ANALYSIS OF TRANSMISSION LINE MODELS WITH NON-LINEAR ELEMENTS FOR OPPOSITE PHASE AND GROUP VELOCITIES

A THESIS SUBMITTED IN FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY
IN
ELECTRONICS AND COMMUNICATION ENGINEERING

By

SALMAN RAJU TALLURI
Enrollment No: 106003

विद्या तत्व ज्योतिसमः

Under the guidance of
Prof. (Dr). S.V. BHOOSHAN

Department of Electronics and Communication Engineering
JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY
WAKNAGHAT, SOLAN (H.P.) - INDIA
June - 2016

Declaration

I hereby declare that the work reported in the Ph.D. thesis entitled "**Analysis of transmission line models with non-linear elements for opposite phase and group velocities**" submitted at Jaypee University of Information Technology, Waknaghat India, is an authentic record of my work carried out under the supervision of Prof. Sunil Bhooshan. I have not submitted this work elsewhere for any other degree or diploma.

Salman Raju .T

Salman Raju Talluri

Department of Electronics and Communication Engineering

Jaypee University of Information Technology

Waknaghat, Solan-173234

India

Certificate

This is to certify that the thesis entitled, "**Analysis of transmission line models with non-linear elements for opposite phase and group velocities**" which is being submitted by **Salman Raju Talluri** in fulfillment for the award of degree of **Doctor of Philosophy** in Electronics and Communication Engineering by the Jaypee University of Information Technology, is the record of candidate's own work carried out by him under my supervision. This work has not been submitted partially or wholly to any other University or Institute for the award of this or any other degree or diploma.

Prof. Sunil Bhooshan
Professor and Head
Department of Electronics and Communication Engineering
Jaypee University of Information Technology
Waknaghat, Solan-173234 India

Acknowledgment

It is a great privilege for me to meet some of the best people in academic circles and I firmly believe that it is a good opportunity for me to meet my supervisor Prof. (Dr).Sunil Vidhya Bhoosan Sir. I would like to thank him from the bottom of my heart for all the support that he has provided me very patiently. Sir's support is not only in my research but in my teaching and more importantly in my personal life as well. The support he has given me is immense and I am very thankful to him. Sir is one of the best people whom I have ever met.

I would like to thank my DPMC members for their support and comments they given me during my presentations so that I will be a much better person than I am now.

I would like to thank all the colleagues of my department in JUIT for their goodness and kindness towards me for all these years.

Personally, I would like to thank Prof. Lamba sir for his most valuable advice sir has given me once.

I would like to thank Prof. Samir Dev sir for his encouragement when I interact with sir.

No matter what I do, it may not be sufficient for all the sacrifices my father and mother made for me. My father is always there for me in all situations. I will never forget the love and care my mother taken for me and for all the prayers she offered to the LORD. I would like to thank my sisters for all the care and concern they had me for all these years. I would like to thank my wife who always encourages me when I needed the most.

I am so thankful to the ALMIGHTY for all his blessings showered on me so far moment by moment as an answer to the prayers offered by HIS people.

Contents

List of Figures

List of Tables

1 Introduction

1.1 Introduction

A medium is required for the transmission of energy or power from one place to another place. Depending on the nature of energy or power, such as mechanical or electrical or optical or electromagnetic energy, different types of media are required. Such commonly used media in communication systems are co-axial cable, parallel wire transmission lines, optical fiber cable and wave guides etc [1]. A transmission medium is selected such that the energy or power of the signal/wave can be transmitted from one point to another point very efficiently with loss as minimal as possible. Hence it is important to study and analyze the properties of the medium for different types of waves.

When a signal/wave is traveling through the medium, it can go through some changes in amplitude or phase or frequency. If the transmitted signal is only varied in the amplitude, it can be equalized with the help of an amplifier. If there are any changes in phases or frequencies, then equalizers [2] are required to cancel the unwanted distortions during the transmission.

For a distortion-less transmission, the transmitted signal can be attenuated in amplitude and can also be delayed but it cannot go through any modifications in phase or frequency. For a distortion-less medium, the input and output relationship [3] can be expressed as:

$$y(t) = Cx(t-T) \tag{1.1.1}$$

where $y(t)$ represents the output and $x(t)$ represents the input of the medium. Usually, $x(t)$ contains multiple frequencies. Here C represents the amplification or attenuation of the channel. It represents attenuation if $C < 1$ and amplification if $C > 1$. If $C < 1$, then the signal power or energy will be dissipated in the medium. Here T represents the delay of the signal. This delay can be defined as the time taken by the signal to travel from input to output side. If the physical length of the medium is L units and v is the velocity of the signal (expressed in the same units as that of length), then T can be equated to L/v.

The above Eqn.1.1.1 represents the channel or medium characteristics in time domain. This can be converted into frequency domain to get the transfer function $H(j\omega)$ of the medium [4] as:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = Ce^{-j\omega T} \tag{1.1.2}$$

where $X(j\omega)$ is the Fourier transform of the input signal and $Y(j\omega)$ is the Fourier transform [5] of the output signal.

From this equation it is observed that for distortion-less transmission, the amplitude variation C, which is also called magnitude response [6] is same for all frequencies of the input signal while the phase variation also called as phase response is a linear function$(-j\omega T)$ of frequency.

If the phase response of the medium is a linear function of frequency, every frequency component in the original input signal will be delayed by the same amount of time and the original transmitted signal through the medium can be reconstructed at the output as the superposition of all equally delayed frequency components. In the phase response, if there are any deviations from this linear relationship then it results in distortion.

This distortion happens due to the fact that different frequencies will be delayed by different amounts of time and sum of these signals at the output deviates from the original signal there by producing the distortion in the signal transmission. Hence for a distortion-less transmission, the phase response must be linear which is an important requirement. If the phase response is a non-linear function of frequency, then the signal gets distorted which is called as dispersion [7].

In a non-dispersive medium, waves can propagate without any distortion. Electromagnetic waves [8] in vacuum are non-dispersive as well as non-dissipative. Due to this reason they travel long distances in vacuum without any distortion. Ideally, all practical media exhibit dispersion for signal transmission. For the wave propagation in a medium rather than using the magnitude and phase responses to characterize the signal transmission, the complex propagation constant is used in the similar manner as that of the frequency response of the medium [9].

