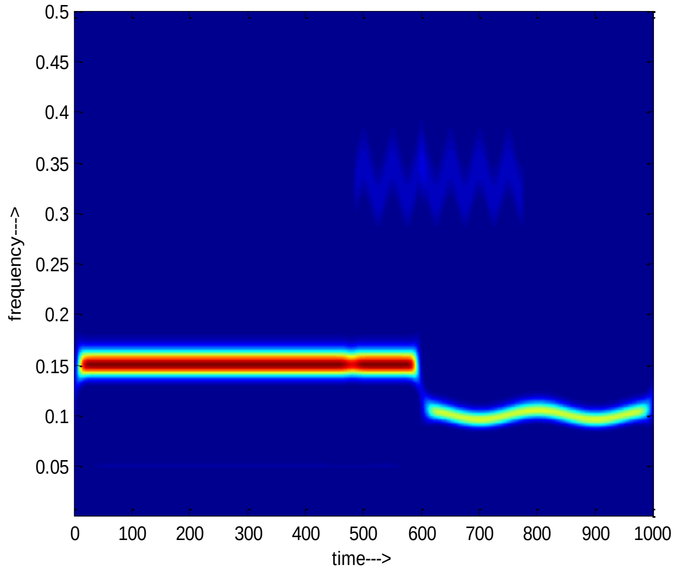


Figure 4.2: TFD generation by ST

The results shows better results than STFT, CT and PCT. The components are separated out very well. We can compare Figure 4.2 , Figure 3.22, Figure 3.23 and Figure 3.24. Conclusion is made that the separation in case of ST is better. Concentration is also a lot better than PCT. Spreading of TFD is also not there in case ST.

4.3.2 EXAMPLE 2

We have taken a multi-component signal in this example, where not only frequency rapidly changes, but also the amplitude of signal varies over the time. Mathematically the signal the signal is written as:

$z_1(t) = sin(2\pi 50t_1) \qquad 0 \leq t_1 \leq 0.3\ sec.$

$z_2(t) = \sin(2\pi \times 50t_2) \qquad 0.3 \leq t_2 \leq 0.6\ sec.$

$z_3(t) = \sin(2\pi \times (50 + 0.2t_3)t_3) \qquad 0 \leq t_3 \leq 0.8\ sec.$

$z_4(t) = \sin(2\pi \times 50t_1) + sin(2\pi \times 350t_1)$

$z_5(t) = \sin(2\pi \times 50t_1)$

$$z(t)=[\, z_1(t) \quad z_2(t) \quad z_3(t) \quad z_4(t) \quad z_5(t)] \qquad (4.7)$$

Equation 4.7 shows that $z(t)$ is the concatenation of the above signals and the signal is sampled at 200 Hz. Figure 4.3 is the graphical representation of the synthetic signal. Signal generated shows that there are two parts where signal limits to zero amplitude. On this signal, we have applied STFT and ST. TFDs are shown in the Figure 4.3 and Figure 4.4.

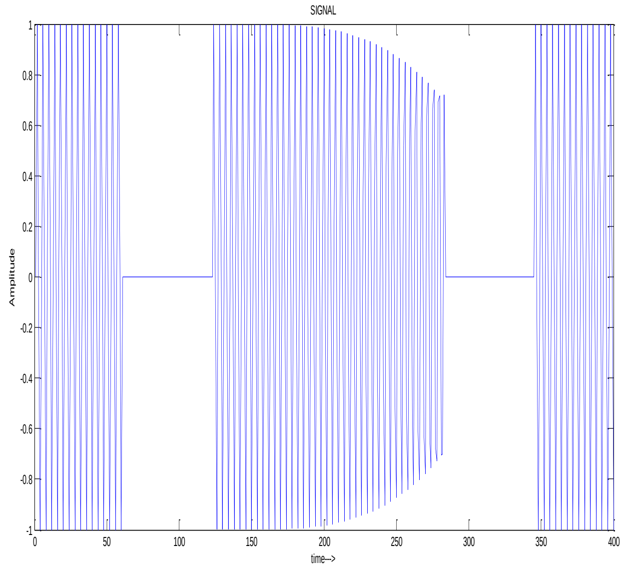


Figure 4.3: Representation of the input signal

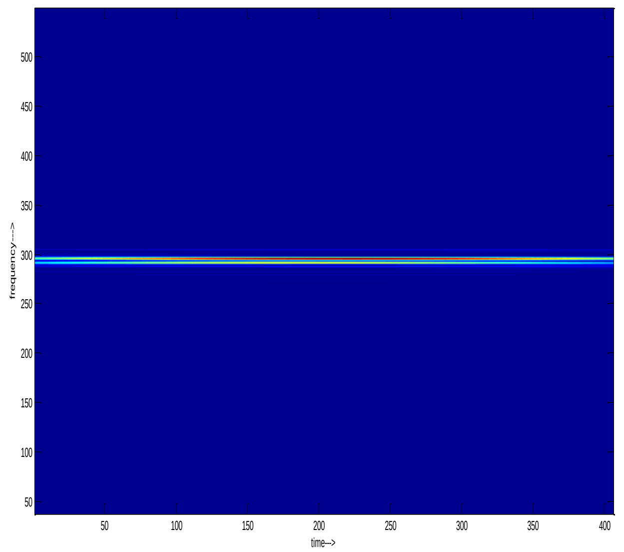


Figure 4.4: TFD generated by STFT

The TFD obtained on applying STFT clearly reflects that signal has no change in frequency all. Therefore giving completely wrong information about the input signal. So on the same signal we have applied ST and notice the TFD to know about the difference between the STFT and ST.

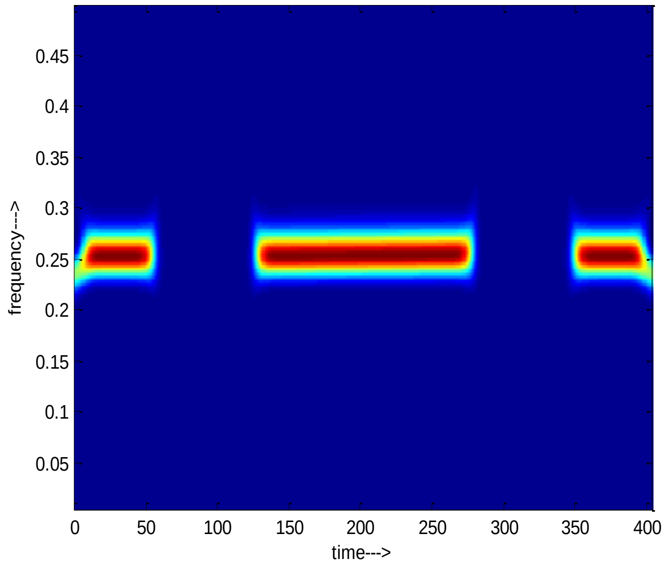


Figure 4.5: TFD generated by ST

The results of analysis using ST are better than STFT. Also the separation is quite good.

4.3.3 EXAMPLE 3

The signal assumed in this case has continuous changes in frequency. In Figure 4.6, we cannot even know at what points the frequency changes.

$$z(t) = \cos(2\pi \times 20t) + \cos(2\pi \times (20 + 50t)t) \tag{4.8}$$

The signal is sampled at 200 Hz. The signal is addition of two components.

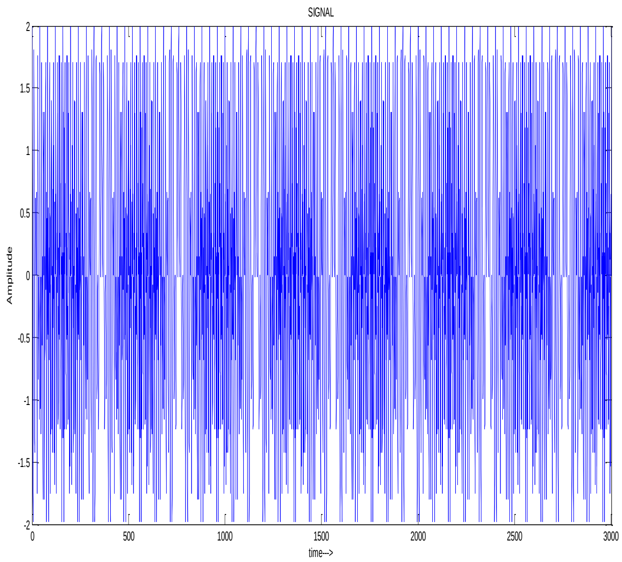


Figure 4.6: Representation of the input signal

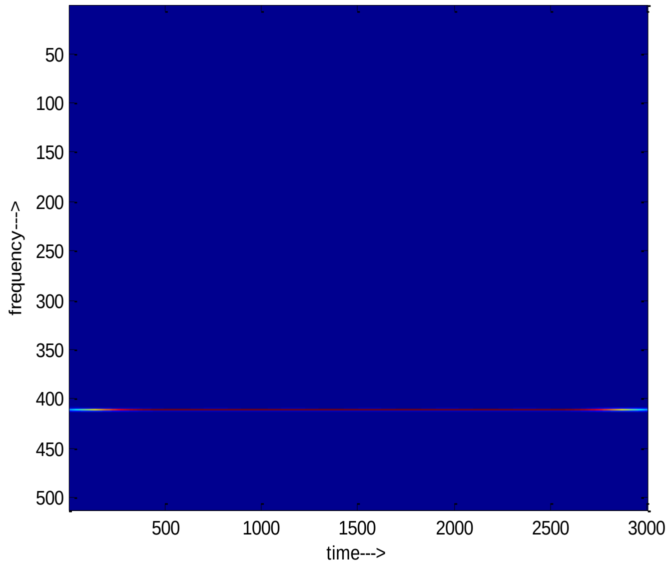


Figure 4.7: TFD generated by STFT

The spectral analysis done by STFT is showing very different results for the multi-component signal. It is because of the fixed window length of the STFT.

