ANALYSIS AND DESIGN OF FOLDED PLATES

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CERTIFICATE

We here by certify that the work which is being presented in the project entitled "**Analysis and Design of Folded Plates**" in the partial fulfillment of the requirement for the award of the Master of Technology and submitted in the department of civil engineering of the Jaypee University of Information Technology (JUIT), Waknaghat, Distt. Solan. (H.P.) is an authentic record of our own work carried out during a period from July 2013 to May 2014 under the supervision of Dr. Ashok Kumar Gupta, Prof. and Head, Civil Department.

This work has not been submitted, partially or wholly to any other university or institute, for the award of this or any other degree or diploma.

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This is certified that the above statement made by the candidates is correct to the best of our knowledge.

Signature of Supervisor   Signature of Student

Dr Ashok K. Gupta

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I also thanks to national and international companies working on Analysis and Design of Folded Plates.

Lastly, but not the least, I wish to thank all my family, friends, teachers and everyone else who help me along the way, thank you for your support.

Signature of the student……………………..

Name of Student……………………..

Date……………………..

ABSTRACT

This work presents the preliminary analysis of folded plates using three procedures .First one is Transverse slab analysis, longitudinal beam analysis and making compatibility of stresses. By using Winter-Pei method with correction analysis and without correction analysis and also design of reinforcement in folded plates and supporting diaphragms. This work also presents ananalysis of continuous folded plate roofs considering the effects of relative joint displacements. For this analysis the normal modes of the lateral beam vibration were used as the form of the deflection curve and the loading was sinusoidal. By using symmetry and anti-symmetry, a possible method of analyzing prismatic folded plate roofs comprising one bay transversely but continuous over two or more spans longitudinally is suggested.

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LIST OF SYMBOLS

Symbol	Description
A_n	cross sectional area of plate n
a_n	Horizontal length of plate n (slab section)
$C_{n,\ n+1}$	carry over factor from joint n to joint n+l
$D_{n,\ n+1}$	stress distribution factor at joint n of plate n+l
ft, fb, fn	longitudinal fiber stresses 1n the plates at top, at bottom and at joint n
E	modulus of elasticity
I	moment of inertia
h	plate height (slab section)
L	span length
M	bending moment
N_o	normal load
N_n	resultant shearing force at joint n
$P_{n,n-1}$	plate loads on plate n due to slab reaction at joint n-1
R	slab reaction
K_n	factor for the actual joint displacement resulting from an arbitrary rotation of plate n
S	section modulus
T_n	longitudinal shearing force at joint n
T	plate thickness
V_n	vertical jo1nt settlement of joint n
V	shearing stresses
y_n	deflection of plate n in 1ts own plane

INTRODUCTION

Folded plate structures have aroused attention in recent years because of their economic advantage and architectural appearance. Longer spans may be due to the inherent stiffness without an increase in material requirement. This type of structures has gained increasing popularity and offers more advantages than more complex structures, such as cylindrical shells, arches and frames.

The ASCE Task Committee on Folded Plate Construction issued a report in 1963 in which they summarized the status of analyses for folded plate structures and provided an extensive list of references on prismatic shells.

Most of the methods are limited to folded plates on simply supported spans.The primary purpose of this investigation is the determination of the stress distribution and the effects of relative joint displacements for folded plate structures continuous over two or more inter mediate supports.

The analysis is based on extension of Gaafar's method which has been modified and recommended as a dependable and satisfactory method of analysis for prismatic folded plates on simple spans by the ASCE Task Committee.

Since one of the assumptions made in folded plate design is that the supporting members (diaphragms, beams frames, etc.) are infinitely in their own planes ,folded plate structures continuous over two spans longitudinally might be considered as two separate spans with one end simply supported and the other built-in. If there are more than two spans, the structure could be analyzed by assuming that the middle spans have both ends built-in and the exterior span has one simply supported end the exterior span has one simply supported end and one built-in end.It is necessary to select a sinusoidal load so as to deflect the slab to confirm with the deformed plates. The distribution of these sinusoidal loads along the structure is according to the normal function of free vibration, which will make the plate deflection proportional to the load distribution. The use of the normal functions results in a considerably simplified procedure for finding the stresses and deflections in continuous structures, regardless of the type of external load acting on the structure.

In analyzing continuous folded plate structures, the following basic assumptions will be followed which are recommended by the ASCE Task Committee.

1. The material is homogeneous, isotropic, and linearly elastic.
2. The actual deflections are minor relative to the overall configuration of the structure. Consequently, equilibrium conditions for the loaded structure may be developed using the configuration of the undeflected structure.
3. The principle of super-position holds; this assumption is actually derivable from the previous two assumptions.
4. Longitudinal joints are fully monolithic with the slab acting continuously through the joints.
5. Each supporting end diaphragm is infinitely stiff parallel to its own plane but is perfectly flexible normal to its plane.
6. The length of each plate is greater than twice its width, and the thickness is small compared to its width.
7. The longitudinal joints are assumed completely monolithic.
8. All plates are rectangular. Each plate has uniform thickness.
9. The structure is supported on end diaphragms which are assumed to be completely rigid in the in-plane direction and perfectly flexible in the direction normal to the plane.

In 1963, the American Society of Civil Engineers suggested a method for analyzing simply supported prismatic folded plate structures. It is based on Gaafar's original paper, and has the following major assumptions:

1. The longitudinal distribution of all loads on all plates is the same.
2. The structural action is considered as a combination of transverse continuous one –way slab action and longitudinal plate action or beam action. The longitudinal stresses are assumed to vary linearly across the plate width.
3. Displacements due to forces other than bending moments are neglected.

CHAPTER - 2

LITERATURE REVIEW

The principle of folded plate construction was first developed by Mr. G. Ehlers and Mr. Creamer in Germany in 1930.They considered the various plate elements as beams supported at the joints and end diaphragms. Along the longitudinal edges, the plates were assumed to be connected by hinged joints. They proposed a folded plate theory based on a linear variation of longitudinal stress in each plate but neglected the effects of the relative displacements of the joints. Since that time, there have been numerous papers written on the subject. Messer's. Winter and Pei published a paper in 1947 in which they transformed the algebraic solution into a stress distribution method, which has the advantage of numerical simplicity over the algebraic procedure.

The first method to take into account the effect of relative joint displacement was proposed by Messer's Gruber and Gruening.For determination of the ridge moments and displacements,Mr. Gruber developed his solution in the form of simultaneous differential equations of the fourth order. Consequently, he concluded that the influence of the rigid connections ought not to be neglected.

Recently, Mr. I. Gaafar and Mr.Yitzhaki have introduced methods which consider separately the longitudinal distribution of transverse moments due to applied loads as distinct from that due to relative joint displacement.

Finally, the ASCE Task Committee on folded plate construction has reported an interesting study of the available methods for the analysis of folded plate structures and recommended a design method for prismatic folded plates on simply supported spans.

A limited amount of work was done on continuous folded plate structures by Mr. Gruber in 1952.He developed a series of simultaneous differential equations of higher order for the solution. From a practical point of view, this work calls for prohibitively extensive mathematical computations.

Portland Cement Association Bulletin suggests two approaches for analyzing folded plates, without further explanation ,the paper mentions the complexity of this theory for folded plate structures which is due primarily to the fact that the transverse distribution of longitudinal stresses is not uniform throughout the length of the folded plate as for simple spans. One expedient way which might be employed to overcome this difficulty is relaxing the requirement of satisfying the condition of compatible deflections at mid span. The deflection of each plate at mid span is determined has been used and has given satisfactory results is to proportion the longitudinal stresses over the support and at mid span on the basis of the moments created in a continuous beam whose spans are equal to those of the folded plate. In this approximation, the transverse distribution is based on an effective span length equal to the distance between the points of inflection of the continuous beam.

Mr.Ashdown presented a complete calculation for a three span continuous prismatic roof but neglected the effect of the relative joint displacement. He assumed that a plate which is continuous over supporting stiffeners can be considered as an ordinary continuous beam for the determination of the longitudinal bending moments at the ends of any span.

As for the continuous folded plate structure considering the effect of relative joint displacement, Mr. D. Yitzhaki originated the particular loading and slope deflection method for analyzing continuous two span folded plate structures.

An analytical solution for the interior panel of a multiple span, multiple bay, ribless prismatic shells was presented by Lee, Pulmano and Lin in February, 1965.The general approach is similar to the treatment of continuous ribless cylindrical shells, but the study is limited to the investigation of the interior panel of loads uniformly distributed in the longitudinal direction.

It is also necessary to solve 8r simultaneous linear equations, where r is the number of plates, for each harmonic of the trigonometric series.

The method developed in this thesis is a synthesis of many methods outlined above. It can be applied to multi-span continuous folded plate under symmetrical loadings which include distributed loads, concentrated loads and inclined loads. In order to make a comparison, the author of this paper used the same assumptions of loading and other conditions of Mr.Yitzhaki and Mr. Ashdown and extended Mr.Gaafar’s method to two and three-span continuous folded plate structures.Important to ensure that each of the project team members understand their responsible in order to achieve the required objectives. Besides that, the annual management review needs to be conduct by the top management to ensure the effectiveness of implementing the Quality Management System. On top of that, the availability of the resources is important for smooth construction process and the top management also needs to plan the optimal usage of the resources.

CHAPTER- 3

NUMERICAL ANALYSIS AND DESIGN

3.1 PRELIMINARY ANALYSIS BY WINTER-PEI METHOD

The preliminary analysis of a V-type folded plate a span of 18 m shown in fig is carried out .The thickness of a horizontal slab is 120 mm and that of inclined slabs, 100 mm .Assuming, applied live loading is 150 kg/m^2. The dead load varies with the thickness of concrete.

Assuming,

Slope of 31^0

Width about 1/5 span

Such that 18/5=3.5 m giving a rise of the system as L/10=1.8 m

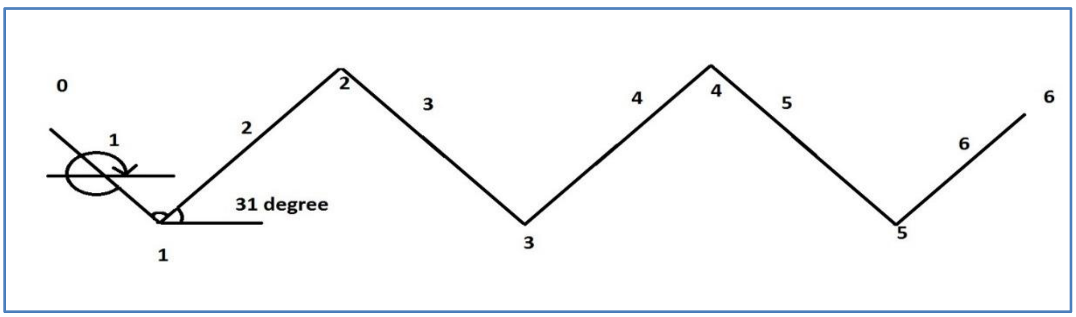


Fig 3.1 V type folded plate

3.1.1 Preliminary Analysis

Plate No.	Width(m)	Platewidth/Hori.projection	Thickness(m)	ϕn(degrees)	αn(degrees)
1	2.1	2.1/1.80	0.12	329	298
2	3.5	3.5/3.0	0.1	31	62
3	3.5	3.5/3.0	0.1	329	298

Table 3.1 Dimensions of folded plates – span-18 m

Symmetric with respect to junction 3

-Φ is the angle made by the plate to the horizontal

-α is the angle between the plate and the next plate

plate no.	cross section area	Transverse analysis	Longitudinal analysis	
	(m^2)	$I_T \times 10^{-3}$(m)	$I_L(10^{-4})$	$Z_L(10^{-3})$
1	0.252	0.144	0.0926	0.0882
2	0.35	0.0833	0.3573	0.2042
3	0.35	0.0833	0.3573	0.2042

Table 3.2 Geometric properties of plates in meter

I_L in plate 1=0.12×2.1^3/12=0.0926

Z is in m^3. For conversion to cm units, multiply by 10^6 when we work in cm units

(Consider unit meter along span)

(Insulation + LL)=150kg/$m^{2.}$ DL for thickness of slab @2400 kg/m^3

Plate no.	Total load	Horizontal span	Support moment
	Kg/m span	(m)	(kg.m)
1	919.8	1.8	(918.8×1.8)÷2=827.8
2	1365	3	(1365×3)÷12= ±341.3
3	1365	3	(1365×3)÷12= ±341.3

Table 3.3 Loads and support moment for transverse analysis

(Kg and m units)

PLATE	1	2	3
Dist. Factor	0	1 1/2	1/2
Support moment	+827	-341 +341 -486	-341 + 341
		-243 +122	122
		61	61
Final values	827	-827 +202	-202 +410

Table 3.4 Transverse analysis by moment distribution supported at joints

($R=W/2+M/d_n\cos\Phi_n$ from both sides)

Description	R_0	R_2	R_3	R_3
UDL from left span	0	919.8	682.5	682.5
UDL from left span	0	682	682.5	682.5
$M_{left}/d_n\cos\phi_n$	0	0	-208.4	69.4
$M_{left}/d_n\cos\phi_n$	0	208.4	-69.4	69.4
Total Reaction	0	1810.7	1087.2	503.8

Table 3.5 Calculation of reactions at supports

$$R_n=\frac{w}{2}\pm\frac{M}{d_{n}\cos\,\phi_n}$$

Trigonometric Values

Angle	Sine	Cos
329	-0.515	0.8572
31	0.515	0.8572
62	0.8829	0.4695
298	-0.8829	0.4695