1.2 Literature review

Even though dispersion is a negative factor of the medium with respect to signal transmission, there are some useful applications of dispersion as well. In optics [10], the separation of white light into color spectrum by prism is one of the main applications of dispersion. The dispersion of light by glass prisms is used to construct spectrometer and spectro radiometers [11]. In electromagnetics, dispersion is used in pulse shaping circuits [12], high power microwave sources (HPM) [13] and in antenna arrays for beam-forming applications.

Linear transmission line theory [14] is extensively used in microwave engineering in designing matching circuits, filter, phase shifters etc. In all these applications, the elements are assumed as linear and less importance is given for dispersion characteristics. Even though the linear transmission lines exhibit dispersion, non-linear transmission lines are widely used to have the dispersion. Non-linear transmission line theory [15] has been reported in literature for many years.

A nonlinear transmission line (NLTL) comprises of a transmission line periodically loaded with varactors, where the capacitance non-linearity arises from the variable depletion layer width, which depends both on the direct current (DC) bias voltage and on the alternating current (AC) voltage of the propagating wave [16]. They are mainly used to produce high power microwave oscillators [17]. Initially, nonlinear capacitors were used followed by nonlinear inductors. The first generation NLTL used low voltage varactor diodes as nonlinear capacitors and operated at low-to moderate RF frequencies. Later on nonlinear ceramics [18] were used in NLTL and the frequency of operation increased up to approximately 100 MHz [19]. This was possible because nonlinear dielectrics, chiefly ceramic barium titanate, had been used in commercial ceramic capacitors for many years.

The radio frequency generation using the non-linear transmission lines is presented in [20]. In this work, they obtained the non-linear transmission line using the varactor diodes. In [21], GaAs planar Schottky varactor diodes are successfully developed to design and fabricate a monolithic phase shifter at 30GHz. There are so many applications of non-linear transmission lines and in all these applications, main emphasis is on only the non-linear capacitor.

In recent years, composite right hand left hand meta-material (CRLH-MTM) [22] attained lot of attention and used very extensively in microwave engineering. The main reason for their popularity is showing negative permeability and permittivity. At the microwave frequencies, a number of transmission line with CRLH MTM characteristics have been proposed so far [23, 24, 25, 26]. Recently LH-NL transmission line has been proposed by [27]. In their work, they proposed a LH-NL transmission line periodically loaded with series non-linear capacitance and linear shunt inductance. In this LH-NLTL, the pulse propagates through negative group velocity while phase velocity is positive.

The use of non-linear inductors in power systems is well established [28, 29, 30, 31]. The use of linear inductors on microstrip version also reported in the literature [32]. Due to the availability of different materials that can be modeled as non-linear medium such as ferromegnetic material, the use of inductors has been increased in the electronic chip fabrication as well. Recently analysis on active inductors are presented in [33].

The use of limited dispersion in optical phased array beam steering is proposed in [34]. Moving from optical waves to electromagnetic waves, photonic crystals fiber array operating in X-band has been proposed in [35]. The application of dispersion in antenna beam steering is presented in the recent literature [36, 37, 38, 39]. In the applications reported in [40, 41, 42], dispersion plays an important role and this dispersion occurs due to the non-linear effects in the medium.

The above mentioned NLTL are analyzed by considering the shunt branch capacitor as non-linear. But due to the availability of non-linear inductors, the NLTL can be more generalized with respect to series branch element as well. Now with both the series element and shunt element as non-linear, it is required to analyze the propagation of electromagnetic waves through generalized NLTL. Propagation characteristics can be obtained fro the dispersion and hence this thesis is aimed at the dispersion characteristics of transmission line models with non-linear elements.

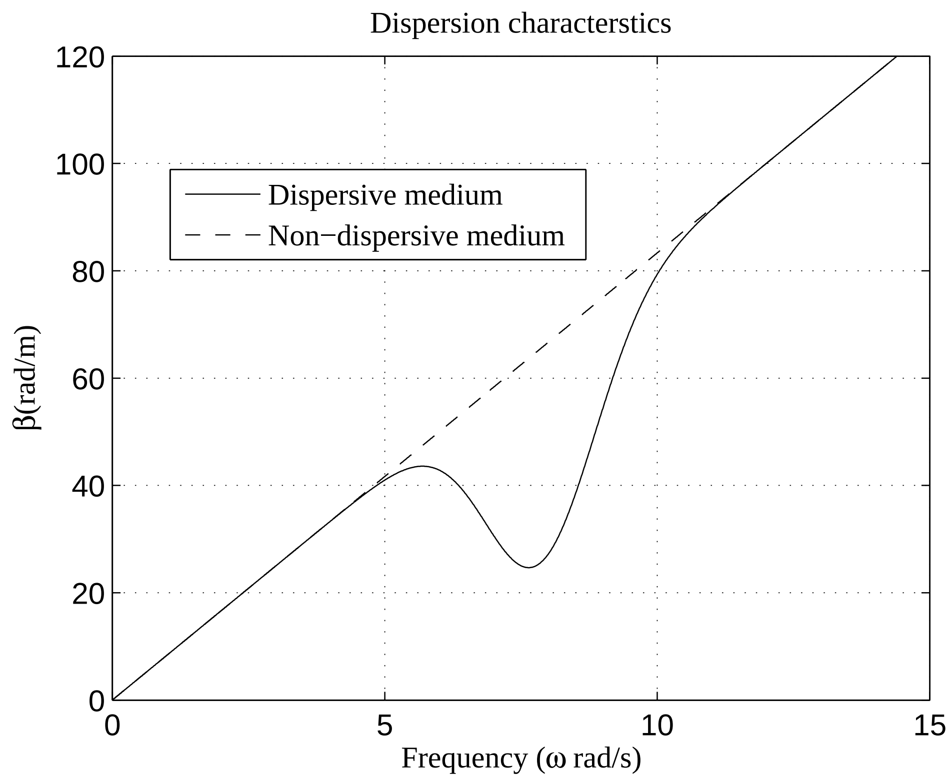

Figure 1.3.1: Typical variations in phase constants (β) for dispersive and non-dispersive media

1.3 Problem formulation

The propagation of electromagnetic waves in a medium is characterized by its complex propagation constant γ. This γ is a complex number whose real part α represents the attenuation constant while imaginary part β represents the phase constant. For an ideal lossless transmission line α is zero while β is a linear function of frequency. This means that on an ideal transmission line, signals with different frequencies travel with the same velocity. But in a real (practical) transmission line, the velocity of propagation varies with frequency which leads to dispersion. A typical dispersion characteristic for dispersive medium is as shown in Fig.1.3.1.