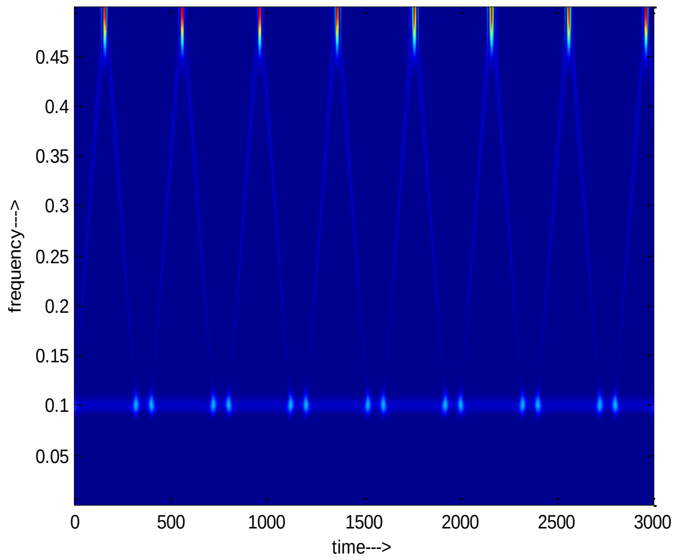


Figure 4.8: TFD generated by ST

The Figure 4.8 shows exact results for changing frequencies. It is due to the varying length of the moving window.

4.3.4 EXAMPLE 4

This example is taken from paper [40] of Bao Han. This is a frequency conversion signal defined as:

$$y(n) = 4\cos\left(\frac{8\pi n}{N}\right) + \frac{N}{5}$$

$$z(n) = \cos(2\pi \times y(n) \times \frac{n}{N}) \tag{4.9}$$

Where n=1,2,3,4....512, N=512 and n is the sampling point.

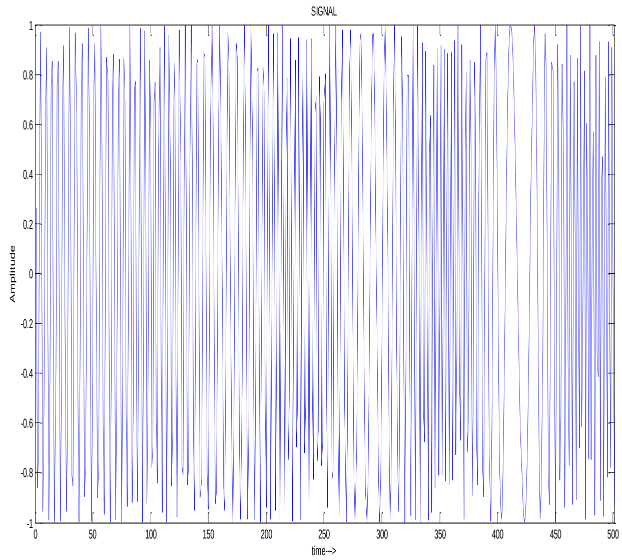


Figure 4.9: Representation of the input signal

Figure 4.10 and Figure 4.11 shows the time frequency distribution using STFT and ST respectively.

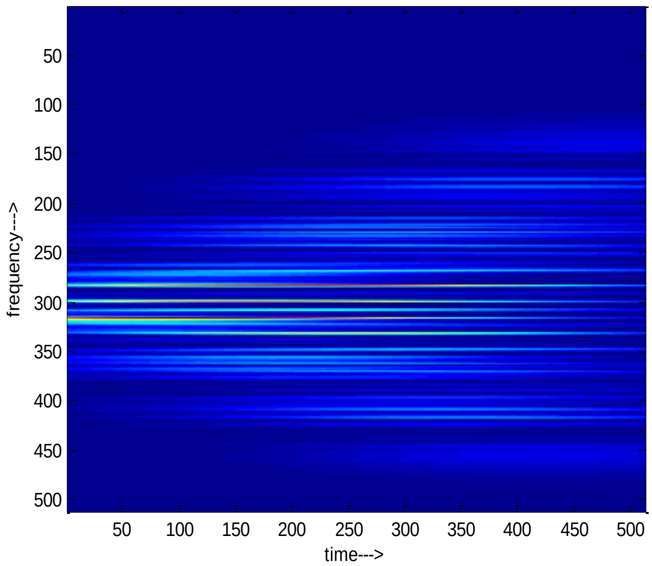


Figure 4.10: TFD generated by STFT

It is evident from Figure 4.10 that STFT has a very poor time frequency resolution. It concludes that STFT has minimum flexibility in window length.

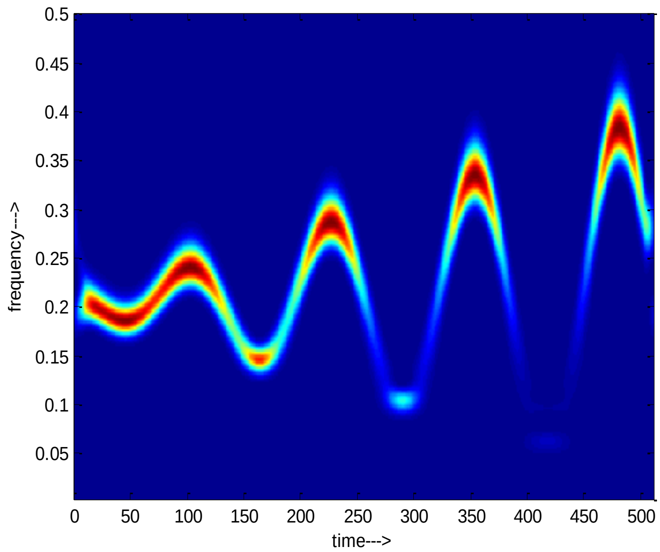


Figure 4.11: TFD generated by ST

The TFD obtained from ST provides good time frequency resolution. The changes in frequency with time are quite evident from the Figure 4.11.

4.3.5 EXAMPLE 5

Now we have considered a third signal which is a multi-component chirp signal is given as:

$$h(t) = sin\left(60\pi t + 12\pi sin\left(\frac{\pi t}{6}\right)\right) + \sin(0.7\pi t^2 + 25\pi t + 25) \qquad (4.10)$$

IF laws of above signal components are $30 + \pi\cos\left({}^{\pi t}/_{6}\right)$ and $0.7t + 12.5$ respectively. This signal has two chirping components.