Table 3.6 Calculation of P loads (in-plane loads in plates)

$$P_1 = R_1\frac{\mathrm{COS}\phi_1}{\mathrm{Sin}\alpha_1} = 1810\frac{\mathrm{COS}\ 31}{\sin 298} = -1758\ \mathrm{kg}$$

$$P_2 = -R_1\frac{\mathrm{COS}\phi_1}{\mathrm{Sin}\alpha_1} + R_2\frac{\mathrm{COS}\phi_2}{\mathrm{Sin}\alpha_2} = 2814\ \mathrm{kg}$$

$$P_3 = -R_1\frac{\mathrm{COS}\phi_1}{\mathrm{Sin}\alpha_1} + R_2\frac{\mathrm{COS}\phi_2}{\mathrm{Sin}\alpha_3} = -2516\ \mathrm{kg}$$

(In kg per meter units)

Plate no.	P load (per meter) (kg)
1	-1758 (jt.0 to jt.1)
2	+2814 (jt.2 to jt.1)
3	-2516 (jt.2 to jt.3)

Table 3.7 Calculation of P forces (in-plane loads in plates

Loads from lower to higher joints are –ve, loads from higher to lower joints are +ve

Plate 1: Load=1758 kg/m =17.58 kg/cm

$$f=\frac{PL^2}{8Z} = \pm\frac{17.58\times1800^2}{8\times888200} = \pm 80.7\ \mathrm{kg/cm^2}$$

Plate no.	p load(kg/cm)	$Z(cm^3)$	$f(kg/cm^2)$
1	17.58	888200	$\mp$80.7
2	28.14	204200	$\mp$55.8
3	25.316	204200	$\mp$49.9

Table 3.8 Stresses (f values) (at ends (L=18m=1800 cm)

Distribution in proportion to inverse of areas, i.e. 1/A and carry over -1/2

Plate	1		2		3	
Distribution	0.419		0.581	0.5	0.5	
Stresses	-80.7	+80.7	+55.8	-55.8	-49.9	+49.9
	-10.4		+14.5	-2.9	-3.0	
	+5.2		1.5	-7.3	1.5	
		-0.6	+0.9	+3.7	-3.6	
Final values	-74.8		+68.8	-57.0	-57.0	+53

Table 3.9 Stress Distribution for compatibility of stresses

Now joints which are not free (restrained) have to be corrected for rotation of joints by Simpson method.

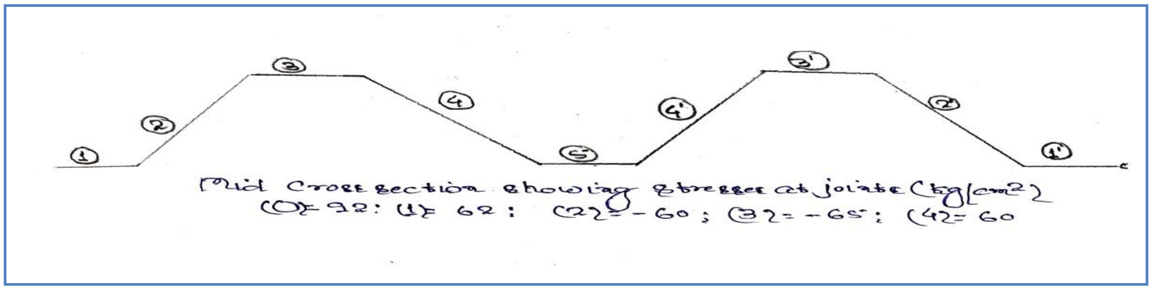
3.1.2 Calculation of Plate Deflections

$$y=\frac{f_{(n-1)}-f_n}{9.6\times d_n}\times\frac{L^2}{E}=\frac{1800^2}{9.6\times 2\times 10^5}\times\frac{f_{(n-1)}-f_n}{d_n}=\frac{1.688\,(f_{(n-1)}-f_n}{d_n}$$

Plate.no	$f_{(n-1)}$	f_n	$f_{(n-1)}$-f_n	d_n	y=(cm)
1	-74.8	68.8	-143.6	210	-1.154
2	68.8	-57	125.8	350	0.607
3	-57	53	-110	350	0.53

Table 3.10 Deflection in Preliminary Analysis

3.2 DESIGN OF FOLDED PLATE FOR SHEAR



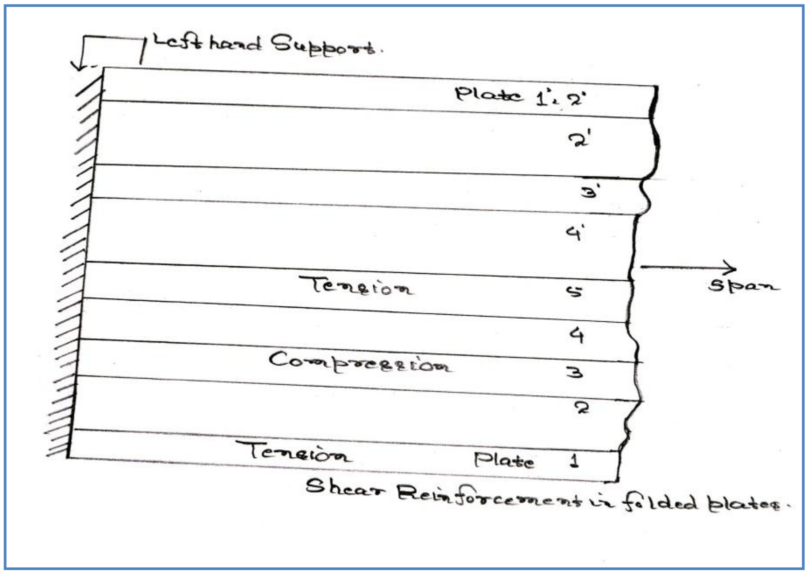


Fig 3.2 Shear Reinforcement in folded plates

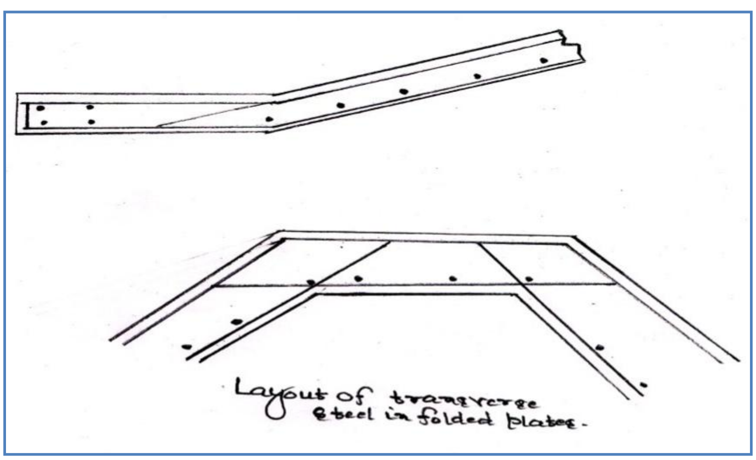


Fig 3.3 Layout of transverse steel in folded plates

Reference	Steps	Calculation
	1	Find shear between joints Take stresses at joints 0 to 4=92,62,-60,-65,60kg/cm^2 $T_n=T_{n-1}+A_n/2(f_{n-1}+f_n)$ Joint 1,$T_1=\frac{20\times10}{2}(92+62)=15400\ kg$ Joint 2,$T_2=15400+\frac{1400}{2}(62-60)=16800kg$ Joint 3,$T_3=16800+\frac{400}{2}(-60-65)$=-8200 kg Joint 4,$T_4=-8200+\frac{1400}{2}(-65+60)$=-11700 kg Joint 5,$T_{5=}$-11700+$\frac{400}{2}(60+60)$=12300kg
	2	Find the point of max. shear between plates and its values Stress at 1=62. Stress at 2=-60 Distance of zero stress from joint 1 X=71cm from joint 1 T_{max}=37410kg/m This is the total tension from the edge to point of zero tension
IS 456 Table 22 F_s= 140	3	Calculation shear stress V=0.83x1000x100= 83000 N In N/mm^2=0.83 N/mm^2 (we designas in cylindrical shells. Steel is placed diagonally (with direction as with diagonal steel in beams) for principal stress taken equal to shear stress as direct stress is small. [take max shear at support in plate 2=.83. it varies as cosine function= .83cosπx/l Design for max shear per meter length(4000mm) and thickness of plate=100mm V=0.83$\times 1000\times 100=83000$N Using 415 steel Area of steel A_S=360mm^2 12mm@200mm gives 565mm^2 provides 12mm at 20cm diagonally

Table 3.11 Folded plate designed for shear

3.3 DESIGN OF STEEL IN FOLDED PLATES IN TRANSVERSE DIRECTION

Reference	Step	Calculations
	1	Find effective depth d=100-15-5=80mm
	2	Check depth(thickness) required Fe415 steel M20 concrete M=460cm kg/cm width=460$\times$ $100N/10mm$ b=10mm;d=($\frac{46000}{0.917\times10}$) $= 70.8 < 80mm$ available d=80mm (thickness 100mm)
	3	Find the area of steel required using elastic design (as it is a roof and crack control is need , we will use elastic design. We can also use limit sate) $A_{S=}\frac{M}{f.j} = \frac{46000}{230\times0.9\times80} = 2.77$mm For B =10mm Spacing of 10mm rods=283mm Adopt 10mm@275mm spacing top and bottom

Table 3.12 Design of Steel in Folded Plates in Transverse Direction

3.4 DESIGN OF STEEL IN FOLDED PLATES IN LONGITUDINAL DIRECTION

reference	step	Calculation
	1	Design in plate (fully in tension) T=$[\frac{92+62}{2}] \times 10 \times 20 = 15400$kg[platebreadth $= 20$cm
	2	Find area of steel required(ELASTIC DESIGN) $A_S = \frac{154000}{230} = 669$ mm^2 4 rods of 16mm gives 804mm^2 These rods are placed equal distance in plate 1
		Design of plate 2(partly in tension and partly in compression)
	1	Find point of zero stress Let it be at x from joint 1. $\frac{x}{62} = \frac{140}{122}$ X=71cm Total tension $= \frac{62}{2} \times 71 \times 10 = 22010kg$ $A_S = \frac{220100\ (N)}{230} = 957$mm^2 5 rods of 16mm gives 1005cm^2 This steel is provided in the tension zone (we can also check in the composite region)
	2	Check stress at compression zone Max stress in compression =60kg/cm^2 (for M_{20}concrete,f_c in varying compression can be up to 7N/mm^2 hence safe)
	3	Provide minimum steel in compression zone. Min.steel=0.12% Total steel=$\frac{(140-71)\times 10 \times 0.12}{100}$=0.828cm^2=83mm^2 Length of compression zone=69cm Provide 5nos. 8mm rods(giving 201mm^2)equally spaced

Table 3.13 Design of Steel in Folded Plates In Longitudinal Direction

3.5 DESIGN OF DIAPHRAGMS (SUPPORTS)

The diaphragm must be designed for self-weight+Pforces in the plates.

Calculate the p forces acting on the diaphragms from the folded slabs.

Value of x	Values for p for unit x=1 m.kg/cm			
	Plate 1	Plate 2	Plate 3	Plate 4
X_1=4.0 X_2=4.26 X_3=0.05	1.11	3.86 -3.90	-7.07 7.05 -1.103	3.89 -3.887 0

Table 3.14 Correction Analysis X and P values

To convert to kg/m run, we multiply above values by 100.

From the above, we get the value of P for total analysis.

No.	Analysis	Plate1	Plate2	Plate3	Plate4
1	Preliminary	-322	998	-116	-908
2	For X_2=4.0	440	1552	-2828	1556
3	For X_3=4.260			3006	-1656
4	For X_4=0.051			-571	
Net value 118 895 -509 -1008					

Table 3.15 P forces in plates

These P forces can be resolved into vertical and horizontal loads on the diaphragm.For a half-length span of 9m, the forces along the plate will be as follows from net values

Plate1=118$\times 9 = 1062\text{kg} = 1.1\text{t}$

Plate2=895$\times 9 = 8055\text{kg} = 8.1\text{t}$

Plate3=509$\times 9 = 458\text{kg} = 4.5\text{t}$

Plate4=-1008$\times 9 = -9072\text{kg} = -9.02\text{t}$

CHAPTER - 4

PROPOSED METHOD OF ANALYSIS OF CONTINUOUS FOLDED PLATE ROOFS

The complexity of the analysis of continuous folded plates is primarily due to the fact that the end restraint of continuous folded plates creates longitudinal stresses at the intermediate supports, which are infinitely stiff in the plane of loads and are assumed as clamped.

In dealing with continuous folded plates with two equal spans, since the loading is symmetrical about the intermediate support, only one span need be investigated. The statical behavior of every span is that of a singleshell, built-in at the middle traverse and freely supported at the outer traverse. The stresses and elastic curves are similar to that of a beam with one end built-in and the other freely supported. For three-span and Multiplan continuous folded plates, the same assumption will be made in exterior spans, and the support condition of intermediate spans will be considered as built-in at both ends.

The analysis is divided into three parts in the same manner as the method of analysis for simply supported shells, and in addition, the effect of continuity over the supports is considered.

4.1 ELEMENTARY ANALYSIS

The first step in the analysis is the computation of the forces and of the transverse and longitudinal stresses acting at the edges of each plate element, neglecting the effect of the relative displacement of the joints. The roof in the transverse direction is considered to be a continuous one way slab supported on rigid supports at the joints. All loads carried transversely to the joints are considered to be transferred longitudinally to the end supporting members by the plates acting as inclined simple beams. The reactions at the joints are resolved into plate loads in the planes of the plates. Longitudinal stresses will be determined from these plate loads, and corrected in a manner similar to the moment distributionmethod. From the equalized edge stresses, the plate deflection at 0.41 of the exterior span and at mid-span of the middle span will be obtained.