The dispersion can be expressed in terms of phase velocity(v_p) and group velocity(v_g).

$$v_p = \frac{\omega}{\beta} \tag{1.3.1}$$

$$v_g = \frac{d\omega}{d\beta} \tag{1.3.2}$$

The phase velocity represents the velocity at which the constant phase wave-front moves in the medium while group velocity represents the velocity at which the entire the envelope of the wave-front moves in the medium. Due to dispersion, different frequency components travel with different velocities and if a pulse is traveling in that medium it gets distorted. If the dispersion is more, then the pulse will be distorted completely and it cannot be distinguished clearly. This is a disadvantage in data transmission [43]. Even though the dispersion is a negative factor in data transmission, dispersion is used pulse shaping circuits and in beam steering of the antenna arrays.

In antenna arrays [44], progressive phase shift is used to steer the antenna pattern in different directions. If the frequency is changed, the progressive phase shift [45, 46] changes and the direction of main beam changes from the desired direction. In such applications it is required to have the same phase constant at different frequencies. This means that the dispersion characteristic is non-monotonic to have same β at different frequencies. To have the same phase constant at two different frequencies, it is required to have non-monotonic variation in the phase constant as a function of frequency. This leads to dispersion with opposite phase and group velocities. This is the advantage of dispersive medium.

Linear circuit elements also exhibit the dispersion. But the dispersion coming from the linear circuit elements is mostly monotonic variation and cannot produce opposite phase and group velocities. Hence this thesis analyses the dispersion characteristics of transmission line models (which supports the electromagnets wave propagation through it) with non-linear element such as non-linear inductor and non-linear capacitor. To observe this, the following objectives have been formulated and analysis is carried out to present the general conclusions.

- Objective:1 Modeling of the non-linear flux dependent inductor as current dependent inductor to obtain its frequency response. Similarly, modeling of non-linear charge dependent capacitor as voltage dependent capacitor and obtaining its frequency response.

- Objective:2 To analyze the unit cell of the conventional transmission line models with non-linear elements to get the opposite phase velocity and group velocity.

- Objective:3 To present a simple circuit model which produces opposite phase and group velocities with minimum number of elements.

1.4 Thesis organization

All these objectives are organized in the following chapters as follows. Chapter 2 gives the basic revision of the transmission line theory. Chapter 3 presents the detailed analysis of the non-linear inductor and capacitor. Chapter 4 gives the analysis on conventional transmission line models. Chapter 5 presents the analysis of conventional transmission line models with Miller loading while Chapter 6 presents the conclusions, future scope and limitation of the present work.

2 Transmission lines

2.1 Introduction

A transmission line is a combination of conductors or arrangement of conductors to carry the signal from one point to another point. Some of the commonly used transmission lines are co-axial cable, microstrip line, twisted pair etc [47]. In microwave engineering [9], transmission lines are used to carry the power from one point to another point in a circuit or to connect the transmitter or receiver with an antenna.

At higher frequencies, the time periods of the currents and voltages become comparable to the time taken by the signal to propagate from one point to another point. In such cases, the length of the line becomes important because the changes in voltage or current at the source are not reflected instantaneously at all points on the line. There will be phase changes from point to point on the line. Due to these differential phase changes, both voltage and current are functions of distance and time on the line. Hence the two-wire transmission line at higher frequencies is treated as a distributed-parameter [1] network where voltages and currents vary in magnitude and phase over its length.

Transmission line theory [14] is extensively used in microwave engineering. The main difference between the transmission line theory and circuit theory is electrical size. Circuit analysis assumes that the physical dimensions of the device are much smaller than the electrical wavelength, while transmission lines may be a considerable fraction of wavelength, or many wave lengths, in size. A common thumb rule is that if the length of the cable is more than $\lambda/10$, where λ is wavelength, then the circuit is treated as distributed.

In many ways the transmission line theory bridges the gap between the field analysis and basic circuit theory. With the help of transmission line the-

Figure 2.1.1: Two wire transmission line as two port network

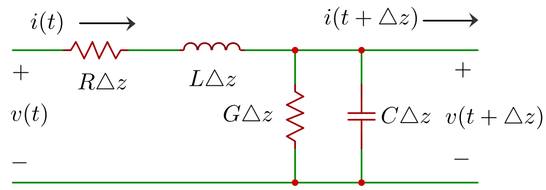

Figure 2.1.2: Equivalent circuit model of transmission line of differential length dz

ory, it is easy to analyze the wave propagation through the medium without exactly solving the Maxwell's [48] equations. Wave phenomenon such as reflection and transmission can be analyzed in circuit quantities (namely voltage and current) rather than in filed quantities (namely electric field intensity and magnetic field intensities).

A transmission line can be analyzed using the Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) [49] in the equivalent distributed circuit model. For the purpose of analysis, the transmission line can be modeled as a two-port network as shown in Fig.2.1.1. This figure represents the uniform distributed transmission line.

The above differential length (dz) transmission line can be represented with the equivalent lumped element circuit model as shown in Fig.2.1.2. Here R , L , G and C represents the distributed resistance, inductance, conductance and capacitance of the transmission line model. These are distributed along the line and hence all these are expressed per unit distance. This circuit model is treated as the unit cell for the conventional transmission line model.

The governing equations for the transmission line model are obtained by writing the KCL and KVL equation for the circuit. Applying KVL the following equation is obtained.

$$v(z,t) = i(z,t)R \triangle z + \frac{\partial i(z,t)}{\partial t} L \triangle z + v(z + \triangle z, t) \tag{2.1.1}$$

Based on the mathematical definition of derivative, the following equation is obtained.

$$v(z + \triangle z, t) = v(z,t) + \frac{\partial v(z,t)}{\partial t} \triangle z \tag{2.1.2}$$

Substituting the Eqn.2.1.2 into Eqn.2.1.1 the following equations are obtained.

$$v(z,t) = i(z,t)R \triangle z + \frac{\partial i(z,t)}{\partial t} L \triangle z + v(z,t) + \frac{\partial v(z,t)}{\partial t} \triangle z \tag{2.1.3}$$

$$-\frac{\partial v(z,t)}{\partial z} \triangle z = Ri(z,t) \triangle z + L \triangle z \frac{\partial i(z,t)}{\partial t} \tag{2.1.4}$$

$$-\frac{\partial v(z,t)}{\partial z} = Ri(z,t) + L \frac{\partial i(z,t)}{\partial t} \tag{2.1.5}$$

