

# Analysis of multi-component signals using covariance based Modified S-transform

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**Abstract--** Now a day's time-frequency based method plays an important role for the analysis of the non-stationary, chirp, and multi-component signals. For analysis of these signals, covariance based S-transform has been proposed and its performance has been compared with short time Fourier transform, and S-transform.

**Index Terms:** Non-stationary signals, chirp signals, STFT, ST

## 1. INTRODUCTION

The extraction of information from the multi-component signal is highly needed because most of the signals present in the nature are multi-component. Analysis of the multi-component signals play an important role in various fields like signal processing, bio-signal processing, and genomics signal processing [1]. Therefore, time-frequency methods, short time Fourier transform (STFT), continuous wavelets transform (CWT), and Stockwell transform have been reported for the analysis of these signals [2]. The STFT has been successfully applied to extract the information from the multi-component signals but it has a major limitation of fixed resolution due to fixed window length [2]. CWT has been applied to overcome the problem of STFT, where as in CWT scale is inversely proportional to frequency, and it unable to provide relation between time and frequency both as well as phase information is also lost [3]. Stock well transform (ST) has been reported to eliminate, problem of fixed time frequency resolution in STFT and phase elimination problem of the CWT. The ST provides the better time frequency resolution as compared to CWT and STFT without the problem of phase removal and fixed resolution respectively. The Gaussian window has applied in ST to provide the time-frequency resolution. The width of this window has been varies with frequency [3]. Therefore, a time-frequency resolution of ST still needed improvement.

In this work, a modified S-transform have been presented in which the width of the Gaussian window varied with respect to covariance of the signal.

In this paper, time-frequency methods have been presented in Section 2, modified S-transform has been explained in Section 3, simulation results and discussions have been done in Section 4, and conclusion has been presented in Section 5.

## 2. TIME FREQUENCY ANALYSIS

Time-frequency methods for the multi-component signal have been presented as follows-

### A. STFT

The  $N$ -point windowed DFT of a signal  $x(n)$  is defined as [3]-

$$x(k) = \sum_{n=0}^{N-1} x(n)w(n)e^{-\frac{j2\pi nk}{N}} \quad (1)$$

where,  $k = 0 \dots, N-1$ ,  $w(n)$  is the rectangular window of fixed length. Using (1), the power spectrum of the signal is -

$$S(k) = |x(k)|^2 \quad (2)$$

Also due to fixed window length, it does not provide good resolution at all the frequencies. Thus STFT is unable to provide better performance than other TFDs.

### B. S-Transform

The S-transform of a signal  $x(t)$  is defined as [2]-

$$S(t, f, \sigma) = \int_{-\infty}^{\infty} x(\tau) G(t - \tau, \sigma) e^{-j2\pi f\tau} d\tau \quad (3)$$

where  $G(t - \tau, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\tau)^2}{2\sigma^2}}$ , is a variable Gaussian window function,  $\sigma$  its standard deviation,  $t$  time,  $\tau$  central position of the window and  $f$  is the frequency. The advantage of the S-transform over the STFT is that the standard deviation  $\sigma$  is a function of frequency  $f$ , i.e.  $\sigma(f) = \frac{1}{|f|}$ , so the length of the Gaussian window will vary with respect to the frequency. Substituting the value of  $\sigma$  in (3) we get[3], [4],[5] and [6]-

$$S(t, f) = \int_{-\infty}^{\infty} x(\tau) \left\{ \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2 f^2}{2}} e^{-j2\pi f\tau} \right\} d\tau \quad (4)$$

The S-transform is defined as a short time Fourier transform with a variable window. The S-transform provides the progressive time frequency resolution but faces the problem of poor energy concentration in the time frequency plane as the standard deviation of the Gaussian window is inversely proportional to the frequency. For higher frequencies, the window length is small, and it contains fewer numbers of periods resulting in poor energy concentration. Similarly, for the lower frequencies, window length is large, and it contains more number of periods within it and the energy concentration is more.

### 3. MODIFIED S TRANSFORM

In this section, S-transform has been modified using covariance of the signal and it is given by -

$$\beta(f) = n \times \gamma \times f \quad (5)$$

In equation (5),  $f$  is frequency,  $\gamma$  is a covariance, and  $n=51$ , the value constant has been chosen experimentally. The relation between  $\beta$  and standard deviation of the Gaussian is given by following relation-

$$\sigma = \frac{\beta(f)}{|f|} \quad (6)$$

Thus window function of the proposed method is represented by-

$$w(\tau - t, f, n, \beta) = \frac{|f|}{\sqrt{2\pi\beta(f)}} e^{-\frac{(\tau-t)^2}{2(\beta(f))^2}} \quad (7)$$

Now, S transform thus becomes modified and is given by the following equation-

$$S(\tau, f, n, \gamma) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{\sqrt{2\pi(n \times \gamma \times f)}} e^{-\frac{(\tau-t)^2 f^2}{2(n \times \gamma \times f)^2}} e^{-i2\pi ft} dt \quad (8)$$

Thus, we have controlled the time and frequency resolution using the standard deviation of the Gaussian window.

### 4. SIMULATION RESULTS AND DISCUSSIONS

Simulation results have been presented using STFT, ST and modified S-transform using following example signals.

#### 4.1 Example 1

The first signal is

$$h(t) = \sin(2\pi(10t + \frac{5}{4}t^2 + \frac{1}{9}t^3 - \frac{1}{160}t^4)) \quad (9)$$

$0 \leq t \leq 15\text{sec}$

The signal is sampled at a sampling frequency of 200 Hz. The time-frequency plot has been generated for the signal using STFT, ST and modified S transform.