4.2 CORRECTION ANALYSIS

The second step in the analysis is to provide for the effect that the relative transverse displacement of the joints has on the transverse and longitudinal stresses. This operation is most easily accomplished by applying arbitrary relative joint displacements successively to each plate. and computing the resulting plate deflections. A number of simultaneous equations equal to the number of restrained plates can be set up from the geometrical relation and solved for the actual relativejoint displacements.

4.3 SUPERPOSITION

The results of the elementary analysis are added algebraically to the corresponding values in the correction analysis to give the final forces, moments, stresses and displacements.

4.4 NORMAL CURVES

The principal problem associated with the analysis of folded plates is that of making the displacements computed from the longitudinal behavior compatible with the displacements obtained from the transverse behavior. A few points along a strip, but the requirement should be satisfied at all points on the surface. To secure this, it is necessary to express the external loads as a sinusoidal load. In the case of single-span roofs symmetrically loaded with respect to the middle of the span, the relative deflections can be represented by half of a s1ne curve, instead of assuming them to vary as the elastic line of the corresponding loaded beam. In the case of multispan roofs, or of roofs on whole the loads are far from being symmetrical about the middle of the span, this sine curve treatment cannot be used withaccuracy, and a specific form of elastic curves, known as the normal modes of lateral beam vibrations have to be adopted. The form of the deflection curve of a folded plate isthe same as that of a beam, which depends mainly on its support conditions, regardless of the longitudinal variation of the load. The use of normal curves would greatly simplify the analytical treatment in continuous folded plate·. Design for- the two most important.

"Normal Mode" of vibration of the beam is a definite shape 1n which the beam will deflect while vibrating harmonically. The mathematical expressions which define the normal modes are called characteristic functions. For each type of beam with specified end conditions there is an infinite set of these functions. The function of the normal modes will be derived from the condition of identity in form of the load and the corresponding elastic curves, expressed in the form:

$$N (X) = ky (X) \tag{1}$$

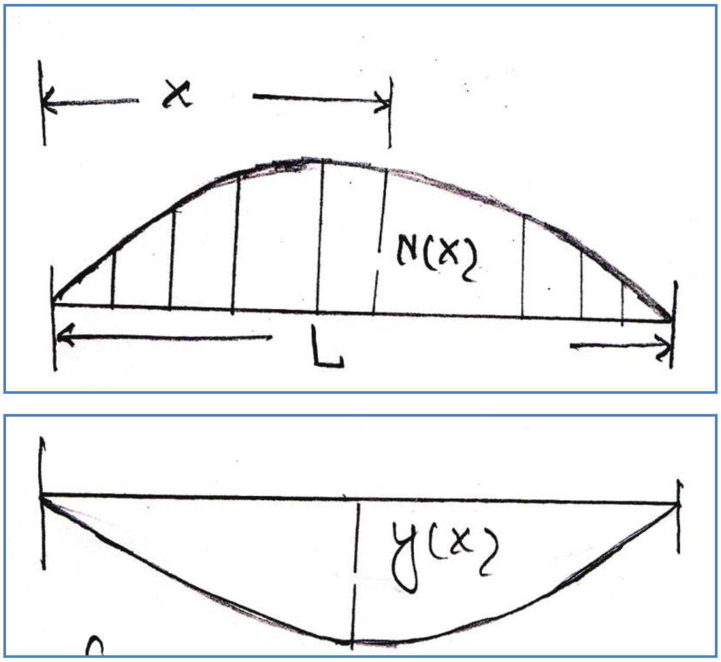


Fig 4.1 Normal curve

Substituting this relation into the differential equation of the elastic curve

$$\frac{N(x) = EI\ d\ y(x)}{dx\ 4} \tag{2}$$

The load and deflection curves will be expressed 1n the form N(x) = N0 f(x), y = y0 f(x), where N0, y0, are the maximum ordinates of the load and deflection curves, f(x) is a function of the coordinate x defining the shape of the normal mode of vibration under consideration, which is referred to as the normal function. Equation (l) becomes

$$EI\ f\ IV\ (x) = kf(x) \tag{3}$$

from which the normal functions for any particular case can be obtained, and the general solution of this equationwill have the following form:

$$f(x) = c_1(\cos nx + \cosh nx) + c_2(\cos nx + \cosh nx) + C_3(\sin nx + \sinh nx) + c_4(\text{s1n}\ nx + \sinh nx) \tag{4}$$

In Eq. (4) c_1 , c_2 , c_3, $c_4$are constants which should be determined in each particular case from the conditions at the ends of the beam.

4.4.1 The beam built-in at one end and freely supported at the other end

Assuming that the left end (x= o) is simply supported, the following end conditions are obtained:

(a) f (x) =0, x=O,)

(b) f(x):O, x=L,

(c) f'(x)=O, x=L,

(d) f''(x)=O, x=O.

The conditions of (a) and (d) yield c_1=C_2=0 1n the .general solution of Eq. (4). The remaining two cond1t1ons give the following equations:

$$c_3(\sin nL + \sinh nL) + c_4 (\sin nL - \sinh nL) = 0 \quad (5)$$

$$c_3(\cos nL + \cosh nL) + c_4 (\cos nL - \cosh nL) = 0 \quad (6)$$

A solution for the constants c_3 and c_4, different from zero, can be obtained only when the determinant of Eqs. (5) and (6) is equal to zero. Therefore,

$$\tanh nL = \tan nL \quad (7)$$

The consecutive roots of this equation are:

n_1L	n_2L	n_3L	n_4L
3.9266023	7.06858275	10.21017613	13.5176878

For purposes of design only the first term needs to be used. The effort of the succeeding term will 'be important only in the vicinity of the supports, and will not produce any significant stresses at the section of maximum deflection and maximum moment 1n the span.Substituting the n1L value into Eq. (5) And Eq. (6). The ratio c_3/ c_4 for the first mode of Vibration can be calculated and the shape of the deflection curve will then be obtained.

$$F(x)= \sin \frac{n1X}{L} + 0.02787494 \sin \frac{n1X}{L} \quad (8)$$

From Eq. (8), it was found that the maximum deflection would occur at approximately x = 0.419L, and the maximum moment at x = 0.383L. It would not make a large difference if 0.4L is selected for maximum moment and maximum deflection. This approximation, while acceptable for determining the critical stresses and moments, tends to obscure the exact distribution of stresses.

When f(x)x=0. 4L = 1.0641)76, the following equations are obtained:

Deflection curve:

$$f_y N = \frac{1}{1.0641376}\left(\sin 3.9266\frac{X}{L} + 0.02787494 \sinh 3.9266\frac{X}{L}\right)$$

Moment curve –

$$f_{MN}=\frac{-10641376\ L^2}{(3.9266)^2 \times 0.93586229} f^{II}{}_{yN}= \frac{L^2}{13.56} f^{II}{}_{yN}$$

$$= \frac{-1}{0\ .93586229}\left(-\sin 3.9266\frac{X}{L} + 0.02787494 \sinh 3.9266\frac{X}{L}\right.$$

Shear curve

$$f_{SN}=\frac{-1.0641376\ L^3}{(3.9233)^3 \times 0.9721251} f^{III}{}_{yN} = \frac{-L^3}{55.3L} f^{II}{}_{yN}$$

$$=\frac{-1}{0.9721251}\left(-0.00s3.9266\frac{X}{L} + 0.02787494\ 00sh3.9266\frac{X}{L}\right)$$

Load curve –

$$fN = \frac{L^4}{(3.9266)^4} f^{IV} yN = \frac{L^4}{237.72} f^{IV} yN$$

$$= \frac{1}{1.0641376}\left(\sin 3.9266\frac{X}{L} + 0.02787494 \sinh).9266\frac{X}{L}\right)$$

Maximum deflection -

$$yo = \frac{N_o L^4}{237.72EI} = \frac{NM_o L^2}{13.56EI} \qquad \text{at } X = 0.4L$$

Maximum moment -

$$M_o = \frac{N_o L^2}{17.60} = \frac{13.56\ y_o\ EI}{L^2} \text{at } X = 0.4L$$

Minimum moment -

$$M_{min} = -1.5105\ M_o = \frac{-\mathbf{N_o L^2}}{\mathbf{11.60}} \text{at } X{=}L$$

Maximum shear–

$$s_o = 55.30\ \frac{-y_o\ EI}{L^3} = \frac{\mathbf{N_o}}{\mathbf{4.30}}$$

4.4.2 The beam with both ends built-in

In the case of a beam with both ends fixed the boundary conditions are

(a) f(x}=O, x=O, (b) f'(x)=O, x=O,

(C) f (X) =0 , X=L , (d) f'(x)=O, x=L,

In order to satisfy the conditions (a) and (b) the Constants c_1 and c_2 should be equal to zero in eq. (4) and from conditions (o) and (d) we obtain

$$C_2(\cos nL - \cosh nL) + c_4(\text{s1n } nL - \sinh nL) = 0 \quad (9)$$

$$C_2(\sin nL + \sinh nL) + c_4(\cos nL + \cosh nL) = 0 \quad (10)$$

in which the frequency equation will be :

$$\cos nL \cosh nL = 1 \quad (11)$$

The first four consecutive roots of this equation are as follows:

n_1L	n_2L	n_3L	n_4L
4.7300408	7.8532046	10.9956078	14.1371655

Substituting the n1L value into Eqn. (9) and (10),the shape of deflection curve will be obtained, when

$$f(x) = 1.58815 \qquad \text{at } X = 0.5L$$

Deflection curve:

$$F_{yN} = \frac{1}{1.58815}\left(\cosh 4.73\frac{X}{L} - \cos 4.73\frac{X}{L}\right) - 0.9825\left(\sinh 4.73\frac{X}{L} - \sin 4.73\frac{X}{L}\right)$$

Moment curve:

$$f_{MN} = \frac{1.58815\ L^2}{(4.73)^2 \times 1.21565} f^{II}{}_{yN} = \frac{L^2}{17.13} f^{II}{}_{yN}$$

$=\frac{-1}{1.21565}\left[\left(\cos 4.73\frac{X}{L}+\cos 4.73\frac{X}{L}\right)-0.9825\left(\sinh 4.73\frac{X}{L}+\sin 4.73\frac{X}{L}\right)\right]$

$f_{yN}=\frac{1}{1.58815}\left(\cosh 4.73\frac{X}{L}-\cos 4.73\right)-0.9825\left(\sinh 4.73\frac{X}{L}-\sin 4.73\frac{X}{L}\right)$

Shear curve –

$$f_{SN}=\frac{1.58815\,L^3}{(4.73)^3\times 1.9650}f^{III}{}_{yN}=\frac{L^3}{130.93}f^{II}{}_{yN}$$

$$=\frac{-1}{1.9650}\left[\left(\sinh 4.73\frac{X}{L}-\sin 4.73\frac{X}{L}\right)-0.9825\left(\cosh 4.73\frac{X}{L}+\cos 4.73\frac{X}{L}\right)\right]$$

Load curve –

$$fN=\frac{L^4}{(4.73)^4}f^{IV}y_N=\frac{L^4}{500.55}f^{IV}{}_{yN}$$

$$=\frac{1}{1.58815}\left(\cos 4.73\frac{X}{L}-\cos 4.73\frac{X}{L}\right)-0.9825\left(\sin h\ 4.73\frac{X}{L}-\sin 4.73\frac{X}{L}\right)]$$

Maximum deflection –

$$yo=\frac{N_o L^4}{500.55EI}=\frac{M_o L^2}{17.1EI}\quad \text{at } X = 0.5L$$

Maximum moment –

$$M_o=\frac{N_o L^2}{29.2}=\frac{17.1\,y_o\,EI}{L^2}\quad \text{at } X = 0.5L$$

Minimum moment –

$$M_{min}=\frac{-N_o L^2}{17.75}\qquad \text{at } X= 0$$

Maximum shear –

$$s_o=\frac{N_o L}{3.75}$$

In comparing the deflection curves caused by the normal curve load and uniform load for different support conditions.(Appendix) it is observed that the discrepancy in the ordinates of the normal curve corresponding to the ordinates of the deflection curve caused by uniform loadis evidently quite small. Hence, the error introduced into the analysis by replacing a deflection curve w1th a normal curve can be neglected. The elastic curve of a beam with one fixed end and one simply supportend that carries a uniform load, having the maximum deflection y0 at 0.4L, is expressed by

$$f_{yw} = 3.86\left(\frac{X}{L} - 3\frac{X^3}{X^3} + \cos 2\frac{X^4}{L^4}\right)$$

The elastic curve produced by a uniform load for a beam with both ends fixed, having the maximum deflection y0 at mid-span, is

$$f_{yw} = 16\left(\frac{X^2}{L^2} - 2\frac{X^3}{X^3} + \frac{X^4}{L^4}\right)$$

4.5 CONTINUOUS FOLDED PLATES WITH TWO EQUAL SPANS

A continuous prismatic folded plate of the shape shown in Figure 2, with two equal spans, and continuous over the middle traverse will be analyzed. Since the loading is symmetrical about the center line support, only one span need be considered.