Following the same procedure and writing the KCL equations at the intersection point of series elements and shunt elements the following equation is obtained.

$$-\frac{\partial i(z,t)}{\partial z} = Gv(z,t) + C \frac{\partial v(z,t)}{\partial t} \tag{2.1.6}$$

Eqn.2.1.5 and Eqn.2.1.6 are coupled equations and they can be separated using algebraic manipulations and the final equation can be obtained as follows

$$\frac{\partial^2 v(z,t)}{\partial z^2} = RGv(z,t) + (RC + LG) \frac{\partial v(z,t)}{\partial t} + LC \frac{\partial^2 v(z,t)}{\partial t^2} \tag{2.1.7}$$

$$\frac{\partial^2 i(z,t)}{\partial z^2} = RGi(z,t) + (RC + LG) \frac{\partial i(z,t)}{\partial t} + LC \frac{\partial^2 i(z,t)}{\partial t^2} \tag{2.1.8}$$

Now these are uncoupled equations and also called as Telegrapher's equations. These equation can be converted into phasors by replacing the time derivative with multiplication of $j\omega$ and the following equations will be obtained.

$$\frac{dV(z)}{dz} = -ZI(z) \tag{2.1.9}$$

$$\frac{dI(z)}{dz} = -YV(z) \tag{2.1.10}$$

$$\frac{d^2V(z)}{dz^2} = -Z\{-YV(z)\} = ZYV(z) = \gamma^2 V(z) \tag{2.1.11}$$

where

$$Z = R + j\omega L \tag{2.1.12}$$

and

$$Y = G + j\omega C \tag{2.1.13}$$

From the above two equations the parameter γ is obtained as:

$$\gamma = \alpha + j\beta = (ZY)^{\frac{1}{2}} = ((R + j\omega L)(G + j\omega C))^{\frac{1}{2}} \tag{2.1.14}$$

Here γ is complex propagation constant of the circuit. This represents whether the medium actually supports the wave propagation through it or not. The real part α is called as attenuation constant and imaginary part β is called as phase constant. For a loss-less transmission line, $R = 0$ and $G = 0$ thus making $\alpha = 0$ and $\beta = \omega\sqrt{LC}$ as a linear function of frequency.

The solution of Eqn.2.1.11 can be obtained by solving this equation as a second order differential equation as:

$$V(z) = V_+ e^{-\gamma z} + V_- e^{+\gamma z} = V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z} \tag{2.1.15}$$

Substituting Eqn.2.1.15 in Eqn.2.1.9 the following equation is obtained.

$$I(z) = \gamma(V_+ e^{-\gamma z} - V_- e^{+\gamma z})/Z \tag{2.1.16}$$

Substituting $\gamma = \sqrt{ZY}$ in the above equation it is possible to express the given equation as

$$I(z) = Y_0(V_+e^{-\gamma z} - V_-e^{+\gamma z}) \tag{2.1.17}$$

where Y_0 represents the characteristic admittance of the Transmission line. Reciprocal of characteristic admittance is defined as characteristic impedance. This is defined as the ratio of complex voltage and the complex current of the same line at any point on the line. If the characteristic impedance is a complex number, then it indicates that the phase of current and voltage are not the same. Two most important parameters of the transmission line models are the complex propagation constant and the characteristic impedance. Depending on the medium, the complex propagation constant can be independent of frequency or can be dependent on frequency.

2.1.1 Different types of transmission lines

There are different types of transmission lines. If the distributed elements are independent of both space and time, that transmission line is considered as linear and uniform transmission line. If these values are changing only with space then, they are called as non-uniform transmission line. If they are changing with only time, then they are non-linear transmission lines. If these values changes with both space and time, they are non-linear non-uniform transmission lines.

- Ideal transmission lines: For an ideal transmission line, the losses are zero. This is obtained when $R = 0$ and $G = 0$. If the line parameters are independent of length, then it is called uniform transmission line. For a uniform transmission line the size of the conductors and spacing between then remains the same there by having constant impedance and admittance. If the elements are assumed as linear then this is called as linear. For this line the attenuation constant α is zero and phase constant is linear function of frequency as $\beta = \omega\sqrt{LC}$

- Non-ideal transmission lines: For non-ideal transmission lines, the propagation constant and is generally complex number and there will be losses Phase constant is no longer a linear function of frequency but a

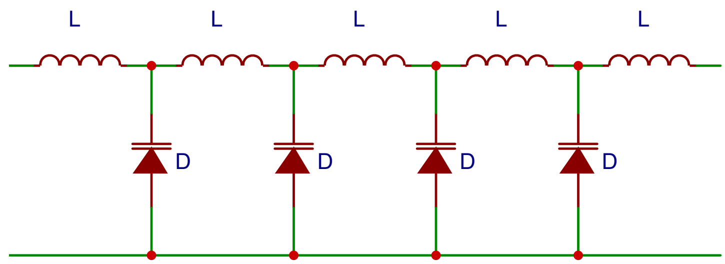

Figure 2.2.1: Non-linear transmission line representation with varactor diodes

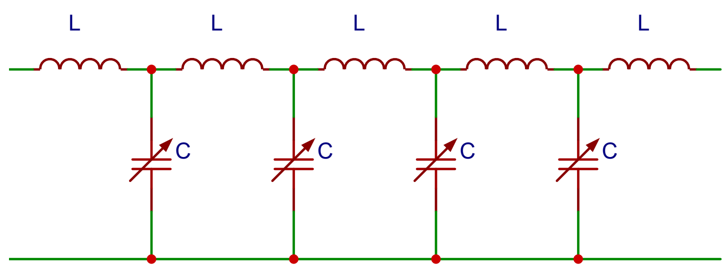

Figure 2.2.2: Equivalent circuit model with variable capacitor

non-linear function of frequency. These non-linear transmission lines distort the signal when they propagate through the medium.

Depending on the nature of elements in the circuit model, the transmission lines can also be considered as linear transmission lines and non-linear transmission lines. If the elements like resistance, capacitance, inductance and conductance are considered to be non-linear then it is treated as a non-linear transmission line. This thesis presents the analysis of the non-linear transmission line models.