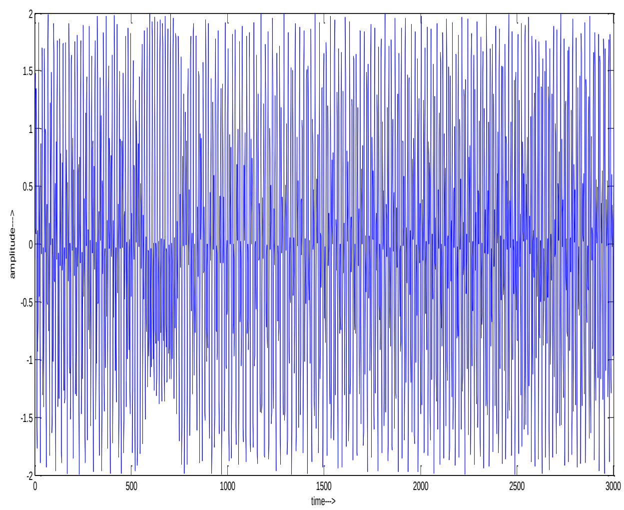


Figure 4.12: Representation of the input signal

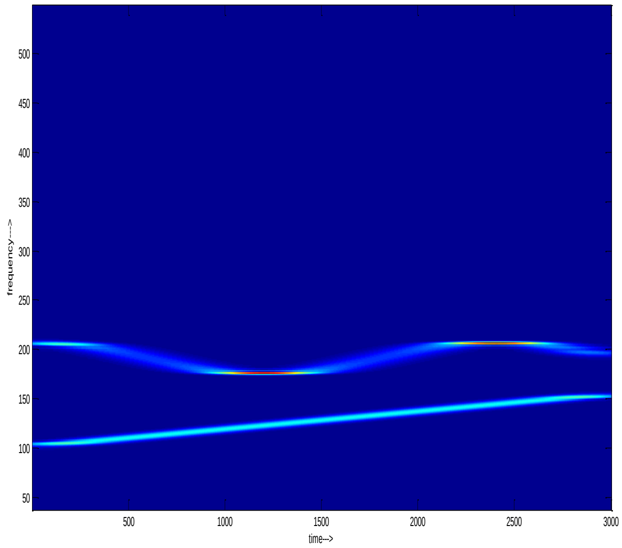


Figure 4.13: TFD generated by STFT

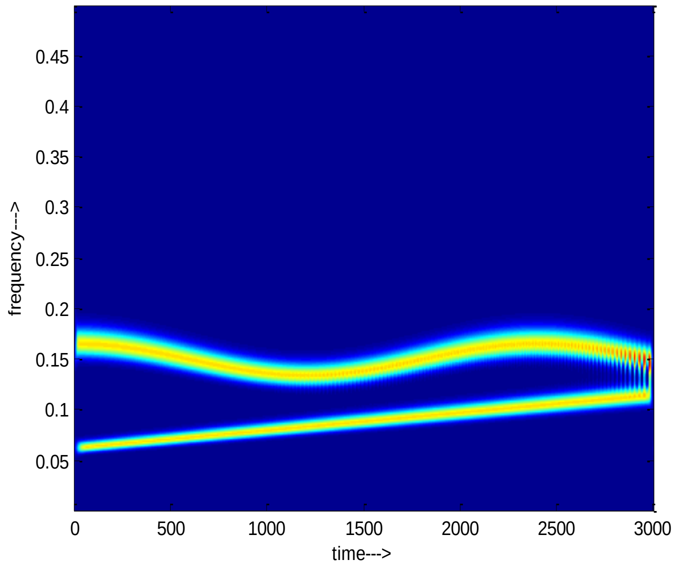


Figure 4.14: TFD generated by ST

From Figure 4.13 and Figure 4.14, it is concluded that IF laws of the signal components do not cross each other. STFT also better resolution in this case and some spreading can be seen in TFD of ST. The comparison with proposed method is also done in next chapter.

4.3.6 EXAMPLE 6

This example explains the working of STFT and ST on a signal in which the IF laws of the components do not cross each other. This multi-component chirp signal is as given below

$$h(t) = \sin(2.5\pi t^2) + \sin(10\pi \sin(\pi t/_{4+\pi}) + 40\pi t - 0.8\pi t^2) \tag{4.11}$$

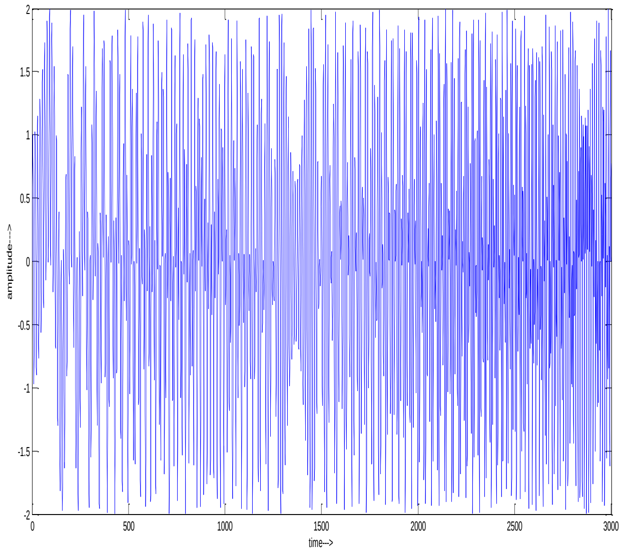


Figure 4.15: Representation of the input signal

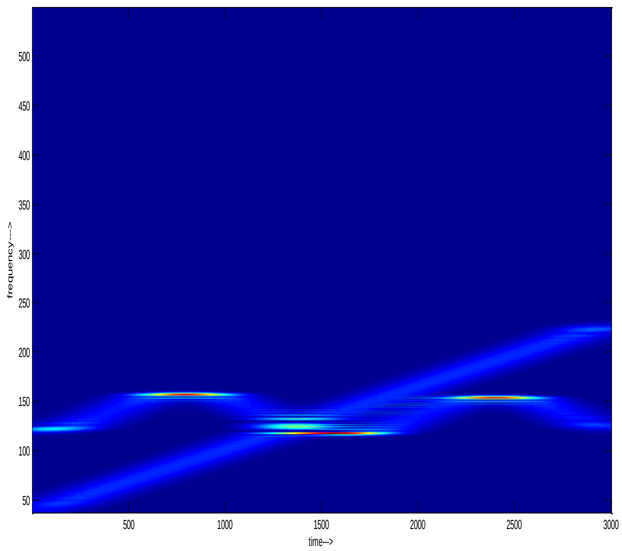


Figure 4.16: TFD generated by STFT

The TFD of the signal in Figure 4.16 shows that IF laws of signal cross each other. But it cannot provide a clear view of the IFs. There is expansion of TFD and concentration is also not good. So it is more interesting to see that whether ST is able to isolate both Ifs and also track them by providing better time frequency resolution.

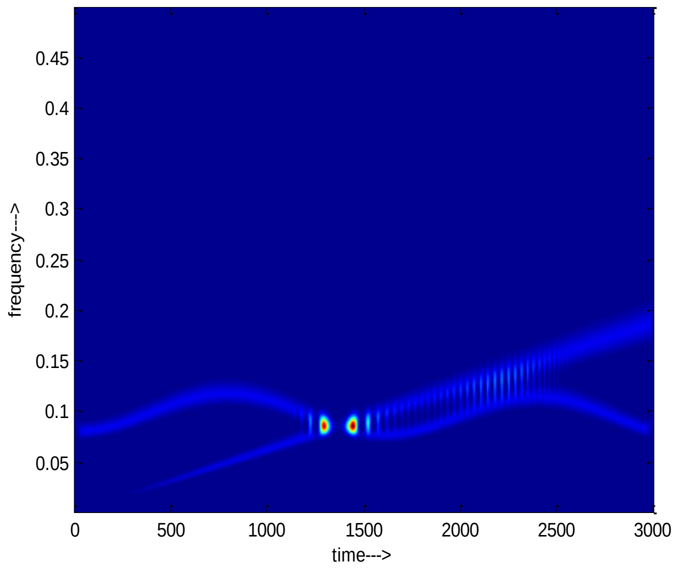


Figure 4.17: TFD generated by ST

The analysis of crossing multi-component by ST shows that there is interference in two components at different times. Also time frequency resolution is poor. In next chapter, we have taken the same signal and modified S transform is applied.

4.3.7 EXAMPLE 7

In this example the performance of the STFT and ST is analysed using some synthetic signal. In this signal a low frequency of 7 Hz, a medium frequency of 25 Hz, and a high frequency of 65 Hz is taken. The components in this signal are short lived and present at different time. The signal is shown in Figure 4.18 and represented by:

$$z_1 = zeros(1,256)$$

$$t_1 = 1{:}\,70\ sec$$

$$z_1(1{:}\,70) = \cos(2\pi \times t_1 \times {}^{7}/_{256})$$

$$z_1(71{:}\,128) = 0$$

$$t_2 = 1{:}\,128\ sec$$

$$z_1(129{:}\,256) = \cos(2\pi \times t_2 \times {}^{25}/_{256})$$

$$t_3 = 30{:}\,60\ sec$$

$$z_1(30{:}\,60) = z_1(30{:}\,60) + 0.5\cos(2\pi \times t_3 \times {}^{65}/_{256}) \tag{4.12}$$

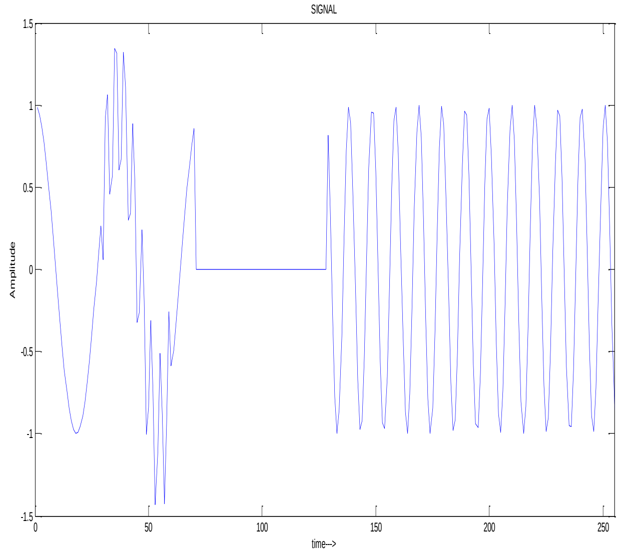


Figure 4.18: Representation of the input signal

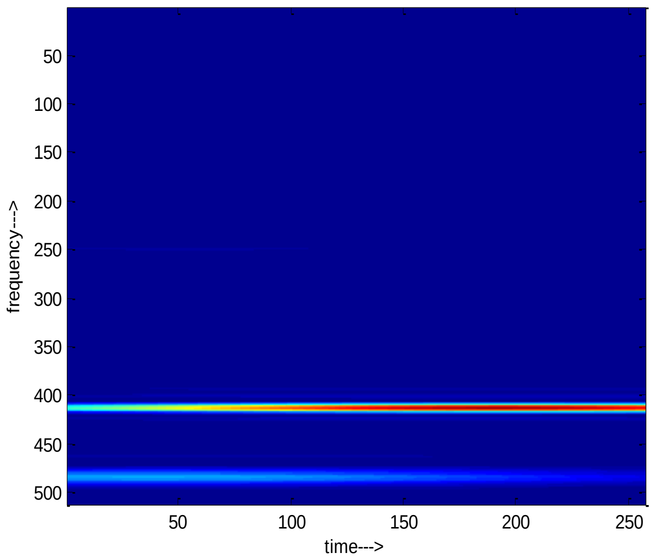


Figure 4.19: TFD generated by STFT

The time frequency resolution provided by STFT is quite poor. The information extracted from Figure 4.19 show that there are two components. But actually there are three. So STFT totally failed in this case. The TFD generated using S transform separates out the components, but they are quite invisible. One component shows good concentration. Other two are in faded form. We can observe from Figure 4.20 that one component of frequency lives for longer period of time than others. The performance of ST is not quite ideal, but still we can evaluate information about the nature of signal.