4.5.1 Resolution of ridge loads

Consider a prismatic folded plate loaded along all joints. Since in the actual structure there are no Supports at the various joints, forces of equal but opposite magnitude to the reactions are applied to the platestructure. These ridge loads are assumed to be resisted by the plates acting long1tud1nal as deep beams. For this purpose the reaction are resolved into components parallel to the plates as shown in Figure

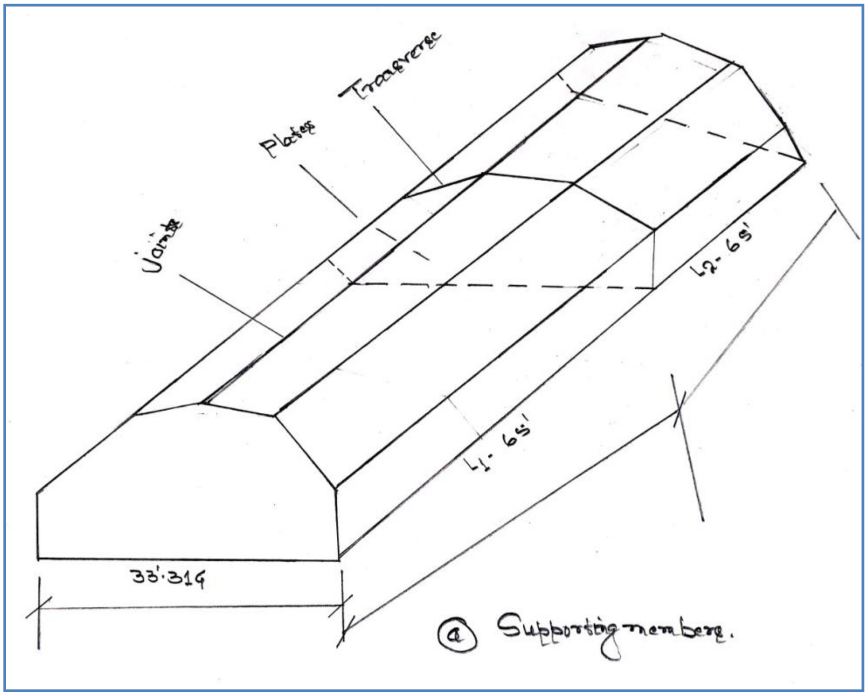


Fig 4.2 (a) Dimensions of Example 1

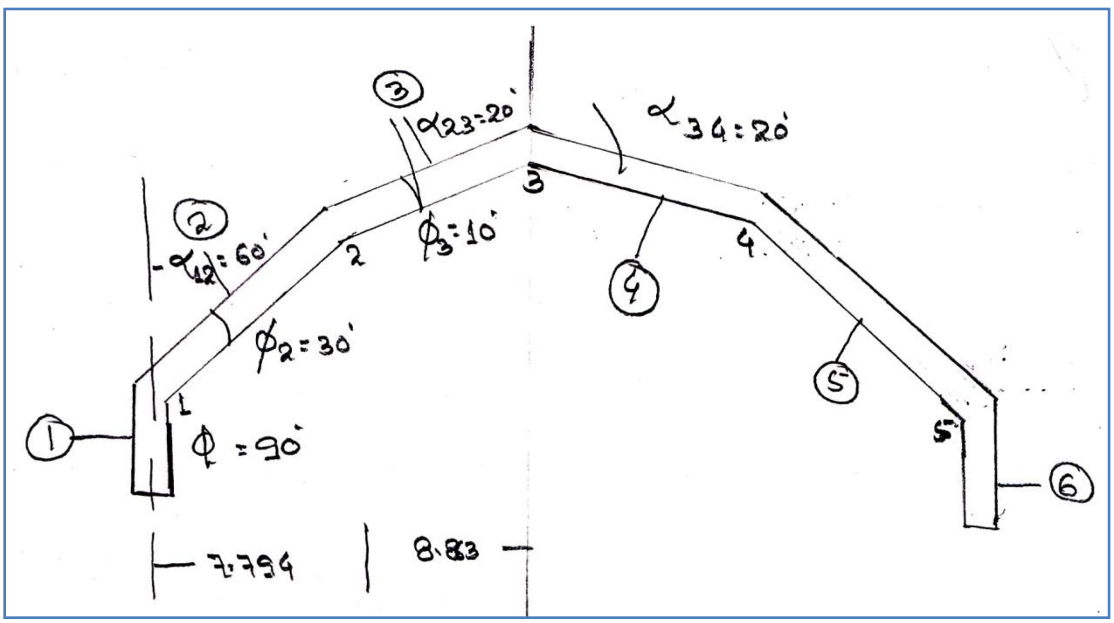


Fig 4.2 (b) Dimensions of Example 1

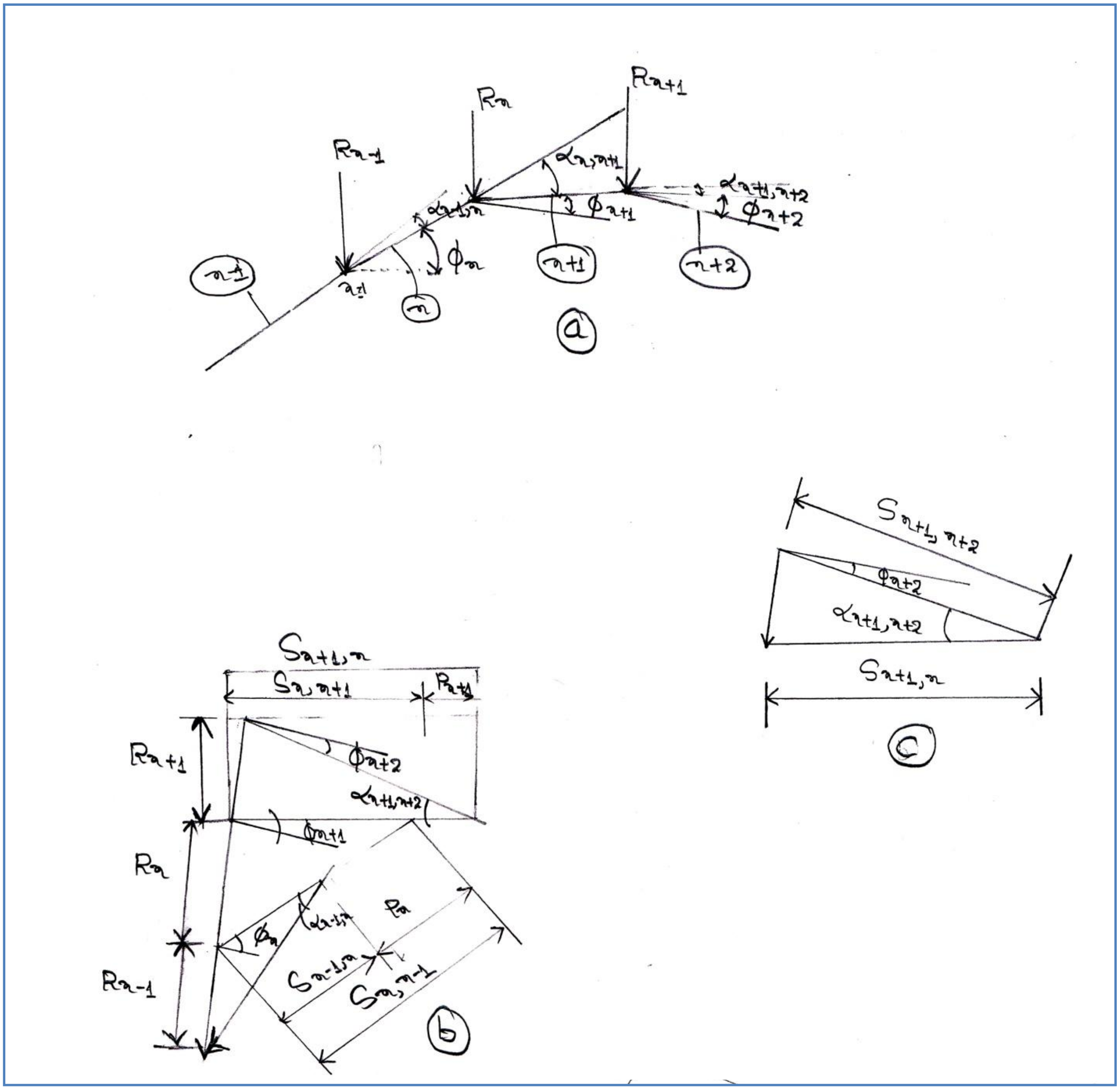


Fig 4.3 Resolution of Ridge Loads

From fig 4.3(c), using the sine law

$$\frac{s_{n+1,n}}{R_{n+1,n}} = \frac{\sin(90^{o} - \emptyset_{n+2}}{\sin\ \alpha_{n+1,n+2}}$$

$$s_{n+1,n} = R_{n+1}\ \frac{\cos_{\emptyset_{n+2}}}{\sin_{\emptyset_{n+1,n+2}}}$$

By the same reasoning R_n is resolved into its components $S_{n,n+1}$ and $S_{n,n-1}$

$$s_{n+1,n} = R_{n}\ \frac{\cos_{\emptyset_{n}}}{\sin_{\emptyset_{n,n+2}}}$$

It 1s seen that the total load acting in the plane of plate n+l is

$$P_{n+l}= S_{n+1,n}- S_{n,n+l}$$

$$P_n = R_n \frac{\cos \phi_{n+1}}{\sin \phi_{n-1,n}} \qquad (12)$$

4.5.2 Stress Distribution method

These plate loads are applied to the plates as loads acting along the entire length as shown in Figure 4. In computing the stresses the plates are assumed at first to act independently of each other. Moreover it is assumed thatthe plates arehomogeneousandtherefore the stress is equal to the moment divided by the section modulus. But this can generally only be possible 1f a longitudinal shearing force T_n isacting along this joint which tends to equalize thestresses 1n both plates meeting at the common Junction.

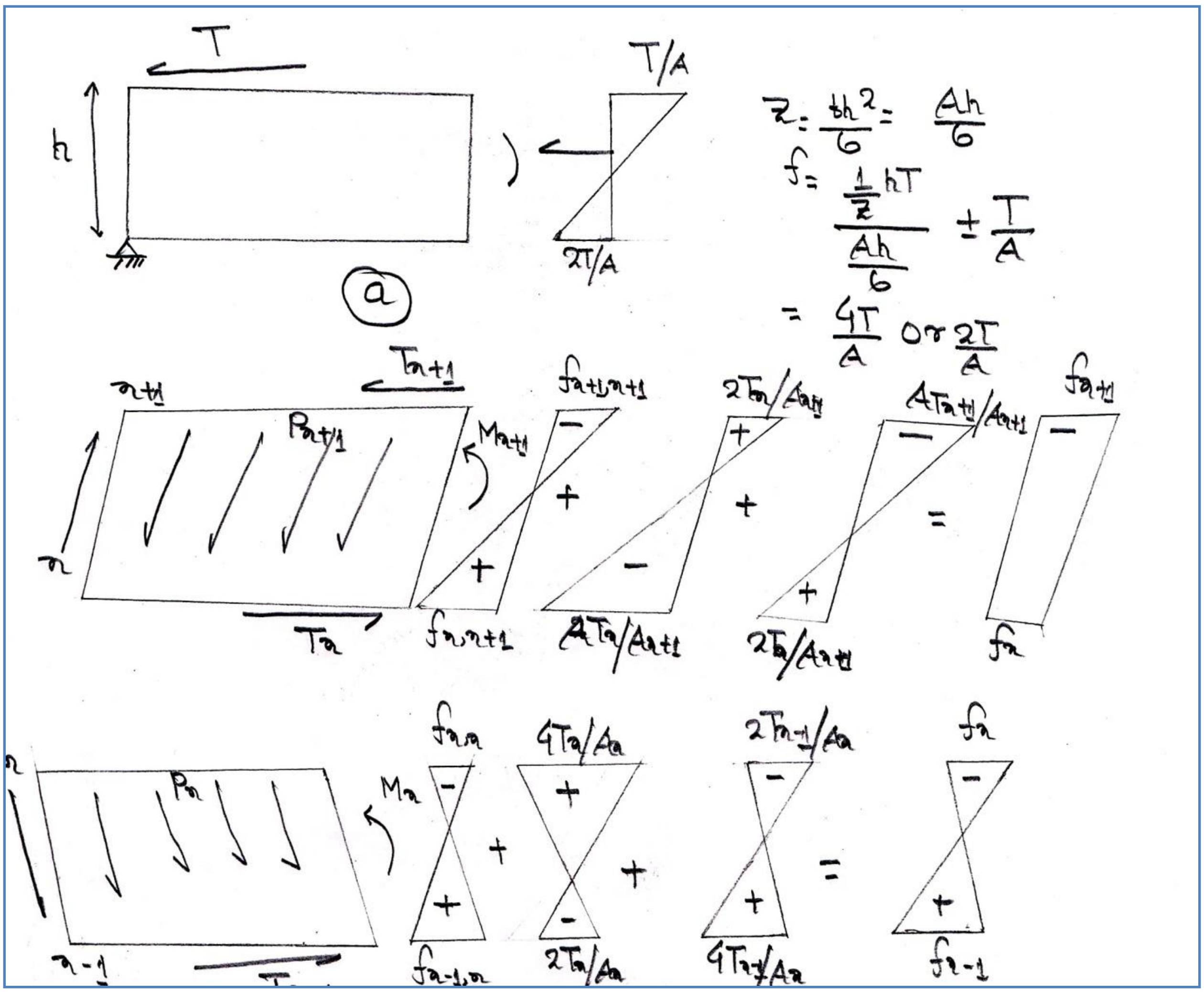


Fig 4.4 Longitudinal stresses at a Joint of Two Adjacent plat*es*

The stress of junction n of plate n due to the moment M_n may be written as:

$$f_{n,n} = -\frac{M_n}{Z_n}$$

Where the minus sign indicates compression.