2.2 Non-linear transmission lines

Non-linear transmission line theory [16] is an extension of the linear transmission line theory. In practical non-linear transmission lines, the shunt branch element is replaced with a non-linear element such as varactor diode [50] as shown in Fig.2.2.1. These non-linear transmission lines are used in different application in microwave engineering.

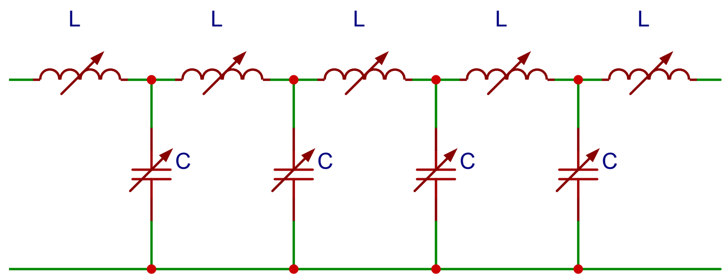

Figure 2.2.3: Considered non-linear transmission line in this thesis

To analyses the non-linear transmission line model, the varactor diode is replaced with the equivalent voltage dependent capacitor [51] as shown in Fig.2.2.2. The analysis of non-linear transmission lines is specific to the non-linearity assumed. There is continuous research is going on these non-linear transmission lines. But all the literature available is only with the capacitor as non-linear but this thesis has addressed the non-linear transmission line with non-linear inductor as well. The general model for the non-linear transmission line with both the elements as non-linear is shown in Fig.2.2.3.

To study the behavior of non-linear transmission lines, it is required to analyze the single non-linear device and the next chapter provides the analysis on the non-linear inductor and capacitor.

3 The non-linear elements

3.1 Introduction

The inductor and the capacitor are the two important passive circuit elements which have the ability to store and deliver finite amount of energy [49]. In an inductor, the energy is stored in the form of magnetic flux and in a capacitor the energy stored in the form of electric charge present on the electrodes [52]. The current-voltage relationships for these elements are time dependent. They are treated as linear elements in many applications. But all real inductors and capacitors exhibit some sort of non-linearity [53].

For an ideal linear inductor, the constitutive relationship between current and magnetic flux is linear. This results in a linear relationship between current and voltage. These relations are converted into frequency domain and parameter impedance is defined as the ratio of complex voltage to complex current through the device. Due to this, there are two types of non-linear inductors such as current dependent inductor and flux dependent inductor. But for a non-linear inductor, the constitutive relationship between current and magnetic flux is non-linear.

For an ideal linear capacitor, the constitutive relationship between voltage and charge is linear. For a non-linear capacitor, the relationship between voltage and current is given by non-linear differential equation. This chapter performs the analysis of non-linear elements namely non-linear capacitor and non-linear inductor. These devices are considered as time independent. The frequency response [4] of the non-linear inductor and capacitor are obtained and compared with the linear inductor and the linear capacitor.

For a linear circuit, the impulse response [6] can be used to find the frequency response. But this is not possible to get the impulse response of the non-linear device. The analysis of non-linear circuit is very specific depending on the type of non linearity. For the non-linear elements, the higher order differential equations are obtained and solved using numerical methods [54] in order to get the exact results.

Another way to analyze the non-linear circuits is by approximating the non-linear relationship with a linear equation using of Taylor's series expansion [55], about an operating point. This linearization [56] is most suitable for small signal. This analysis is not valid if the signals are large, in the sense that they change the operating point [57] of the device from its quiescent value.

This chapter does the analysis without any linearization. This chapter presents the theoretical study and simulations for a non-linear inductor and non-linear capacitor in time domain and frequency domain. The non-linear inductor is assumed to be flux dependent and non-linear capacitor is assumed to be charge dependent.

3.2 The inductor

3.2.1 Linear inductor

A current carrying conductor produces a magnetic field and this magnetic field is linearly related to the current that produces it. Michael Faraday discovered that a changing magnetic field can induce a voltage in a neighboring circuit [58]. He proposed that the induced voltage is proportional to the time rate of change of current producing the magnetic field. The constant of proportionality is defined as inductance and symbolized with L expressed in Henries. The current-voltage relationship for an ideal linear inductor is given in Eqn.3.2.1

$$v(t) = L\frac{di(t)}{dt} \tag{3.2.1}$$

The mathematical model of an ideal linear inductor is given in Eqn.3.2.1. In an ideal inductor the impedance is purely reactive and proportional to the inductance. In an ideal linear inductor the phase of the current lags the phase of voltage by 90^0. The impedance of the ideal linear inductor is given by $Z_L = j\omega L$. Practically, inductors are constructed using a coil of wire winded on a core made of ferromagnetic material, such as ferrite. Due to this, for a real inductor, it will have small resistance in series. This resistance represents the dissipation of energy or power. An ideal inductor acts as a short circuit to the direct currents.

3.2.2 Non-linear inductor

While inductors are modeled as linear components for the analysis purposes, all real inductors exhibit non-linear behavior [59]. Practical inductors are constructed with the ferromagnetic materials [60]. Ferromagnetic material has a non-linear relationship between the magnetic field intensity and magnetic flux density. The magnetic flux density saturates after applying large magnitudes of the magnetic field intensity. In other words, the magnetic field is not increasing proportionally with respect to current applied which is represented as a non-linear relationship between magnetic flux density and current passing through it.

For an inductor on iron core [61], the relationship between magnetic flux $\varphi(t)$ and current $i(t)$ passing through the loop are related as:

$$i(t) = \frac{N}{L}\phi(t) + A\phi^3(t) \tag{3.2.2}$$

where N is the number of turns of the coil, A is the cross sectional area of the core and L is the self inductance. Using the Faraday's law of induction [62] along with Lenz's law [63], the induced voltage can be obtained as:

$$v(t) = -N\frac{d\phi(t)}{dt} \tag{3.2.3}$$

The minus sign represents that the induced voltage opposes the inducing current. This equation is more accurate at low frequencies and large currents. For an ideal linear inductor the coefficient $A = 0$.

$$\phi(t) = \frac{L}{N}i(t) \tag{3.2.4}$$

Hence equation (3.2.1)is derived from the above equation.

$$v(t) = L\frac{di(t)}{dt} \tag{3.2.5}$$

which is a linear relationship between current passing through the inductor and induced voltage across the linear inductor.