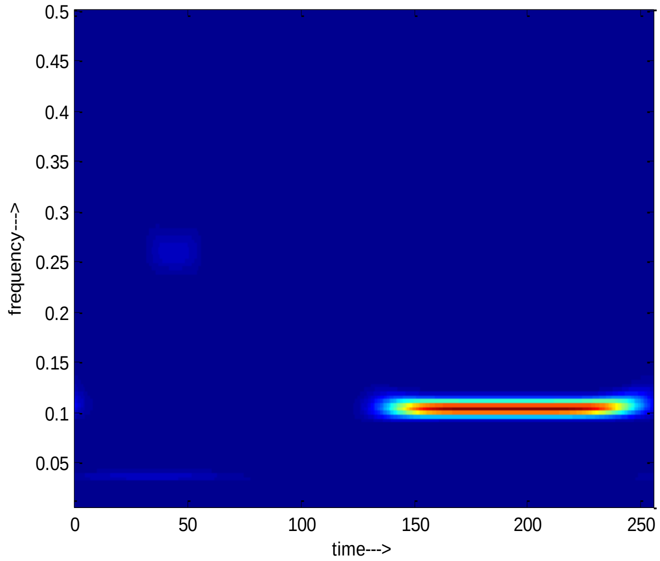


Figure 4.20: TFD generated by ST

The summary of whole chapter is that ST overcomes the demerits of STFT and WT. It provides the best properties of STFT and WT. In next chapter, we have proposed a new method to analyse both chirp and multi-component signals. Also comparison with other methods is done.

CHAPTER 5
PROPOSED WORK

5.1 INTRODUCTION

The development of STFT gives boost in the area of time frequency analysis. In STFT, localization in time is done by using a window function and localization in frequency is applied by using Fourier transform. Due to fixed length of window, STFT has poor resolution in time frequency domain. On the other hand WT does not retain the phase information and provides time scale plots which are difficult to analyze. The S transform is developed by Stockwell, which is a combination of both STFT and WT. Based on the idea of S transform we have proposed a new TFA method called as modified S transform. In this method, we have introduced a parameter to change the width of window according to the covariance of the input signal which is discussed in detail in the following section. This method provides better results for both chirp and multi-component signals.

5.2 MODIFIED S TRANSFORM

In this proposed method, we have inserted a parameter β to change window length in S transform. The parameter β is given as

$$\beta(f) = n \times \gamma \times f \tag{5.1}$$

where γ =covariance of the input sgnal.

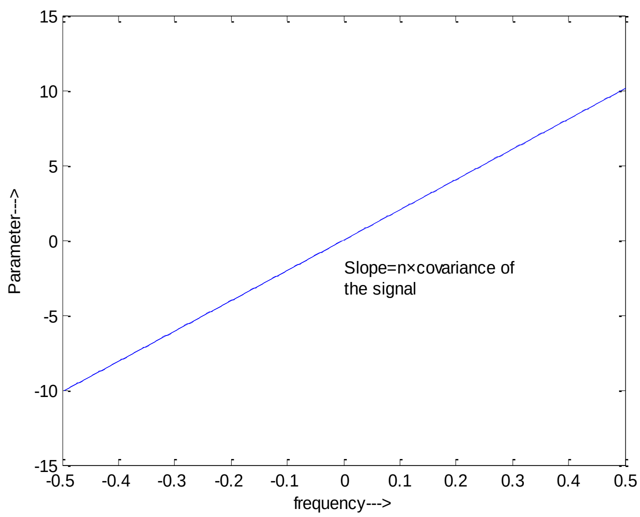


Figure 5.1: Change in parameter w.r.t. frequency

In equation (5.1), n is a constant that is experimentally decided by doing several iterations. It also shows that resolution in time and frequency depends directly on the covariance of the signal. Standard deviation of the window is thus updated as follows:

$$\sigma = \frac{\beta(f)}{|f|} \tag{5.2}$$

The modified S transform thus becomes

$$S(\tau, f) = \int_{-\infty}^{\infty} h(t) w(\tau - t, f, n, \gamma)\, e^{-j2\pi f t} dt \tag{5.3}$$

where w is a window function of the proposed method, represented by

$$w(\tau - t, f) = \frac{|f|}{\sqrt{2\pi}\beta(f)} e^{-\frac{(\tau-t)^2 f^2}{2(n\gamma f)^2}} \tag{5.4}$$

From (5.3) and (5.4), we get

$$S(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{\sqrt{2\pi} \text{n}\gamma\text{f}} e^{-\frac{(\tau-t)^2 f^2}{2(n\gamma f)^2}} e^{-i2\pi f t} dt \tag{5.5}$$

Thus we have controlled the time and frequency resolution using the standard deviation of the Gaussian window.

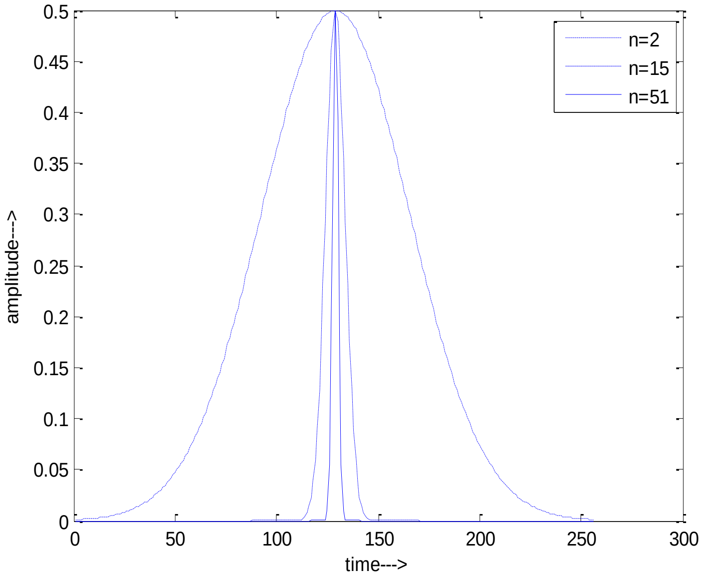


Figure 5.2: Variation of window width with n

From figure 5.2, the variation of window width is shown for different values of n. It shows that the values of n should be selected with care to get better energy distribution of time frequency distributions. The other value on which β depends is the covariance of the input signal. Thus the width of window directly depends on the covariance of signal also. Thus making it to track the IF law of the chirp and multi-component signals.

5.3 SIMULATION RESULTS AND DISCUSSIONS

In this section, we have compared modified S Transform with all other methods discussed in chapter 3 and 4.