Similarly,

$$f_{n,n+1} = \frac{M_{n+1}}{Z_{n+1}}$$

It is observed from Figure 4.that the longitudinal stress at junction n will be

$$f_n = f_{n,n} + \frac{4T_n}{A_n} = f_{n,n+1} - \frac{4T_n}{A_{n+1}} \tag{13}$$

From which T_n can be determined:

$$T_n = (f_{n,n+1} - f_{n,n})\frac{A_n A_{n+1}}{4(A_n + A_{n+1})} \tag{14}$$

When the value of ~n from Eq. (14) is substituted into Eq. (l3), the stress can be obtained

$$F_n = f_{n,n+1} + (f_{n,n+1} - f_{n,n})\frac{A_n A_{n+1}}{A_n + A_{n+1}} \tag{15a}$$

$$= f_{n,n+1} - (f_{n,n+1} - f_{n,n})\frac{A_n A_{n+1}}{A_n + A_{n+1}} \tag{15b}$$

Eqs. (15a,b) provide the basis for the stress distribution method by which the stresses can be determined without knowing the shearing forces Tn.

The distribution factor for plate n at Junction $D_{n,n}$is:

$$D_{n,n} = \frac{A_n}{A_n + A_{n+1}} \tag{16a}$$

For plate n+l at junction n the distribution factoris:

$$D_{n,n+1} = \frac{A_n}{A_n + A_{n+1}} \tag{l6b}$$

Now it is seen from Figure 4 that the shearing force Tn causes at junction n-1 of plate n the stress -2Tn/An and at junction n+l of plate n+l the stress 2Tn/An Comparing these stresses with those caused by T_n at junction n, it will be found that they are minus one-half of their magnitude. This denotes that the carry-over factor is -1/2.).

4.5.3 Shearing stresses

For a complete design, it is necessary to check the shearing stresses. The shearing stresses v at any point in the folded plates are induced by the shearing forces T, which can be calculated from the equilibrium of the horizontal forces (Figure .5)

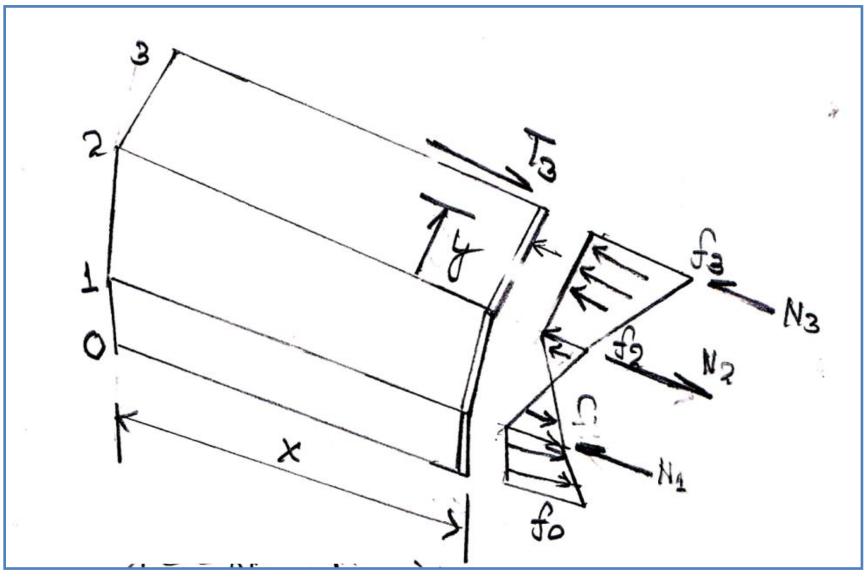


Fig4.5 Equilibrium of Horizontal Forces

$T = \int fsA$

The resultant shearing forces N can be obtained by

$N = \int Tdx$

Thus beg1n1ng at the left edge, the resultant forces at the ridges will be:

$N_1 = -1/2\ (f_0 + f_1)\ A_1$

$N_2 = N_1 - 1/2\ (f_1 + f_2)\ A_2$

$N_3 = N_2 - 1/2\ (f_2 + f_3)\ A_3$

The general form can be written as:

$$N_n = N_{n-1} - 1/2\ (f_{n-1} + f_n) A_n \tag{17}$$

The longitudinal shearing force N_y at any point between joints is

$$N_Y = N_2 - 1/2\ (f_2 + f_y)\ ty \tag{18}$$

Or

$$N_y = N_{n-1} - 1/2\ ty(f_{n-l} + f_{n-1}\frac{h-y}{h}) - 1/2\ tyfn\frac{y}{h} \tag{19}$$

The resultant shearing force at the middle of the plates can be written:

$$N_y = \frac{Nn-1 + Nn}{2} - A_n/8\ (f_{n-1} - f_n) \tag{20}$$

Since the variation of the. Longitudinal shearing force N_y is similar to the moment M_n due to the load P_n it varies parabolic ally.

For a simply supported structure, subjected to a uniformly distributed load,

$M_{max} = wL^2/8$, and the moment at any distance x from the support is

$$M_x = \frac{wx\,(L-x)}{2} = M_{max}\frac{4x\ (L-x)}{L^2} \tag{21}$$

$$N_y = N_{max}\frac{4x\ (L-x)}{L^2} \tag{22}$$

Because'N_y is proportional to M_x' then

$N_y = (N_{max}/M_{max})M_x$' and

$$v = \frac{1}{t}\frac{dN_y}{dx} = \frac{4N_{max}\ (L-2x)}{tL^2} \tag{23}$$

$$v_{max} = \frac{1}{t}\frac{N_{max}}{M_{max}}\frac{dM_x}{dx} = \frac{N_{max}\ v_x}{tM_{max}}$$

$$= \frac{N_{max}\ wL/2}{t\ wL^2/8} = \frac{4^N max}{tL} \tag{24}$$

and if loaded by a sine curve load the shearing stress becomes:

$$M = M_{max}\sin\frac{\pi x}{L} \tag{25}$$

$$N = N_{max}\sin\frac{\pi x}{L} \tag{26}$$

$$V = N_{max}\pi\cos\frac{\pi x}{L} \tag{27}$$

Therefore, combiningEqs. (22), and (26), the total shearing stress can be obtained. For practical design the shearing stress obtained by the sine curve load or normal curve load is quite small compared with the value obtained by the elementaryanalysis, hence, the second .term, Eq, (26), can beneglected.

Theoretically, for a beam fixed at one end, supported at the other, subjected to an uniform distributed load, the shearing stresses can be obtained from the following derivations. From Table I of the Appendix,

$$M_{max} = wL^2/14.28$$

$$M_x = \frac{3wL}{8}x - \frac{3wL^2}{2} = M_{max}\left(\frac{5.36x}{L} - \frac{7.14\ x^2}{L^2}\right) \quad (28)$$

$$N_y = N_{max}\left(\frac{5.36x}{L} - \frac{7.14\ x^2}{L^2}\right) \quad (29)$$

$$V = \frac{1}{t}N_{max}\left(\frac{5.36}{L} - \frac{14.28\ x^2}{L^2}\right) \quad (30)$$

For a beam fixed ·at both ends, subjected to a uniform distributed load, the shearing stress will be expressed as follows: from Table II of the Appendix,

$$M_{max} = wL^2 / 24$$

$$M_x = \frac{w}{12}(6Lx - L^2 - 6x^2)$$

$$= M_{max}\left(\frac{2x}{L} - 2 - \frac{12x^2}{L^2}\right) \quad (31)$$

$$N_y = N_{max}\left(\frac{2x}{L} - 2 - \frac{12x^2}{L^2}\right) \quad (32)$$

$$V = \frac{1}{t}N_{max}\left(\frac{2}{L} - \frac{24x}{L^2}\right) \quad (33)$$

Practically, as the shearing stresses are small throughout the entire structure, the valueswill be obtained by considering the plate as a simply supported beam for convenience and simplicity.

4.6 EXAMPLES

4.6.1 Example 1

The folded plate roof with two equal spans shown in Figure 2 will be analyzed for its own weight only. The loading was computed as follows:

Weight of plate = 1/4 x 150 = 37.5 psf

Weight of edge beam = 150 x 7/12 x 4 = 350 lb/ft.

Table I provides the general data of the crosssection.

4.6.1.1 Elementary Analysis

4.6.1.1.1 **Transverse slab analysis**

A unit strip taken from the folded plates is assumed to act as a continuous one way slab on unyielding supports. The transverse slab moments are determined in Table II, and the· reactions at each joint are computed.

The moment distribution factors at joint 2 are

$$D_{21} = \frac{3/4}{1+3/4} = 0.428$$

$$D_{23} = \frac{1}{1+3/4} = 0.572$$

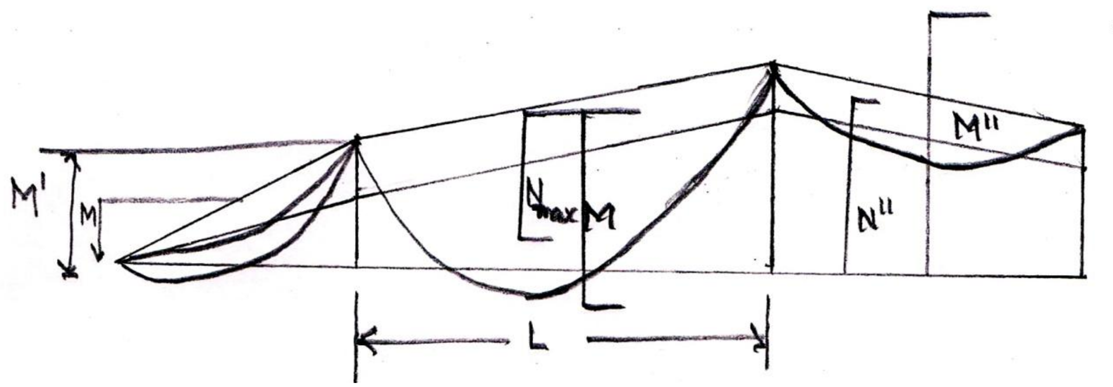


Figure 6(a). Basic Loading of Example 1 The fixed end moment will be

M_{F21} = 1/8 x 7.794 x 9x 37.5 = 328.8 ft – lb / ft.

$M_{F23} = M_{F32}$ = 1/12 x 8.863 x 37.5 = 249.18 ft – lb / ft.

General data

(a) Plates							
Plate no.	H, in feet	T, in In.	A,in SQ.FT	S,in Cu.ft	ϕ	sinϕ	cosϕ
1 2 3	4.0 9.0 9.0	7 3 3	2.233 2.250 2.250	1.556 3.375 3.375	90^0 30^0 10^0	1.00 0.50 0.174	0 0.866 0.985

(b) Joints			
Joint	∝	sin∝	cot∝
0	0	0	0
1	60	0.866	0.576
2	20	0.342	2.750
3	20	0.342	2.750

c) Moment distribution constants			
Joint	Plate	Relative Stiffness	Distribution
O	1	0	0
	1	Kl0 = 0	0
1	2	Kl2 = 4	1
	2	K21 = 3/4(4) = 3	0.428
2	3	K23=4	0.572
	3	K32 = 4	0.500
3	4	K34=4	0.500

Table 4.1 General Data of Example 1

1	2		3	Joint
10 12	21	23	32	Member
	0.428	0.572	0	Dist. Factor
	328.8 -34.1 	-249. 2 -45.5 	249.2 -22.8	F. E. Moment Distribution Carry over
	294.7	-294.7	226.4	Final moment
-37.8	37.8	7.7	-7.7	M/hcos
168·.8	168·.8	168·.8	168·.8	wh/2
480.9	383.08		322.1	Joint reaction

Table 4.2 Shears and Joint Reactions in TransverseOne Way Slab at 0.4L from the Outer Support

4.6.1.1.2 Longitudinal plate analysis

4.6.1.1.2.1 Plate Loads

Thevertical joint reactions are resolved into components parallel to the contiguous plates by using Eq. (12). The plate loads acting on each plate are tabulated in Table III.

Resolution of Ridge Loads

Joint	(1) Reaction Lb./ft.	(2) $Cos\o_n+l/sin\propto_n$	(3) =(1) x(2)	(4) $Cos\o_{n-1}/ sin\propto_{n-1}$	(5) $R_{n-1\times(4)}$	(6) Plate Loads Lb./ft.
1	480.90					480.90
2	383.0	2.877	1101.95	0	0	1101.95

Table 4.3 Resolution of Ridge Loads

4.6.1.1.2.2 Free edge stresses

It is assumed temporarily that each plate bends independently due to plate loads. The maximum stress and deflection occur approximately at 0.4L from the outer support. The moment due to a uniform load will be (refer to Table I of the Appendix)

$$M_{0.4L} = PL2/14.28 \quad (34)$$

$$f = \frac{M}{S} = PL^2/14.28S \quad (35)$$

The free edge stresses are tabulated in Table 4.4

4.6.1.1.2.3 Free edge stress distribution

The free edge stresses are distributed in order to determine the actual edge stresses, which must be equal at the joint.