For a non-linear inductor, it is little difficult to come up with a simple equation of $\phi(t)$ in terms of $i(t)$ not involving the radical sign. If the values of A,N and L are known, then with the help of curve fitting [64]it is easy to invert the same function and express $\phi(t)$ as some function of $i(t)$. If the modeling of the iron core inductor is more accurate, then it is possible to come up with the relationship between flux and current in the higher order terms (not only degree three polynomial but higher degrees) as well. But in practice, first few order terms are sufficient to represent the non-linear behavior of the device and hence non-linear devices are approximated with lower order polynomials.

The two main non-ideal characteristics of the inductor are resonance of the inductor and magnetic saturation [65]. At higher frequencies, the inductor goes through a resonance peak and the impedance then falls and a voltage phase shift of -90^0 is observed indicating the change in reactance from inductive to capacitive. This resonance happens in the practical device due to the parasitic capacitance that comes from the leads. The frequency at which the reactance changes from inductive to capacitive is called as self resonating frequency of the device. Some of the properties of the self resonating frequencies are listed below.

- The input impedance is at its peak value.
- The phase angle of the input impedance is zero.
- The effective inductance is zero.

For a non-linear inductor, the frequency is obtained in the following section and a similarity is established between non-ideal inductor characteristic and non-linear inductor.

3.2.3 Frequency responses of linear and non-linear inductors

For analyzing the frequency response of the non-linear inductor (which is flux dependent), a general expression of the following form has been considered.

$$i(t) = a_0 + a_1\phi(t) + a_2\phi^2(t) + a_3\phi^3(t) + \cdots \tag{3.2.6}$$

For a linear inductor all a_i s are zero except a_1. The significance of the even ordered terms and odd ordered terms in this non-linear relationship can be compared with physical nature of the device. If the hysteresis is neglected, the current has to reverse the direction if the magnetic flux direction is reversed. If only even ordered terms are considered in the relationship, by reversing the flux direction the current direction is not reversed. In reality, the current direction reverses, if the flux direction is reversed [62]. If this has to happen, then the coefficients of even terms must be of smaller in comparison with the odd ordered terms. Due to this reason, even though the above equation represents the more general case of a non-linear inductor, it is reasonably a good approximation to consider only odd terms in the non-linear relationship. Then the simplified formula for the non-linear inductor can be expressed as:

$$i(t) = a_1\phi(t) + a_3\phi^3(t) + a_5\phi^5(t) + \cdots \tag{3.2.7}$$

Usually, the practical devices are operated in the linear region and driven in to non-linear region within the device's overall operating regions. It is again possible to express the original non-linear relation accurately with fifth order polynomials. This chapter considered the analysis with this approximation. Again using the Faraday's law of magnetic induction, the relationship between voltage and current can be obtained by using the above equation.

To get the frequency response of the non-linear inductor, the constitutive relation considered is:

$$i(t) = 10\phi(t) + 1\phi^3(t) + 0.02\phi^5(t) \tag{3.2.8}$$

Here these values are chosen to see the effect of relatively small non-linearity on its impedance. These values can be scaled down in order to see the actual effects of the non-linearity in the devices at high frequencies. Because of this, the conclusions do not change even if the simulations are carried out at higher frequencies. This equation is represented as a flux dependent inductor. But to use the Faraday's law of induction, it is required to have flux as a function of current. Using the mathematical procedure of curve fitting, the flux dependent inductor can be converted into current dependent inductor with an approximate relationship as given below. This is the modeling of the inductor as current dependent device. This is being done to get the relationship between current and voltage in differential equation.

$$\begin{aligned}\phi(t) = & -9.133\times10^{-16}+0.08105i(t)+5.273\times10^{-18}i^2(t) \\ & -1.574\times10^{-5}i^3(t)-9.401\times10^{-22}i^4(t)+1.456\times10^{-9}i^5(t)\end{aligned} \tag{3.2.9}$$

From this the relationship between voltage and current is obtained as

$$v(t) = L(i)\frac{di(t)}{dt} \tag{3.2.10}$$

From this equation, it is observed that the non-linear inductor is acting like a variable inductance whose inductance depends on the current passing through the inductor.

The relationship between the original equation and inverted equation for the non-linear device and for the linear device are shown in Fig.3.2.1.

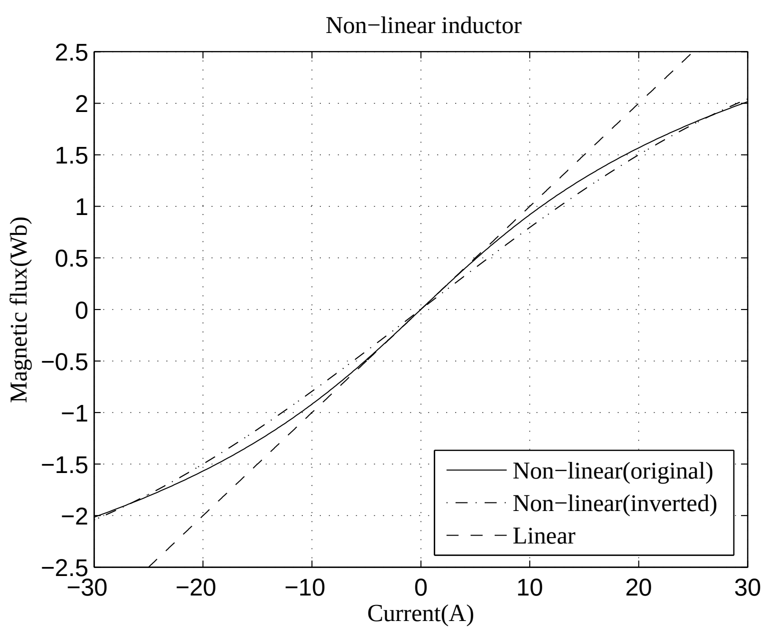

Figure 3.2.1: Relationship between current and magnetic flux for linear and non-linear inductors.

The frequency response for this non linear inductor is obtained by applying a Gaussian current pulse [66] as:

$$i(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t-\mu)^2/2\sigma^2} \tag{3.2.11}$$

with $\sigma = 0.04$ and $\mu = 2$.

Then the voltage across the inductor is obtained by solving the differential equation. The variations of currents and voltages for a non-linear and linear inductor are shown in Fig.3.2.2. From the graph, it is observed that the variations on time domain are very small. The variation of the inductance as a function of current is shown in Fig.3.2.3. To see the effect of this in frequency domain, its impedance has been obtained using the Fourier transform theory.