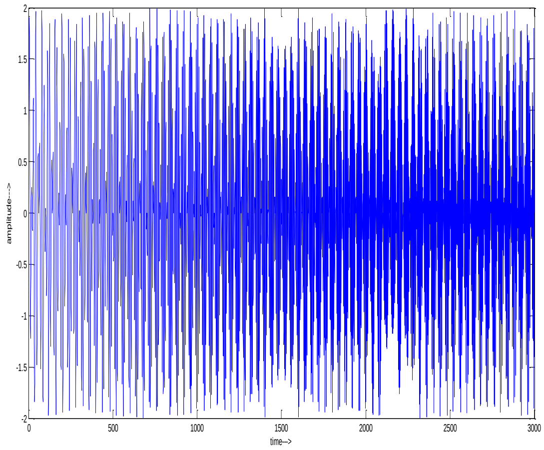
5.3.1 EXAMPLE 1

In the first test signal, we have considered linear multi-components, which is given as

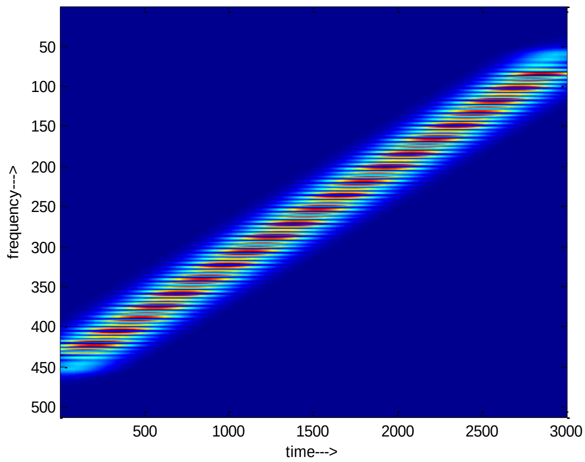
$$h(t) = \sin(2\pi(10 + 2.5t)t) + \sin(2\pi(12 + 2.5t)t)$$

$$(0 \leq t \leq 15s). \quad (5.6)$$

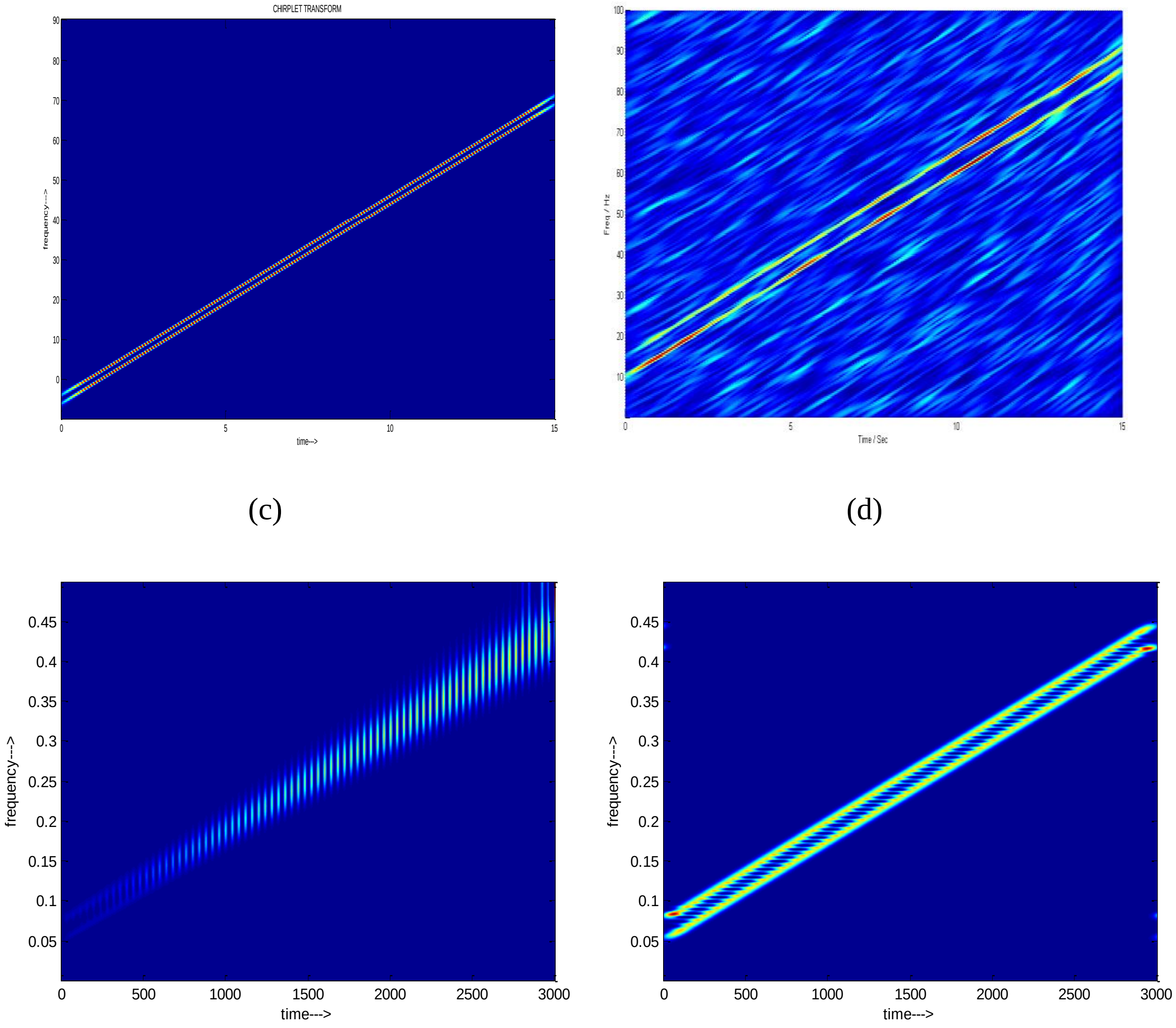
The signal is sampled at a sampling frequency of 200 Hz. The TFDs are shown in Figure 5.3(b)-(f). Conclusion can be made by observing the figure that only CT, PCT and Modified S Transform are able separate the components. Figure 5.3(b) concludes that two components are not separated by STFT. We can say that STFT works poorly on this signal. There is interference between the IF laws of two components due to fixed window length. We have also used S transform having varying window. The result of TFD generated by ST is also unable extract the information. out of signal and separation is again poor in this case. Figure 5.3(e) shows the results of TFD generated by ST.



(a)



(b)



(c) (d)

(e) (f)

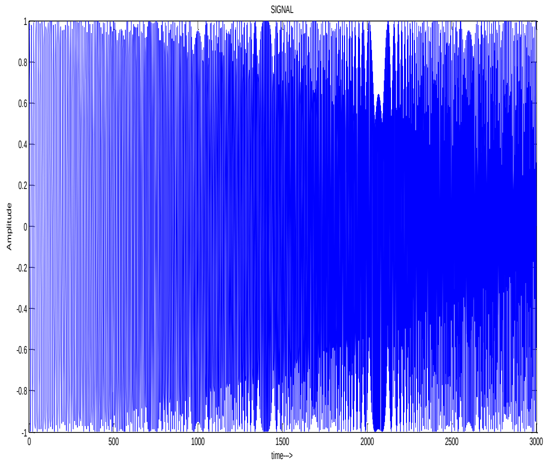
Figure 5.3: Time frequency plot (a) Signal (b) STFT (c) CT (d) PCT (e) ST (f) Modified ST

This example is taken surely to show that there are some signals whose components cannot be distinguished by STFT and ST. In this case, our proposed method gives very good results. It separates the components and provides good concentration as well. Figure 5.3(f) shows the time frequency analysis of this signal using proposed transform. The modified transform kernel tracks both the components because its window is dependent on covariance of input signal. Obviously it is not providing good concentration as shown in Figure 5.3(f), but is far more better than ST and STFT.

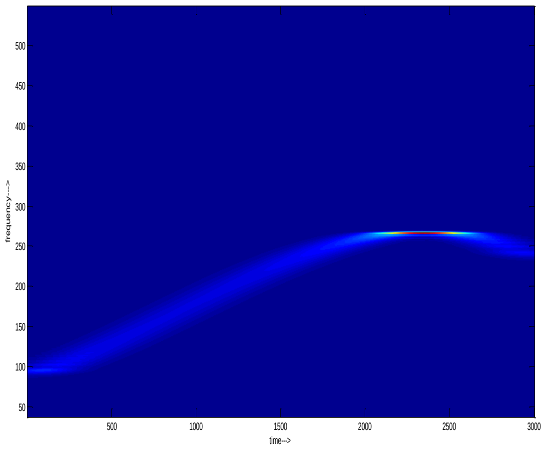
5.3.2 EXAMPLE 2

In this example, we have taken example in section 3.3.2 as reference. This example shows that we can use modified S transform in case of mono-component signals also. Further the comparison between STFT, CT, PCT, ST and Modified ST is also done. Figure 5.4(e) shows that ST provides good concentration, but broadens at the end. It is showing same properties as CT, which can be seen in Figure 5.4(c).But it does not provide better concentration and tracking of signal as in case of PCT.

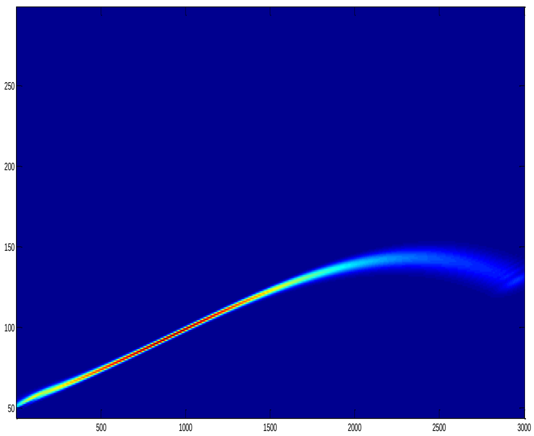
In Figure 5.4(f), we can see that our proposed method give good concentration in case of chirp signals as well. Also its kernel follows the IF law of chirp signal. TFD generated by Modified S transform isolates noise from the input signal as compared to PCT given in Figure 5.4(d). PCT is providing excellent feature extraction of the signal as its TFD provides best concentration than all others in this case.