Free Edge Stresses

Plate	Plate Load lb/ft.	S cu. ft.	$\frac{L^2}{14,28} = \frac{65^2}{14.28}$	fb = -ft kip/sq. ft.
1	480.9	1.556	295.87	91.45
2	1101.9	3.375	295.87	96.60
3	-44.36	3.375	295.87	-3.89

Table 4.4 Free Edge Stresses Resulting From the Elementary Analysis

The free edge stress distribution is shown in Table V; and is plotted in Figure 6. The stress distribution factors, by using Eq. (16), are

$$D_{11} = A_2 / (A_1 / A_2) = \frac{2.250}{2.333 + 2.250} = 0.4909$$

$$D12 = A_1 / (A_1 / A_2) = \frac{2.333}{2.333 + 2.250} = 0.5091$$

$$D_{22} = A_2 / (A_2 / A_3) = \frac{2.250}{2.250 + 2.250} = 0.500$$

$$D_{23} = 0.500$$

Stress Distribution

0	1		2		3	JOINT MEMBER
	11	12	22	23		
	0.491	0.509	0.50	0.50		DIST.FACTOR
-0.5		-0.5		-0.5		C.O.FACTOR
91. 4.5	-91.4.5 92.33	-96.60 -95.72	-96.60 46.36	-3.89 -46.36	3.89	F.E. Stress Distribution
-46.17	-11.38	-23.18 11.80	47.86 -23.93	23.93	23.18	Carry Over Distribution
5.69	5.88	11.97 -6.09	-5.90 2.95	-2.9.5	-11.97	Carry Over Distribution
-2.94	-0.73	-1.48 0.75	3.05 -1.52	1.52	1.48	Carry Over Distribution
0.36	0.37	0.76 -0.39	-0.38 0.19	-0.19	-0.76	Carry Over Distribution
-0.19	-0.05 -0.05	0.05	0.19 -0.10	0.10	0.09	Carry Over Distribution
48.23	-4.99	-4.99	-27.84	-27.84	15.86	Final Stresses

Table 4.5 Stress Distribution

Since the moment at the intermediate support due to a uniform load is 1/8 wL^2, the stresses are proportiona1 to the bending moment, but of. Opposite sign.

$$F_{x=L} = f_{x\ 0.4L} x \ \frac{-14.28}{8} \tag{35a}$$

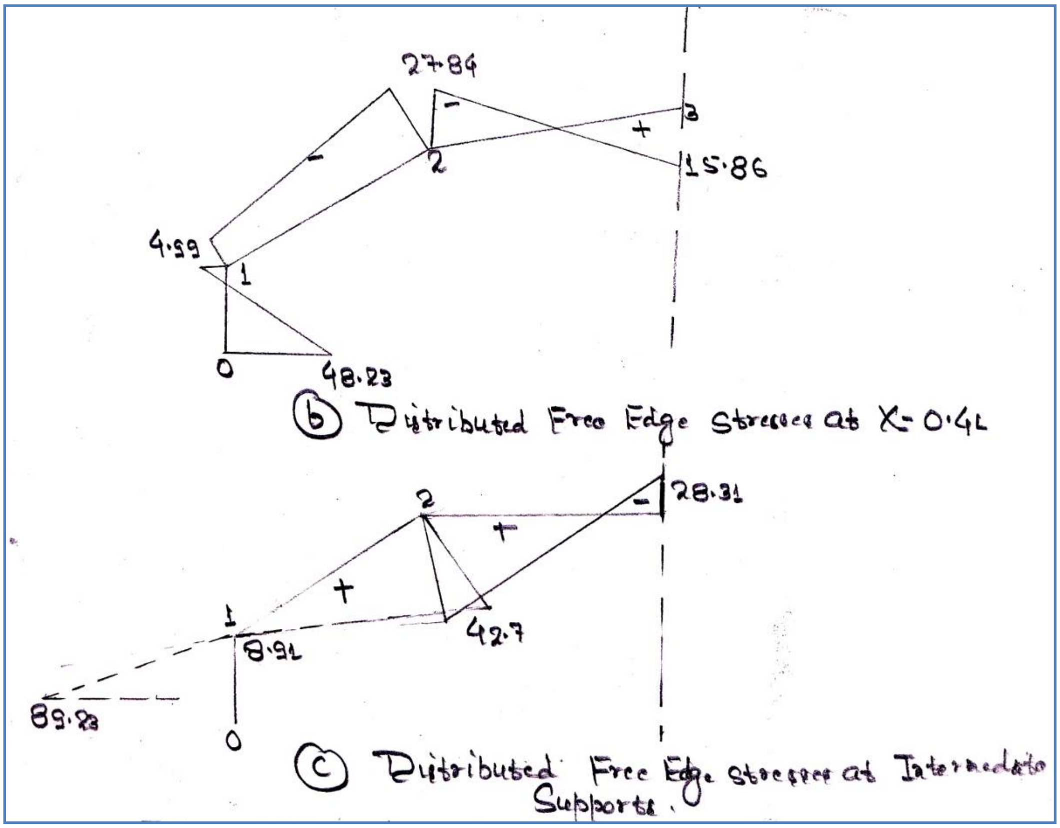


Fig 4.7 Longitudinal Stresses from Elementary Analysis

4.6.1.1.2.4 Plate deflections

From the equalized edge stresses, the plate deflections at 0.4L can be computed. For a uniform load, the deflection is

$$y_{0.4L} = \frac{M L^2}{12.99\ EI} \tag{36}$$

in which the moment at x = 0.4L is

$$M_{0.4L} = \frac{f_b - f_t}{2}\, s \tag{37}$$

Substituting Eq. (36) into Eq. (35)

$$y_{0.4L} = \frac{1}{12.99} \left(\frac{f_b - f_t}{2}\right) \frac{SL^2}{EI}$$

For a rectangular plate S/I = 2/h

$$y_{0.4L} = \frac{1}{12.99}\left(\frac{f_b - f_t}{h}\right)\frac{L^2}{E} \quad (38)$$

Assuming E is 105 kip/sq.ft., the plate deflections in terms of the free edge stresses at X = 0.4L are found as follows:

y_{30} = (-27.84 – 15.86) x 65^2/ 12.99 x9 x E = - 0.0162 ft.

y_{20} = (-4.99 + 27.84) x 65^2 / 12.99 x9 x E = 0.00826 ft.

y_{10} = (48.38 + 4.99) x 65^2 / 12.99 x 4 x E = 0.0433 ft.

4.6.1.2 Correction Analysis

4.6.1.2.1 Transverse slab analysis

The analysis is made for an arbitrary rotation of the plate at the section o.4L from the outer support. The fixed end moment at edge 2, with edge 1 free to rotate, equals to $EI\Delta/h^2$ = -3 ft – k. The fixed end moment at plate 3 is $6EI\Delta/h^2$= - 6ft-k.

By moment distribution, the transverse moments, shears and joint reactions may be computed as in Table VI.

4.6.1.2.2 Longitudinal plate analysis

The same procedure as the elementary analysis will be repeated for plate loads, stresses and deflections caused by the rotations of those plates. Longitudinal moments due to normal curve loads will therefore be

$$M_{0.4L} = PL^2/ 17.53S \quad (39)$$

And,

$$f_{0.4L} = M/S = PL^2 / 17.53\ S \quad (40)$$

$$y_{0.4L} = ML^2/ 13.56\ EI = \left(\frac{f_b - f_t}{13.56}\right)\frac{L^2}{Eh} \quad (41)$$

$$F_{x=L} = f_{0.4L} \times 17.53/ 11.60 \quad (42)$$

Plate loads, free edge stresses and the stress distribution are shown in Tables, VIII, and IX. The plate deflections are computed from Eq. (40).

For (a) an arbitrary rotation of Plate 2

$$y_1' = \frac{(68.27 + 90.73) \times 65^2}{13.56 \times 4 \times E} = 0.124\text{ft.}$$

$$y_2' = \frac{(-90.73 - 146.50) \times 65^2}{13.56 \times 9 \times E} = -0.082\text{ft}$$

$$y_3' = \frac{(146.50 - 178.99) \times 65^2}{13.56 \times 9 \times E} = 0.113\text{ft}$$

(a) For an arbitrary rotation of plate 2				
0 1	2		3	Joints
10 12	21	32	32	Members
1.000	0.428	0.572	0.5000	Dist. Factor
	-3.000 1.286	 1.714	 0.857	F.E. moment Distribution carry over
	-1.714	1.714	.857	Final moment
0.220	-0.220	-0.290	0.290	Without
0.220	-0.510		0.580	Joint reaction
For an arbitraryrotation of plate 3				
0 1	2		3	Joints
10 12	21	23	32	Member
1.000	0.428	0.572	0.500	Dist. Factor
	2.568	-6.000 3.342	-6.000 1.716	F.E. moment Distribution carry over
	2.568	-2.568	-4.284	Final moment

-0.329	-0.329	-2.568	-0.773	a/hope
-0.329			-1.546	Joint reaction

Table 4.6 Slab Action And Plate Loads Due To An Arbitrary Rotation

Resolution of Joint Reactions

(a) For an arbitrary rotation of Plate 2						
Plate	(1) Reaction	(2) $Cos\o_n$+l/$sin\propto_n$	(3) =(1)×(2)	(4) $Cos\o_{n-1}$/ $sin\propto_{n-1}$	(5) $R_{n-1\times(4)}$	Plate Loads k/ft
1	0.22					0.22
2	-0.51	2.877	-1.467	0	0	-1.467
3	0.58	2.877	1.669	2.535	-1.29	2.961
(b) For an arbitrary rotation of Plate 3						
1	-0.329					-0.329
2	1.102	2.877	3.l7	0	0	3.170
3	-1.546	2.877	-4.448	2.535	2.79	-7.242

Table 4.7 Resolution of Joint Reactions for the Correction Analysis

Free Edge Stresses for an Arbitrary Rotation

Table 4.8 Free Edge Stresses foran Arbitrary Rotation

Plate	Plate Load k/ft	S Cu.ft	L 2 652 17.53=17.53	Fb=-ft Kip/sq.ft.
(a) For an arbitrary rotation of plate 2				
1 2 3	0.22 -1.467 2.961	1.56 3.375 3.375	241.02 241.02 241.02	45.87 -104.76 211.45
For an arbitrary rotation of plate 3				
1 2 3	-0.329 3.170 -7.242	1.556 3.375 3.375	241.02 241.02 241.02	-68.59 226.38 -517.17

(a) For an arbitrary rotation of plate 2

	1		2		3	Plate
0.5	0.491 0.5	0.509 0.5	0.50 0.5	0.50 0.5	0.5	Disat. Factor c. o. Factor
45.87	-45.87 -28.92	-104.76 29.98	104.76 53.35	311.5 -53.35	-211.5	F.E. stress Distribution
14.46	-13.10	-26.67 13.58	-14.99 7.49	-7.49	26.67	Carryover distribution
6.5	-1.84	-3.75 1.91	-6.79 3.40	-3.40	3.75	Carry over distribution
0.92	-0.83	-1.70 0.85	-0.95 0.48	-0.8	1.70	Carry over distribution

0.42	-0.12	-0.24 0.12	-0.43 0.21	-0.21	0.24	Carry over distribution
0.06	-0.05	-0.11 0.05	-0.06 0.03	-0.03	0.11	Carry over distribution
68.27	-90.73	-90.73	146.50	146.50 -	178.99	Total
(b) (b)for an arbitrary rotation of plate 3)						
-68.69	68.59 77.48	226 -80	-226.38 -145.40	-517.17 145.40	517.17	F. E. stress distribution
-38.74	35.70	72.70 -37.00	40.16 -20.08	20.08	-72.70	Carry over Distribution
-17.85	4.93	10.04 -5.04	18.50 -9.25	9.25	-10.04	Carry over Distribution
-2.47	2.27	4.63 -2.36	2.55 -1.28	1.28	-4.63	Carry over Distribution
-1.14	0.31	0.64 -0.33	1.18 -0.59	0.59	-0.64	Carry over Distribution
-0.16	0.15	0.30 -0.15	0.16 -0.08	0.08	-0.30	Carry over Distribution
-0.07	0.02	0.04 -0.02	0.08 -0.04	0.04	-0.04	Carry over Distribution
-129.01	189.44	189.4	-340.45	-340.45	428.83	Total

Table 4.9 Stress Distribution Resulting From an Arbitrary Rotation

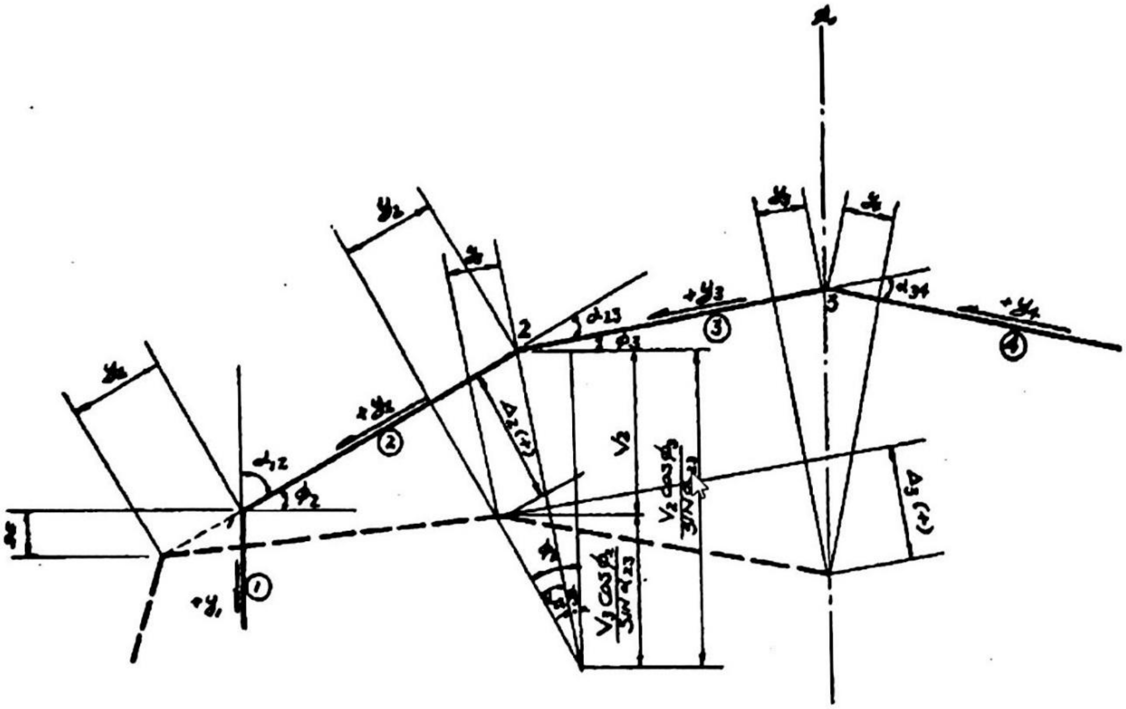


Fig 4.8Williot Diagram for Relative Joint Displacement

For (b) an arbitrary rotation of Plate 3

$$y_1'' = \frac{(-129.01 - 189.44)\times 65^2}{13.56\ \times 4 \times E} = -0.1238\text{ft.}$$

$$y_2'' = \frac{(189.44 - 340.45)\times 65^2}{13.56\ \times 9 \times E} = 0.1834\text{ft}$$

$$y_3'' = \frac{(-340.45 - 428.83)\times 65^2}{13.56\ \times 9 \times E} = -0.2663\text{ft}$$