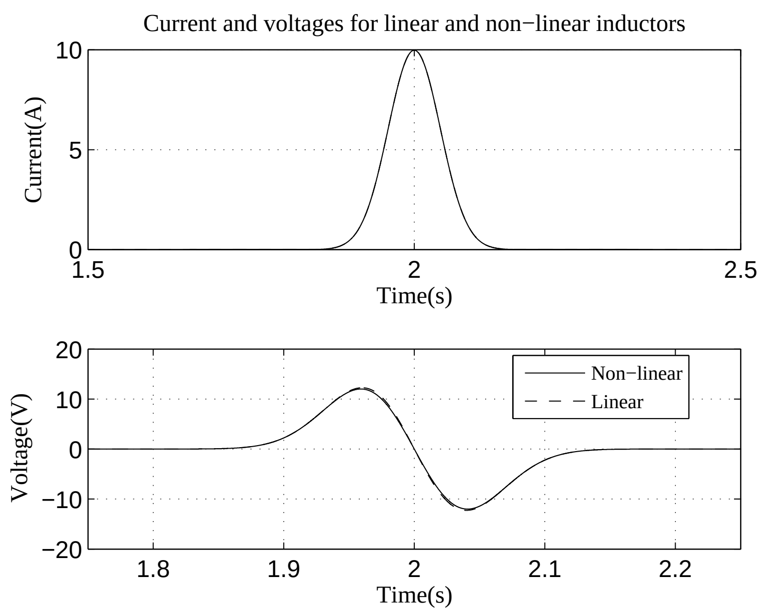

Figure 3.2.2: Time domain signals applied for linear and non-linear inductor.

The impedance of the non-linear inductor has been obtained using Fast Fourier Transform (FFT) [67] and shown in Fig.3.2.4. From the graph, it is observed that the non-linear inductor is changing its nature from inductive

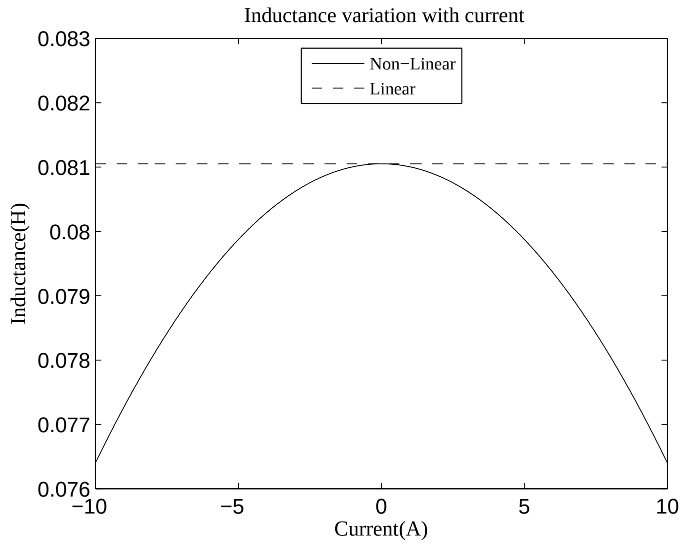

Figure 3.2.3: Variation of inductance with applied current.

to capacitive at 12 Hz for this non-linearity. This is the self resonating frequency of the non-linear inductor. The self-resonant frequency of an inductor is defined as the frequency at which the inductors reactance becomes negative, meaning it starts to behave like a capacitor. Therefore, the self-resonant frequency of an inductor establishes the absolute upper frequency limit [33] on its useful operating frequency range. From this figure, it is also observed that the reactance is decreasing and then increasing suddenly after the self resonating frequency.

3.3 Capacitor

3.3.1 Linear capacitor

In a linear capacitor, charge on electrodes and potential difference between them is related with a linear equation as:

$$v(t) = \frac{q(t)}{C} \tag{3.3.1}$$

with the proportional constant C being defined as the capacitance expressed in Farads.

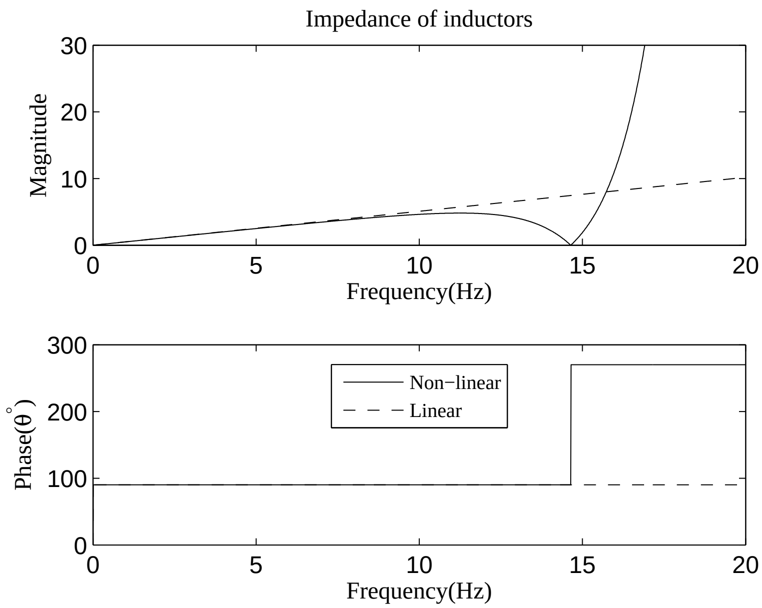

Figure 3.2.4: Impedance of non-linear and linear inductors.

Using the relationship between charge and current, the relationship between voltage and current can be obtained as:

$$i(t) = \frac{dq(t)}{dt} = C\frac{dv(t)}{dt} \tag{3.3.2}$$

Similar to the inductor, the impedance of the capacitor is obtained as the ratio of complex voltage to complex current which is equal to $Z_C = 1/j\omega C$. For a non-linear capacitor, this is not valid and impedance has to be obtained by solving the non-linear differential equation.