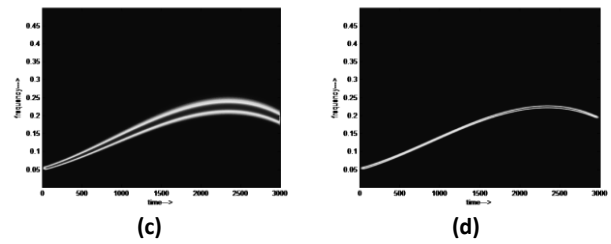
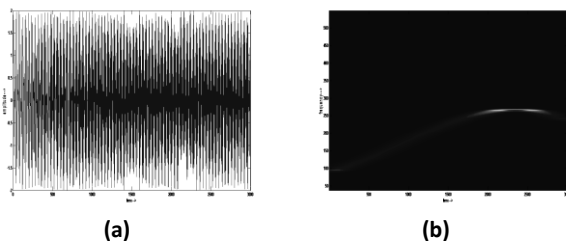


Figure1. Time frequency plot (a) Input signal (b) STFT (c) ST (d) modified ST

Figure.1 (b), 1(c), and 1(d), shows the time-frequency plot using STFT, ST and modified S-transform. From these results, it has been clear that ST performs well as compared to STFT because it gives highlighted region for all the frequency present in it but frequency resolution has been suffered due to the spectral width of the highlighted region. Using modified S-transform the spectral width has been reduced significantly and frequency resolution also improved.

#### 4.2 Example 2

Here, we have considered two linear multi-components signal and it is given by-

$$h(t) = \sin(2\pi(10 + 2.5t)t) + \sin(2\pi(15 + 2.5t)t) \quad (10)$$

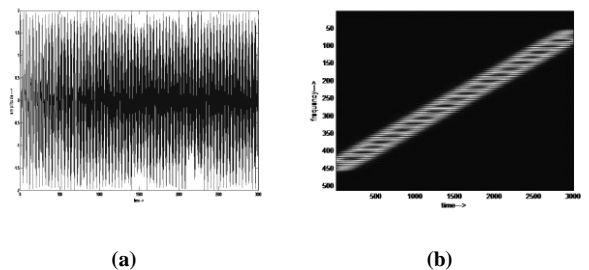
$(0 \leq t \leq 15\text{sec})$

The signal is sampled at a sampling frequency of 200 Hz. As the components are very close to each other and it shown in Figure2 (a). The time-frequency plots have been plotted using 2(b), Figure2(c) and Figure2 (d). From these results it has been clear that the modified S-transform perform better than STFT and ST to separate the multi-component signal.

#### 4.3 Example 3

Here, a multi-component chirp signal has been taken for the analysis and it is given by-

$$h(t) = \sin(60\pi t + 12\pi \sin(\frac{\pi t}{6})) + \sin(0.7\pi t^2 + 25\pi t + 25) \quad (11)$$



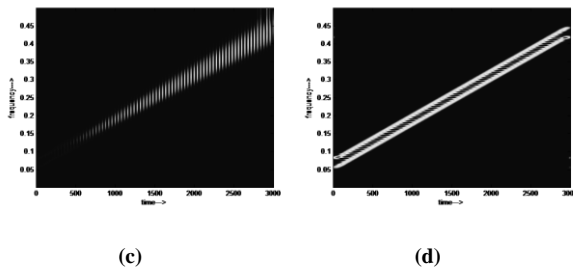


Figure2. Time frequency plot (a) Input signal (b) STFT (c) ST (d) modified ST

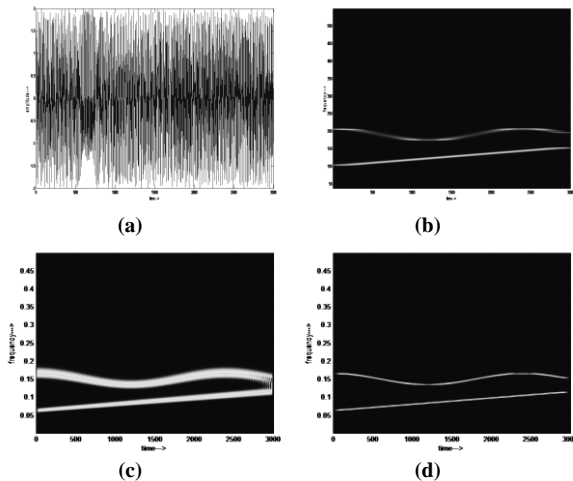


Figure3. Time frequency plot (a) Input signal (b) STFT (c) ST (d) modified ST

The time-frequency plot for the signal given in (11) Figure3(a) have been shown in Figure3 (b), Figure3(c) and Figure3(d). From these results it has been clear that modified s-transform shows the significant improvement in the frequency resolution and separation of the multi-component signals.

## 5. CONCLUSION

In this research work covariance based modified S-transform has been applied for the analysis of multi-component signals. On the basis of the simulation results it has been concluded that proposed method provides better resolution and separation of the multi-component signal as compared to reported method STFT and ST. Presently, the work in progress to explore its application for the analysis of biomedical signals, radar signals processing.

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## Author Profile



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He has done B. Tech in Electronics and Communication from I.E.E.T. Baddi in 2012, M. Tech in Electronics and Communication from Jaypee University of Information Technology, Wanknaghat in 2016. His current area of

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