The general expression of the geometrical relationship between deflections and rotations, as shown in Figure 7, is

$$\Delta_n = -\frac{y_{n-1}}{\sin \propto_{n-1}} + y_n(\cot \propto_{n-1} + \cot\propto_n) - \frac{y_{n+1}}{\sin \propto_n} \qquad (43)$$

The final deflections must be expressed in terms of numerical results obtained from the elementary analysis, y_{10}, y_{20}, y_{30}' plus those for the various ·rotation solutions, each multiplied by an unknown factor kn.

The arbitrary rotation was

$$\frac{EI_n \Delta}{h_n^2} = 1^{\text{ft-k}} \text{ x } k_n \qquad (44)$$

Hence,

$$\Delta_2 = -\frac{h_2^2 \times 1}{EI_2} K_2 = \frac{9^2 \times 1 \times 12}{(\frac{1}{4})^3 \times 10^5} k_2 = 0.622\ k_2$$

$$\Delta_3 = 0.622 k_3$$

by geometrical relationships, using Eq. (42)

$$\Delta_1 = 0.622k_2 = -l.l.5y_1 + 3.32y_2 - 2.92y_3$$

.

$$= -l.l.5\ (y_{lO} + y_1k_2 + y_1k_3) + 3\ .32\ (y_{20} + y_2k_2 + y_2k_3)$$

$$\Delta_2 = 0.0238 - 0.7442k_2 + 1.6724k_3 = 0.622\ k_2 \qquad (45)$$

Similarly,

$$\Delta_3 = -0.1571 + 1.1887k_2 - 2.7790k_3 = 0.622k_3 \qquad (46)$$

Solving eqn. (43) and (44)

$$K_2 = -0.0726$$

$$K_3 = -\ 0.0701$$

4.6.1.3 Superposition

The final value of the longitudinal stresses, transverse moments and deflections by combining the elementary analysis and the correction analysis are. Summarized in Tables X, XI, and XII, and are plotted in Figures 8, 9 and 10. The stresses at the middle support can be obtained using Eqs. (35a) and (42).

(a) longitudinal stress at 0.4L					
joints	Elementary Analysis	Correction Analysis		Total correction	Final values k/sqrt
		Rotation of plate2	Rotation of plate 3		
0	48.23	-4.96	9.04	4.09	52.32
1	-4.99	6.59	-13.28	-6.69	-11.68
2	-27.84	-10.64	23.8	13.23	-14.61
3	15.86	12.99	-30.07	-17.08	-1.22

(b) Longitudinal stress at the middle support.					
0	-89.23	7.49	-13.57	-6.18	-95.40
1	8.91	-9.95	20.03	10.11	19.03
2	49.70	16.07	-36.07	-19.99	29.71
3		19.54	45.45	25.81	-2.50

Table 4.10 Final Longitudinal Stresses Of Example 1

joints	Elementary analysis	Correction analysis		Total correction	Final values
		Rotation of plate 1	Rotation of plate 2		
1	0				
2	-294.72	-124.44	180.02	55.58	-239.1
3	-226.4	62.00	-300.31	-238.09	-464.4

Table 4.11 Final Transverse Moments Of Example 1

Joints	Elementary analysis	Correction analysis		Total correction	Final values
		Rotation of plate 1	Rotation of plate 2		
1	0.0433	-0.001	0.0087	-0.0003	0.043
2	0.0083	0.006	-0.0129	-0.0069	0.0014
3	-0.0162	-0.0082	0.0187	0.0105	-0.005

Table 4.12 Final Deflections of Example 1

In the above analysis the intermediate supporting stiffener is assumed to be a rib. A tie between point 2 and 6 would effect a saving, but could be omitted because of headroom and appearance.. As the shearing stresses .are small, only a nominal amount of. Reinforcement is provide to resist shear. The rib must be designed for bending moments combined with direct forces.

4.6.1.4 Folded Plates Continuous Over Three Spans

Since the loading and span are symmetrical about the center line, only the center and left exterior spans will be investigated. These will be considered individually. The exterior spans have the same behavior as that analyzed in the previous example of two equal spans, and the center portion has both ends fixed. There exists a considerable difference in the determination of the longitudinal moment over each of the two inner traverses in comparison with the moment over the middle traverse of two equal spans. That is, the end moment, wL2/8, of a single beam subjected to a uniformly distributed load with one simply supported end and one fixed end is exactly equal to the moment at the middle support of a continuous beam with two equal spans. Therefore, the stresses at the middle traverse are proportional to the maximum stresses at 0.4L of the span. But the longitudinal moment over each of the two inner traverses in a continuous folded plate with three spans is not the same as the end moment of a beam with one end simply supported and one end fixed. It is known that the effect of continuity over the supports on stresses in shells is similar to the effect ofcontinuity on stresses 1rt.ord1nary beams. Thus, for the purpose evaluating the stresses on each of the inner traverses, the bending moment will have to becalculated from the three moment equation or one of the other acceptable methods in common use.The shearing stress in multi-span continuous foldedplates will' be further investigated and will be emphasizedin Example 2. Figure 11 shows a moment diagram for a uniformly loaded folded plate with three continuous spans.

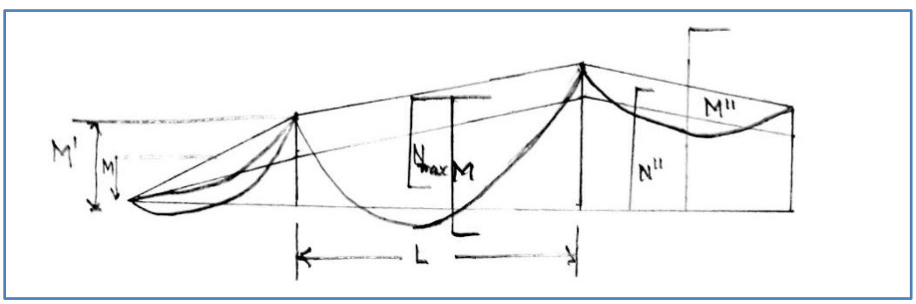


Fig 4.9 Relationship between Moments and Shearing Forces for a Uniformly Loaded Plate with Three Continuous Spans

As explained before, the shearing force N can becalculated since it is proportional to the bending moment.N_{max} represents the shearing force at mid-span. The bendingmoment on the plate at a distance x from the firstinterior support, considering continuous beam action only, is

$$M_x = \frac{WX}{2}\,(L\text{-}x) + M' + (M'' \text{-} M')\frac{X}{L} \tag{47}$$

$$N_x = N_{max}\,\frac{4x(L-X}{L} + N' + (N'' \text{-} N')\frac{X}{L} \tag{48}$$

$$V = \frac{1}{t}\frac{dN_x}{dx} = \frac{4N_{max}}{tL^2}(L\text{- } 2x) + (N'' \text{- } N')/L \tag{49}$$

For the exterior span,

$$V = \frac{4N_{max}}{tL^2}(L\text{- } 2x) \text{ - } N'/L \tag{50}$$

4.6.2. EXAMPLE 2

Figure 12 shows the dimensions of the roof, the longitudinal spans of which are respectively 30ft. (L1), 40ft. (L2) and 30ft. (L3).

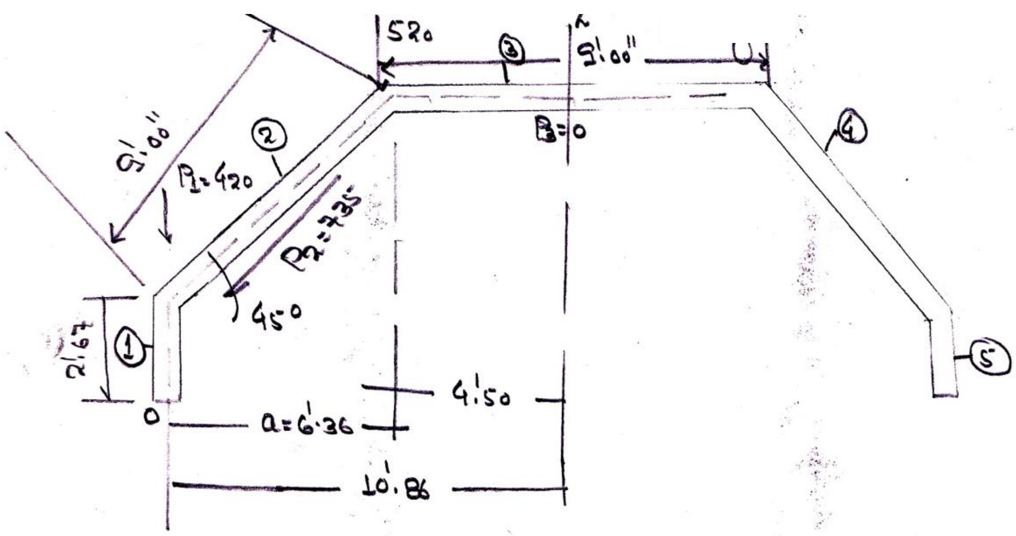


Fig 4.10 Dimension of Example 2

The general properties of the system are given in Table XIII.

4.6.2.1 Elementary Analysis

The transverse moment distributions are shown in Table XIV. The resolved plate loads are also shown in Figure 12. The moment distribution factors at· joint 2 are 0.428 and 0.572.The fixed end moments are computed as follows:

General Data

Plate	H,inft	T,in inch	A,insq inch	S,incub inch	Ø	Sinø	Cosø
1	2.667	5	160	853	90^0	1.00	0
2	9.00	3	324	5832	45^0	0.707	0.707
3	9.00	3	324	5832	0^0	0	1.00

Table 4.13 General Data of Example 2

Slab Moments Due to External Loads

1	2		3	Joint
12	21	23	32	Member
	0.428		0.572	Dist. Factor
	7.16w -0.17	-6.75w -0.23	6.75w 0.23	F.E.M Dist.
	0 -0.05	0.12 -0.02	-0.12 0.07	C.0. dist.
	0 -0.01	0.03 -0.02	-0.03 0.02	C.0. dist.
	6.91w	-6.91w	6.91w	Final moments
-1.09w	1.09w			m/hcosø

Table 4.14 Slab Moments Due To External Loads

$$M_{F22} = WL^2/12 = 6.75w \text{ ft –lb/ft.}$$

$$M_{F21} = Wah/8 = 7.155w \text{ ft- lb /ft}$$

The moment M over each of the two stiffeners is obtained by the theorem of three moments

$$2M\,(L1 + L_2) + ML_2 - PL_2^3/4 - PL_2^3/4$$

Then,

$$M = P(L_2^3 + L_2^3)/\,4(2L_1 + 3L_2) = -126.39\ P \text{ ft-lb} = -1516.7P \text{ in –lb}$$

From the foregoing data, the calculation of the free edge stresses can be tabulated thus:

Free Edge Stresses from the Elementary Analysis

Plate	1	2	3
Plate load, lb/ft.	420	735	0
s, cubic in.	853	5832	5832
(1)Free edge stresses at the intermediate support			
Fb = - ft= M/S, Lb./sq. ft.	-747	-192	0
(2) Free edge stresses for the exterior span at 0.4L			
fb =- ft = $(PL_1^2 \times 12)/14.28$	372.4	95.3	0
(3) Free edge stresses for the center span at mid-span			
fb =-ft= $(PL_2^2 \times 12)/24$	394	101	0

Table 4.15 Free Edge Stresses fromthe Elementary Analysis

Stress Distribution Factors:

$$D_{11} = 324/\ 160+324 = 0.67$$

$$D_{12} = 1- 0.67 = 0.33$$

$$D_{22} = D_{23} = 0.50$$

The stress distributions are performed in Table XVI. In determining the deflections, E is assumed to be 2 x 106 psi. For a uniform load, the deflections at 0.4L in the exterior span are:

$$y_{20} = \frac{(-63.4+6.4)}{12.99 \times 9 \times E} \text{ x } 30^2 \text{ x } 12 = -\ 0.00263 \text{ in.}$$

$$y_{10} = \frac{(218.0+63.4)}{12.99 \times 2.67 \times E} \text{ x } 30^2 \text{ x } 12 = -\ 0.04380 \text{ in}$$

and at mid-span of the middle span are as follows:

$$y_{20} = \frac{(-66.7+6.9)}{16 \times 9 \times E} \text{ x } 40^2 \text{ x } 12 = -\ 0.003986 \text{in.}$$

$$y_{10} = \frac{(230.5+66.7)}{16 \times 2.67 \times E} \text{ x } 40^2 \text{ x } 12 = -\ 0.06678 \text{ in.}$$