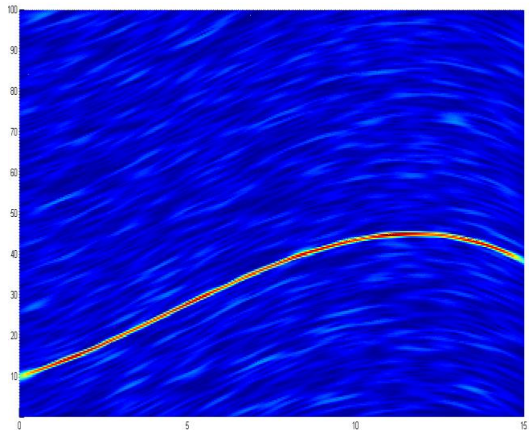
(a)



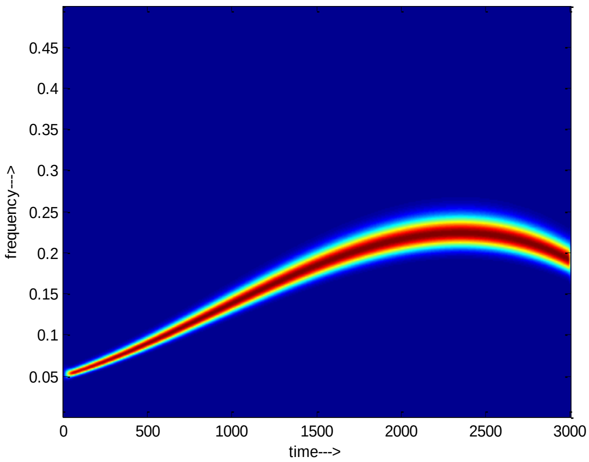
(b)



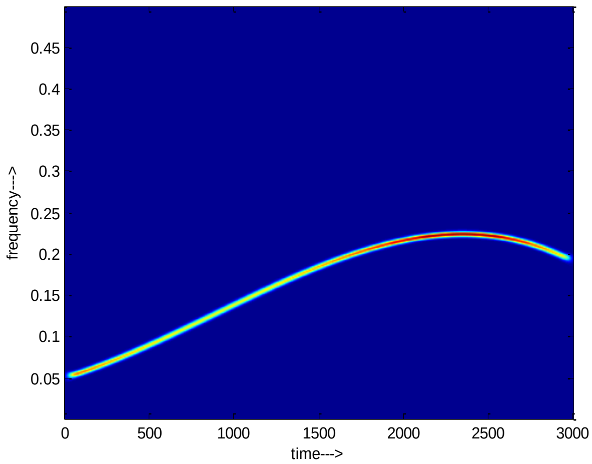
(c)



(d)



(e)



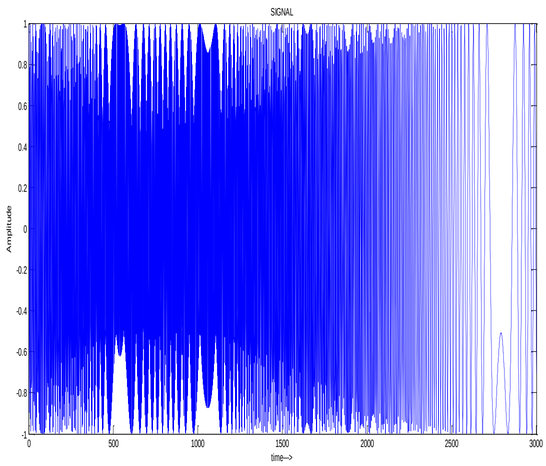
(f)

Figure 5.4: Time frequency plot (a) Signal (b) STFT (c) CT (d) PCT (e) ST (f) Modified ST

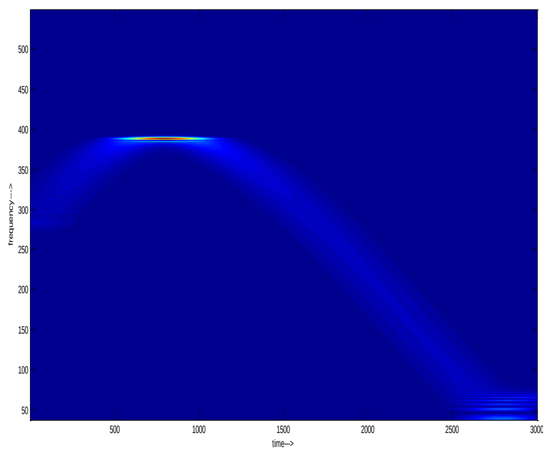
This implementation of transforms on this signal is obviously taken to see if Modified version of S Transform works accurately on non linear chirp signals. TFD using Modified ST shows very good results.

5.3.3 EXAMPLE 3

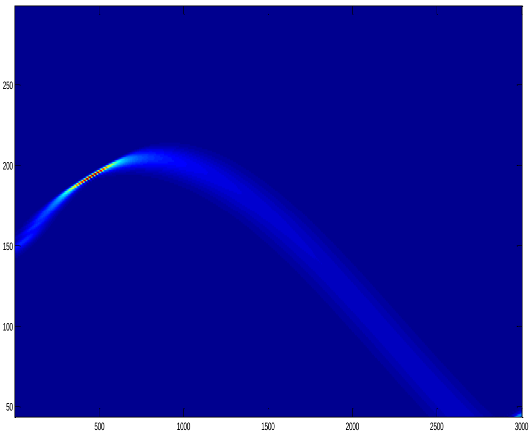
In this case, section 3.3.3 is taken as reference. As we know in that example emphasis is given on the point that the IF trajectory of the signal in the last part of signal is followed or not. Figure 3.18, Figure 3.19 and Figure 3.20 are compared to Figure 5.10. It shows that there is ideal following of IF in case of Modified ST.

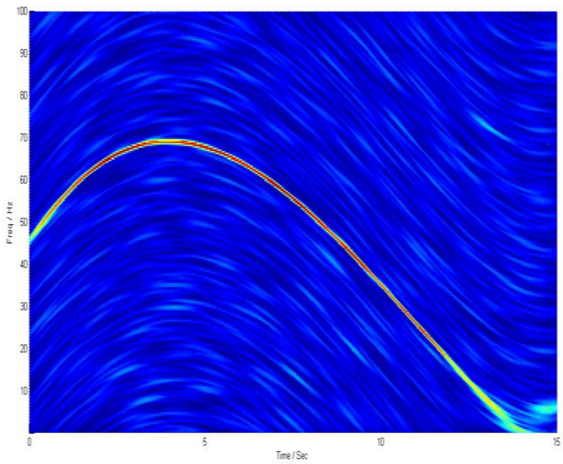


(a)



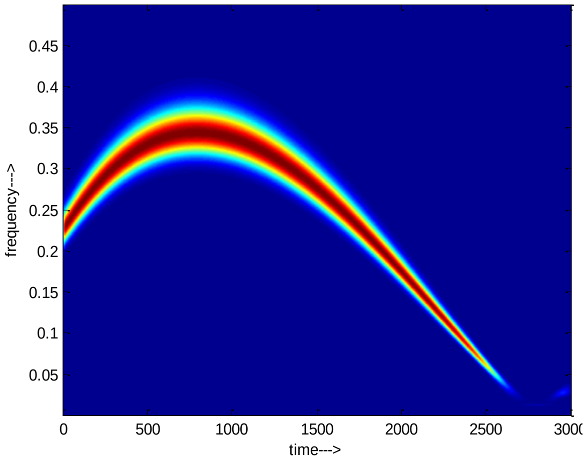
(b)

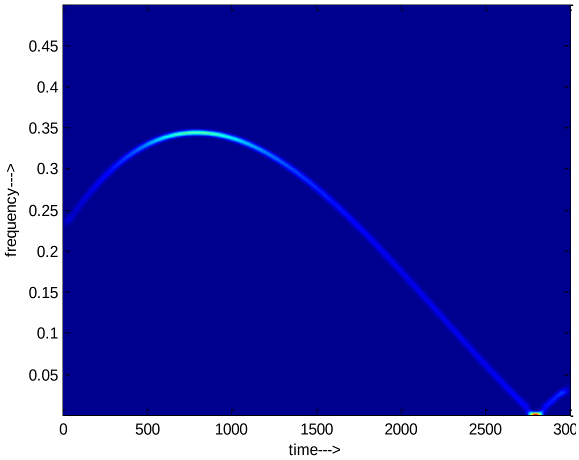




(c)

(d)