4.6.2.2 Correction Analysis

In determining the effect of the relative displacements of the joints, a unit transverse strip is considered, and the ·fixed end moment at edge 3 is

$$M_F = \frac{3EI\Delta}{h_2^2} = \frac{3 \text{ x } 2 \text{ x } 103 \text{ x } 144 \text{ x } 1/12 \text{ x } (3/12)3 \text{ x } 1/12}{9^2} = 1.5474 \text{ ft. kip per ft}$$

Stress Distribution Resulting from the Elementary Analysis

01 10		12 21		23	Member
0.67		0.33 0.5		0.5	Dist. Factor
-0.5		-0.5			C.O.Factor
(a) Stress distribution for the intermediate support					
747	-747.0 629.1	192.0 -309.9	-192.0 96	0 -96.0	F.E.Stress Distribution
- 314.6	-32.2	-48.0 15.8	155 -53.5	48.0 53.5	Carry Over Distribution
16.1	18.0	26.8 -8.8	-7.9 -9.5	-26.8 9.5	Carry Over Distribution
-9.0	3.2	4.8 -1.6	4.4 -4.6	-4.8 4.6	Carry Over Distribution
-1.6	1.5	2.3 -0.8	0.8 -1.5	-2.3 1.6	Carry Over Distribution
-0.75	0.5	0.8 -0.3	0.4 -0.6	-0.8 0.6	Carry Over Distribution
-0.25	0.2	0.3 -0.1	0.2 -0.2	-0.3 0.2	Carry Over Distribution
436.8	-126.7	-126.7	-13.0	-13.0	Final Stress
(b) Stress distribution for the intermediate support					

372.4	-372.4 313.4	95.3 -154.3	-95.3 47.7	0 -47.7	F.E. Stress Distribution
- 156.7	-16.0	-23.8 7.9	77.2 -26.7	23.8 26.7	Carry Over Distribution
8.0	9.0	13.3 -4.4	-4.0 -4.7	-13.4 4.7	Carry Over Distribution
-4.5	1.6	2.4 -0.8	2.2 -2.3	-2.4 2.3	Carry Over Distribution
-0.8	0.8	1.2 -0.4	0.4 -0.8	-1.2 0.8	Carry Over Distribution
218.0	-63.4	-63.4	-6.4	-6.4	Final Stress
(C)Stress distribution for the intermediate support					
394.0	-394.0 332.0	101.0 -163.0	-101.0 50.5	0 -50.5	F.E. Stress Distribution
-166.0	-17.0	-25.3 8.3	81.5 -28.1	25.3 28.1	Carry Over Distribution
8.5	9.4	14.1 -4.7	4.2 -4.9	-14.1 4.9	Carry Over Distribution
-4.7	1.7	-2.5 -0.8	2.4 -2.4	-2.5 2.4	Carry Over Distribution
-0.85	0.9	1.3 -0.4	0.4 -0.9	-1.3 0.9	Carry Over Distribution
-0.45	0.3	0.5 -0.2	0.2 -0.4	-0.5 0.4	Carry Over Distribution
230.5	-66.7	-66.7	-6.9	-6.9	Final Stress

Table 4.16 Stress Distribution Resulting From the Elementary Analysis

The free edge stresses due to the rotation at plate 2 can be obtained.

For the exterior span:

$$\text{Plate 2: } f_b' = -f_t' = = \frac{-148 \times 30^2 \times 12}{17.53 \times 5832} = \text{-15.63 psi}$$

$$\text{Plate 1: } f_b' = -f_t' == \frac{105 \times 30^2 \times 12}{17.53 \times 853} = 75.40 \text{ psi}$$

For the middle span:

$$\text{Plate 2: } f_b' = -f_t' == \frac{-148 \times 40^2 \times 12}{29.2 \times 5832} = -60.7 \text{ psi}$$

$$\text{Plate 1: } f_b' = -f_t' == \frac{105.0 \times 40^2 \times 12}{29.2 \times 853} = 80.5 \text{ psi}$$

These free edge stresses again show incompatibilities which must be removed by stress distribution (Table 4.17).

Stress Distribution Resulting from an Arbitrary RotationStress

0	1		2		Joint Dist. Factor
0.67	0.33		0.5	0.5	
(a) Exterior span:Stress distribution for $\Delta_{2=1.in}$					
75.4	-75.4 40.1	-15.6 -19.7	15.6 -7.8	7.8	F.E. Stress Distribution
-20.0	2.6	3.9 -1.3	9.9 -6.9	-3.9 6.9	Carry Over Distribution
-1.3	2.3	3.5 -1.1	0.7 -2.1	-3.5 2.1	Carry Over Distribution
-1.2	0.7	1.0 -0.3	0.6 -0.8	-1.0 0.8	Carry Over Distribution
-0.4	0.3	0.4 -0.1	0.2 -0.3	-0.4 0.3	Carry Over Distribution
52.4	-29.4	-29.4	9.0	9.0	Final Stress

(b) Center span: Stress distribution for $\Delta_{2=1.\text{in}}$					
80.5	-80.5 42.7	-16.7 -21.1	16.7 -8.4	0 8.4	F.E. Stress Distribution
-21.4	2.8	4.2 -1.4	10.6 -7.4	-4.3 7.4	Carry Over Distribution
-1.4	2.5	3.7 -1.2	0.7 -2.2	-3.7 2.2	Carry Over Distribution
11.25	0.7	1.1 -0.4	0.6 -0.9	-1.1 0.9	Carry Over Distribution
-0.35	0.3	0.5 -0.2	0.2 -0.4	-0.5 0.4	Carry Over Distribution
56.0	-31.5	-31.5	9.6	9.6	Final Stress

Table 4.17 Stress Distribution Resulting From an Arbitrary Rotation

The calculated deflections due to the rotation of Plate 2 are as follows:

For exterior span:

$$y_2' = \frac{(-29.4-9.0)\times 30^2 \ \times 12}{29.2 \times 5832} = \text{-0.001697 in}$$

$$y_1' = \frac{(52.4+29.4)\times 30^2 \ \times 12}{13.56 \times 2.67 \times E} = 0.01220\text{in}$$

For center span:

$$y_2' = \frac{(-31.5-9.6)\times 40^2 \ \times 12}{17.1 \times 9 \times E} = \text{-0.00256 in}$$

$$y_1' = \frac{(56.0+31.5)\times 40^2 \ \times 12}{17.1 \times 2.67 \times E} = 0.0184\text{in}$$

Therefore, the total deflections of these plates will be expressed in terms of the deflections of the elementary analysis and the relative transverse displacementsΔ. By using Eq. (43), the values can be computed from the geometrical relations.

Exterior span:

$$y_2 = -0.00263 - 0.001697\,\Delta_2$$

$$y_1 = \ \ 0.04380 + 0.01220\Delta_2$$

from eqn. (43)

$$\Delta_2 = \frac{-y_1}{\sin 45^0} + y_2\,(\,2\cot 45^0) = \frac{-y_1}{0.707} + 2\,y_2 \qquad (44)$$

Substituting y_2 and y_1 into eq. (44)

$$\Delta_2 = -\,0.0657$$

Centre span:

$$y_2 = -0.00399 - 0.00256\Delta_2$$

$$y_1 = \ \ 0.06678 + 0.0184\Delta_2$$

Similarly using eq. (44)

$$\Delta_2 = -\,0.099385$$

4.6.2.3 Superposition

The final results of the analysis will be determined by combining the elementary solutions and each of them correction solutions multiplied by its respective Δn. The final results are shown in Tables XVIII and XIX.

The value of the deflection which is parallel to the plate element, shown in Table XIX, is a relative value because an arbitrary modulus of elasticity was used. The vertical deflection of any joint can be calculated from the plate deflections. The relationships between these deflections are shown in Figure 7 and are expressed as follows:

$$V_n = y_n \frac{\cos \emptyset_n}{\sin \propto_n} - y_{n+1}\,\frac{\cos \emptyset_n}{\sin \propto_n}$$

The shearing forces N along the joints may be calculated from Eq. (17)

$$N_1 = -1/2\,(\,-436.8 + 126.7\,)x\ 160 = 24800\ lb$$

$$N_2 = 24800 - 1/2\,(126.7 + 13.0)\ x\ 324 = 2200\ lb$$

The shearing stresses are computed from Eq. (49) and Eq. (50) as follows.

Plate 1. - At the supports of the exterior spans, the positive simply supported bending moment is $P_2 = 30^2/8 = 112.5P2$ ft-1b.

Then,

$$N_{max} = 24800\ x\ 112.5/126.4 = 22100\ lb$$

From eq. (50)

$$V_1 = \frac{4 \times 22100}{4.5 \times 360} - \frac{24800}{4.5 \times 360} = 54.7 - 15.3 = 39.41 \text{ lb/ sqft.}$$

V= 0

At the inner support x = L1

$$V_1 = -54.7 - 15.3 = -70.0 \text{ lb/sqft}$$

At each support of the center span the simple span moment is $P_2\ 40^2/8 = 200\ P_2$ ft-lb

$$N_{max} = 24800 \text{ X } 200/\ 126.4 = 39{,}400$$

From Eq. (49)

$$V_1 = 39400 \text{ x } 4/\ 4.50 \text{x } 480 = 73 \text{ lb/ sqft.}$$

Plate 2. - At the support of the exterior spans at Joint 1

$$N_{max} = 22100 \text{ lb}$$

$$V_1 = \frac{4 \times 22100}{3 \times 360} - \frac{24800}{4.53 \times 360} = 82 - 23 = 59 \text{ lb/ sqft.}$$

and at Joint 2

$$N_{max} = 2200 \text{ x } (112.5/\ 126.4) = 1960 \text{ lb.}$$

$$V_2 = \frac{4 \times 1960}{3 \times 360} - \frac{2200}{3 \times 360} = 7.28 - 2.04 = 5.24 \text{ lb/ sqft.}$$

At $x = L_1$

$$V_1 = -82 - 23 = -105 \text{ l / sqft.}$$

$$V_2 = -7.28 - 2.04 = -9.32 \text{ lb/sqft.}$$

At each support of the center span, and at Joint 1,

$$N_{max} = 24800 \text{x} (200/126.4) = 39200 \text{lb}$$

$$V_1 = (4 \text{ x } 39200) / (3 \text{ x } 380) = 109 \text{ lb/ sqft.}$$

And at joint 2, N_{max}=2200 x (200/ 126.4) = 3480 lb

$$V_2 = (3480 \text{ x } 4) / (3 \text{ x } 480) = 9.65 \text{ lb/ sqft.}$$

Longitudinal Stresses

(a)Longitudinal stresses at the intermediate support				
Joints	Elementary analysis	Correction analysis	Total correction	Final values psi
0	-436.8			-436.8
1	126.7			126.7
2	13.0			13.0
(b)Longitudinal stresses for the exterior span at				
0	218.0	52.4	-3.44	214.6
1	-63.4	-29.4	1.93	-61.4
2	-6.4	9.0	-0.59	7.0
(c)Longitudinal stresses for the center span at mid-span				
0	230.5	56.0	-5.6	224.9
1	-66.7	-31.5	3.13	-63.6
2	-6.9	9.6	-0.95	-7.9

Table 4.18 Final Longitudinal Stresses of Example 2

(I) Transverse Moments				
(a)Transverse moments for the exterior span at 0.4L1				
Joints	Elementary Analysis	Correction analysis	Total Correction	Final values
2	-355.0	664.0	-43.0	-398.6
(b)Transverse moments for the center span at mid-span				
2	-355.0	664.0	66.0	421.0
(II) Deflections				
(a)Deflections for the exterior span at 0.4L1				
1 2	0.04380 -0.00263	0.01220 -0.001697	-0.008 0.0001	0.043in -0.0025in
(b)Deflections for the center span at mid-span				
1 2	0.06678 -0.00399	0.01840 -0.00250	-0.0018 0.0003	0.064in -0.0037in

Table 4.19 Final Transverse Moments and Deflections of Example 2

CHAPTER -5

CONCLUSIONS

The proposed method of analysis of folded plates developed in this paper yields satisfactory results for the analysis of continuous folded plate roofs in comparison to the values obtained by Yitzhaki's slope-deflection method. Although the study presented herein suggests a practical method to design continuous folded plate roofs with symmetrical loading, it can also be applied to symmetrical folded plate roofs, unsymmetrical loaded, by dividing the unsymmetrical load into symmetrical and anti-symmetrical loads. The final stresses and deflections will be obtained by superimposing the results of the two cases. The determination of the spacing of the intermediatesupports would be based on an economic study. The thickness, depths, the magnitude of the angles between the individual plates, and the rigidity of the transverse stiffener are all important factors which will affect the spacing of the intermediate supports. The stiffeners must be designed to carry their own dead load plus the reactions imparted to them by the shearing forces from the adjoining plates. The stresses and the design of the intermediate stiffener need to be further investigated. The loads of folded plates have been assumed to be transmitted to the joints by transverse moments. All loads are finally carried by one-way slab action to the end support. Torsional stresses due to twist1ng of the plates may be ignored in this analysis.

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