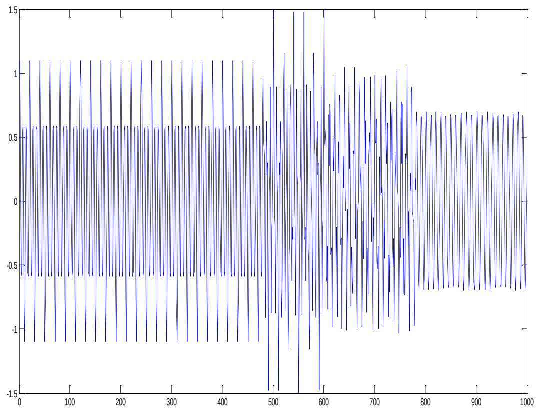


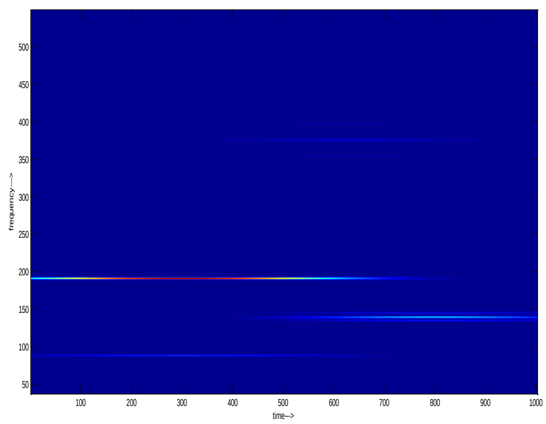
(e)

(f)

Figure 5.5: Time frequency plot (a) Signal (b) STFT (c) CT (d) PCT (e) ST (f) Modified ST

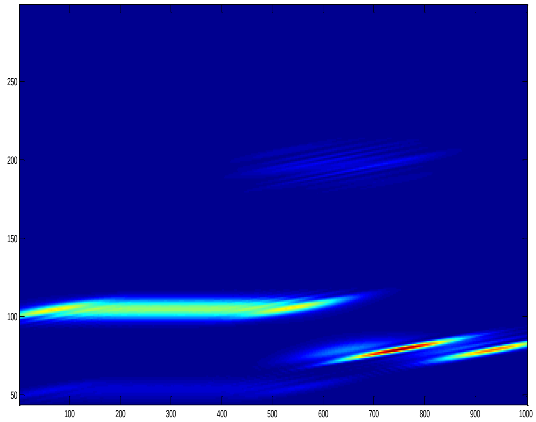
5.3.4 EXAMPLE 4

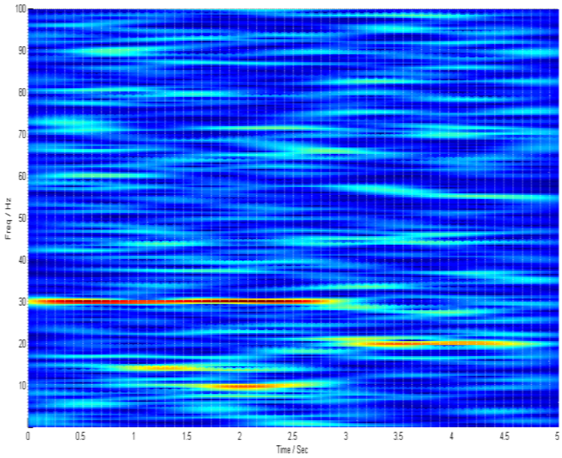




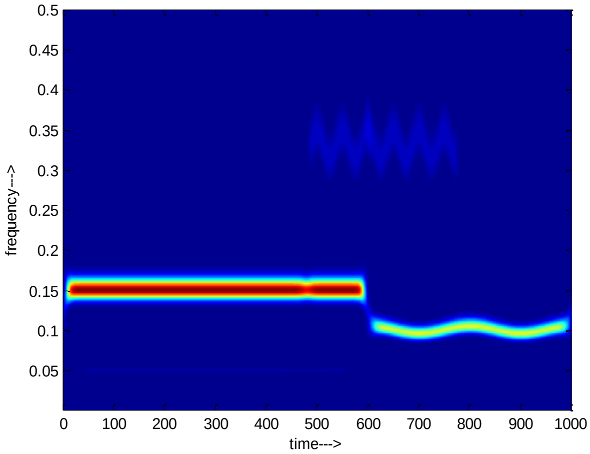
(a)

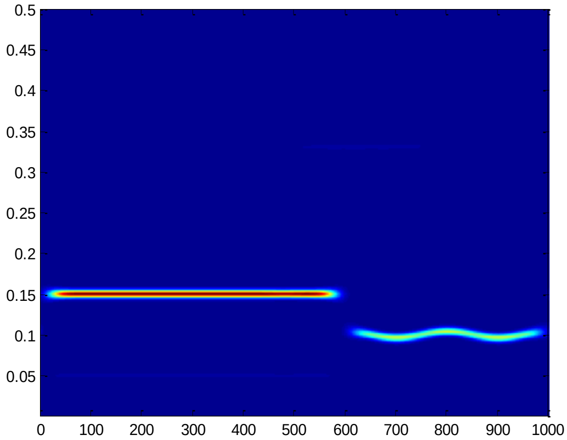
(b)





(c) (d)





(e) (f)

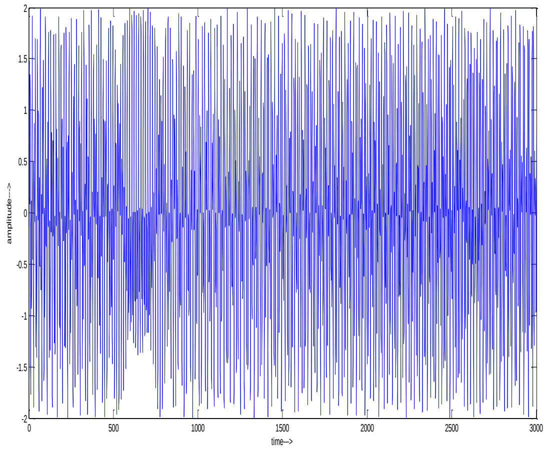
Figure 5.6: I Time frequency plot (a) Signal (b) STFT (c) CT (d) PCT (e) ST (f) Modified ST

This example is taken from section 3.3.4 of chapter 3. The comparison of TFD generated by all transform including Modified S transform is done in Figure 5.6. It ultimately leads us to result that Modified S transform provides best results in all. In CT, there is spreading in the TFD. PCT adds up noise in the TFD. Input signal is multi-component signal. Therefore it is necessary that the applied transform separates out components and provides no interference between the components. From the results in Figure 5.6, we can see that Modified S Transform gives smoother distributions and good time frequency resolution. Thus it can be useful in applications where smoother separation is required. Here STFT is also providing good results.

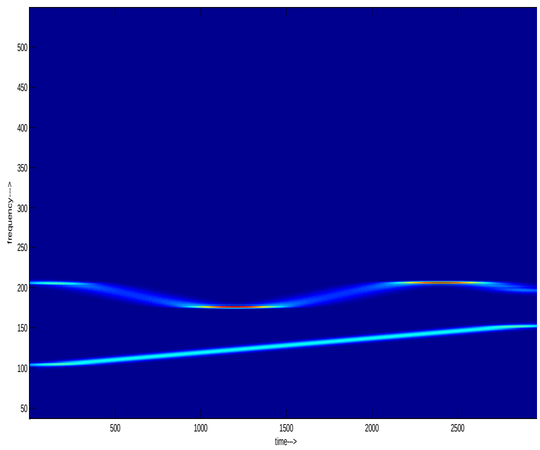
5.3.5 EXAMPLE 5

The signal have two components whose IF laws do not cross each other. This case we have taken form section 4.3.5.

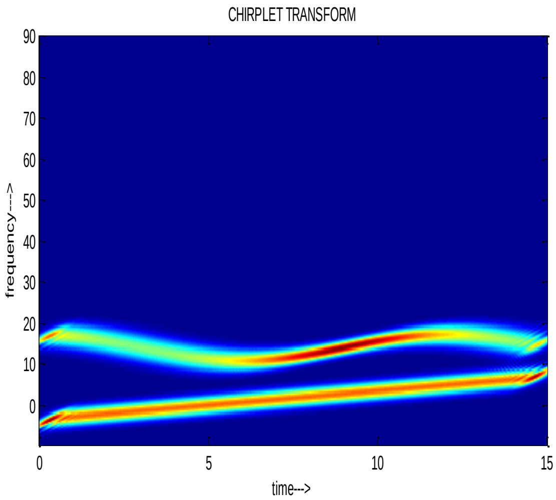
Clearly results shown in Figure 5.7 shows that again our proposed method is the best among all others. All aspects of good TFD are generated by Modified ST. It provides good separation and IF laws of components are followed well. PCT has disadvantage of added noise. CT and ST provides expanded TFDs. STFT gives good results but energy concentration is poor.



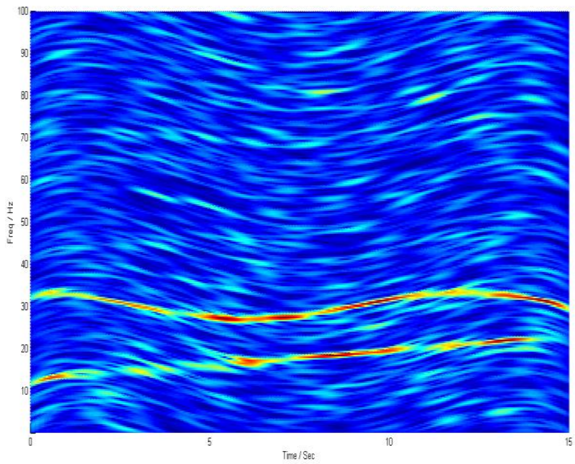
(a)



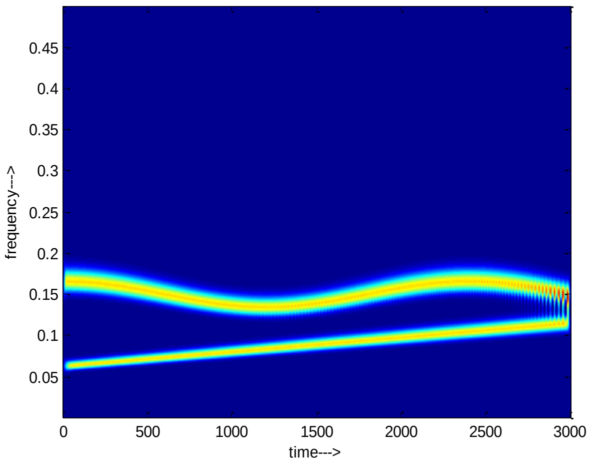
(b)



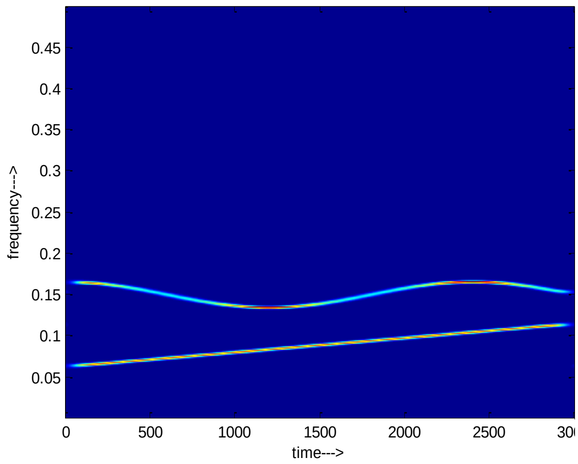
(c)



(d)



(e)



(f)

Figure 5.7: Time frequency plot (a) Signal (b) STFT (c) CT (d) PCT (e) ST (f) Modified ST

5.3.6 EXAMPLE 6

In above example we have taken signal with crossing IF law and now to evaluate the performance of proposed transform, we have taken signal with crossing IF laws. To analyze the signal, we have applied all the methods discussed in this dissertation. Results show better performance of Modified ST. This multi-component chirp signal is as given below

$$h(t) = \sin(2.5\pi t^2) + \sin(10\pi \sin(\pi t/4 + \pi) + 40\pi t - 0.8\pi t^2) \tag{5.7}$$

The TFDs of given signal are shown in Figure 5.8. Clearly form figure it is understood that Modified S Transform separates the components. There is no interference between the two components. PCT gives poor time frequency resolution. It adds up noise and there is no clarity at starting and end points of signal. We cannot recognize which frequency belongs to what time. Rather STFT, CT and ST provides better information, but separation at meeting points is not quite good. The TFD is useful method to extract information out of the signals. Signal with crossing frequencies is really good object on which we can measure the performance of time frequency analysis method. It is clear from the examples that Modified ST is useful for the analysis of both chirp and multi-component signals. Proposed modified S transform is able to generate the TFD with energy concentration closely along the IF laws of the signal which is always necessary for isolation of components.

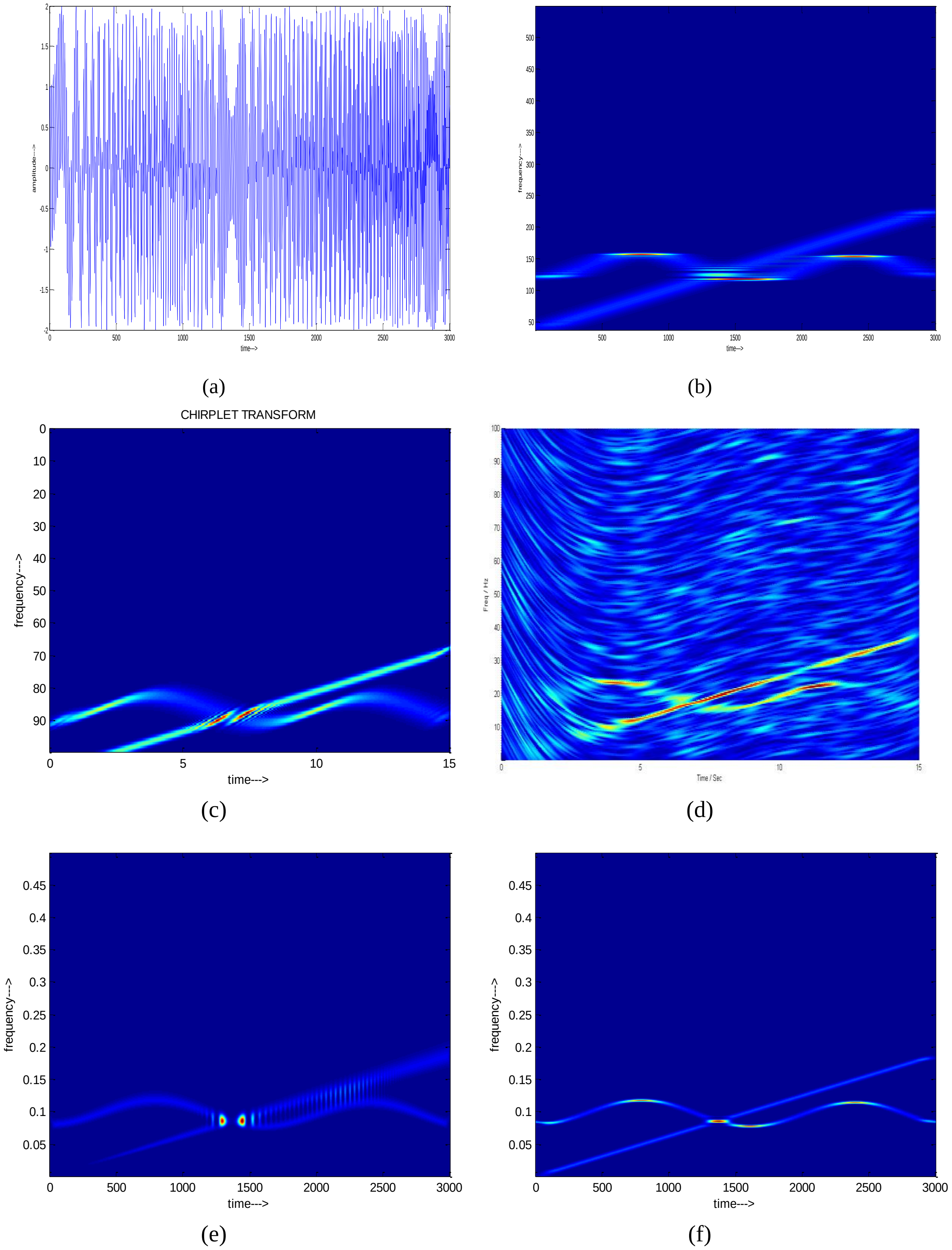


Figure 5.8: Time frequency plot (a) Signal (b) STFT (c) CT (d) PCT (e) ST (f) Modified ST

The summary of chapter is that most of the times Modified S Transform has provided better results. Even better than the transform form which it is derived.

CHAPTER 6

CONCLUSION AND FUTURE SCOPE

6.1 CONCLUSION

Energy concentration of the S transform can be controlled better by the effective variation of the width of the window. This is achieved by using a parameter β in the window, which varies according to the covariance of the input signal and thus varies the S transform kernel according to the variations in the input signal. The proposed method is compared with the original S transform, PCT, CT and STFT by using a set of synthetic signals. The comparison is done by using all five methods and it shows that the proposed modified S transform provides improved results as compared to standard S transform and STFT. The proposed method provides better time and frequency resolution also. Thus the proposed method can be applicable where good time and frequency resolution is needed. We also found that S transform is better than STFT because of its varying window. In some cases, it is unable to separate the components of the signal. PCT is also a good method for chirp signals but it has a disadvantage of contamination by Gaussian noise. For some signals PCT could not able to provide better time frequency resolution than proposed method. But it is far better than CT and STFT in case of non linear IF law following signals.

6.2 FUTURE SCOPE

In future, we can optimize parameter β of the Modified S Transform, rather than fixing it ourselves. Also we can use the proposed method in several numbers of applications like ultrasound imaging, image processing, digital watermarking, radar detection, micro doppler shift detection etc.

LIST OF PUBLICATIONS

1. Jitain Sharma, and Sunil Datt Sharma, "*Analysis of the multi-component signal using modified S transform*", International Journal of Applied Engineering Research. (Communicated)

2. Jitain Sharma, Vivek Thakur, and Sunil Datt Sharma, *"Role of transforms for the analysis of chirp signal-a review"*, ICSP, 2016. (Communicated)

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