3.3.2 Non-linear capacitor

Using the same arguments made in the previous sections, the capacitors are assumed to be charge dependent as shown in Eqn.3.3.3 for different values of $b_1, b_2$and b_3.

$$v(t) = b_1 q(t) + b_3 q^3(t) + b_5 q^5(t) \tag{3.3.3}$$

These equation are inverted using MATLAB [68]with the help of curve fitting tool and obtained the general equation as shown in Table.3.1.This curve fitting is done with fifth order polynomial. Here three different non-linearity are considered to compare their frequency responses.

$$q(t) = c_0 + c_1 v(t) + c_2 v^2(t) + c_3 v^3(t) + c_4 v^4(t) + c_5 v^5(t) \quad (3.3.4)$$

S.No	b_1	b_3	b_5	$c_0(10^{-16})$	c_1	$c_2(10^{-17})$	c_3	$c_4(10^{-19})$	$c_5(10^{-6})$
1	1	0.01	0.0001	-9.981	0.8624	-1.059	-0.002317	1.382	3.209
2	1	0.05	0.0005	-4.634	0.6758	-2.533	-0.002237	1.064	3.329
3	1	0.1	0.001	17.54	0.5889	-0.7998	-0.002074	-0.3165	3.164

Table 3.1: Invertion of constitutive relationships of non-linear capacitors.

The non-linear relationships for these three cases are represented in Fig.3.3.1 while the variations of the capacitance with respect to voltage are as shown in Fig.3.3.2. The time domain voltage and current for this non-linear inductor are represented in Fig.3.3.3. From this figure, if the non-linearity is more, then the pulse has been distorted more. To see the effect of non-linearity in frequency domain its frequency response is obtained and compared with the linear inductor as well.

3.3.3 Frequency response of linear and non-linear capacitors

In the same manner as followed in the previous section, the frequency repose of the capacitor is obtained by exciting the non-linear capacitor with a Gaussian voltage pulse and then obtaining the current passing through the capacitor. The values taken for the input voltage are $\sigma = 0.03$ and $\mu = 2.0$. Then these time domain signals are converted in to frequency domain to obtain the impedance. This is performed on three different non-linear capacitors with the same applied voltage to analyses the effect of linearity on the nature of impedance. The impedance variations of the non-linear capacitors are represented in Fig.3.3.4.

A close observation on the self resonating frequency of the capacitor has been analyzed and Fig.3.3.5 represents the variations in self resonant frequen-

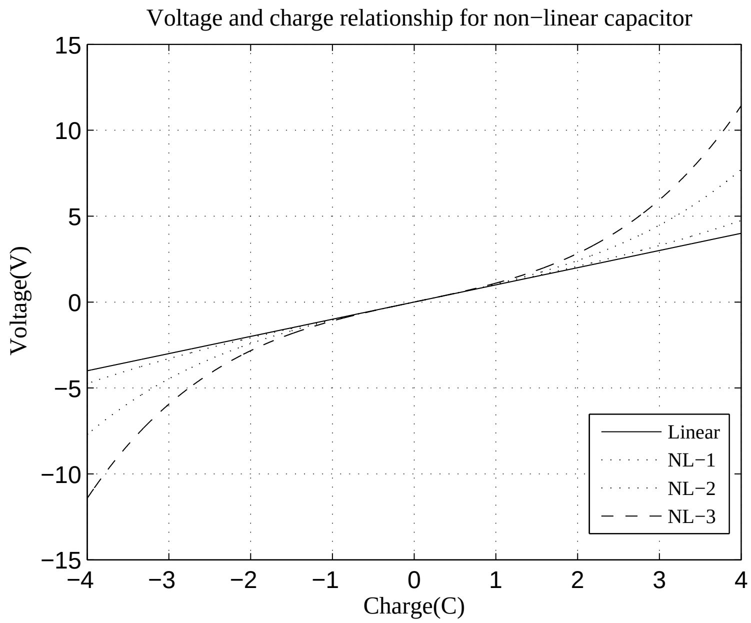

Figure 3.3.1: Non-linear capacitor constitutive relationships

cies of the non-linear capacitor. From this figure, it is observed that the self resonant frequency is occurring at lower frequencies if the non-linearity in the device is more. Hence, it can be concluded that the higher non-linear circuit can be used if the resonance has to happen at low frequencies depending on application.

3.4 Conclusions

From this analysis, it is observed that the non-linear inductor can be used as a capacitor beyond its self resonating frequency and vice-versa for the non-linear capacitor.. A non-linear inductor can be modeled as a variable inductor and non-linear capacitor as a variable capacitor. It is also observed that the considered non-linearity for the inductor is producing a resonance frequency and at this resonant frequency, it is acting like a short circuit, which means that the non-linear inductor can be modeled as a series resonating circuit.

Similarly, the non-linear capacitor is acting like a variable capacitor and exhibiting the resonant frequency. At this resonant frequency, this non-linear

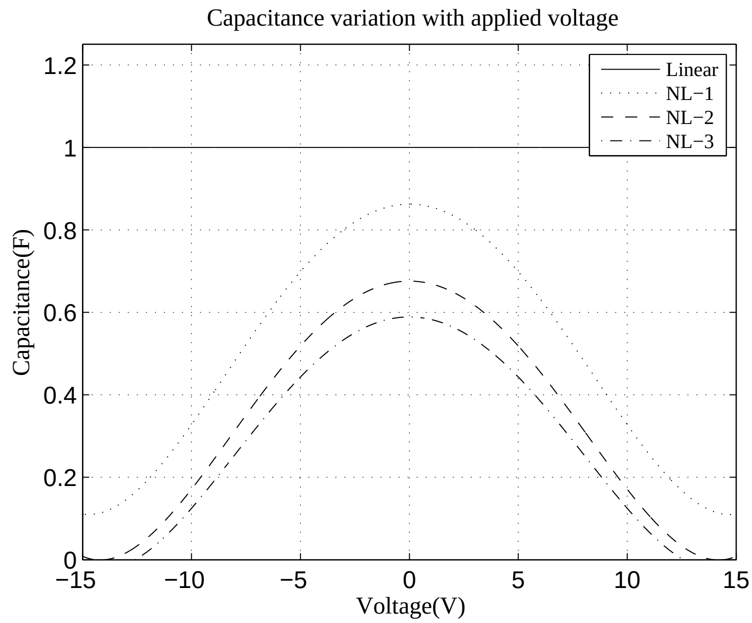

Figure 3.3.2: Variation in capacitance for different non-linearities

capacitor is showing high impedance. Hence this can be modeled as a parallel resonant circuit. It is also observed that, the magnitude of the resonating frequency is decreasing when the non-linearity in the device is increasing. Since a single element is able to produce resonance based on the applied signal, these non-linear elements can be used in filter designs if these devices are practically available with the assumed non-linearity.

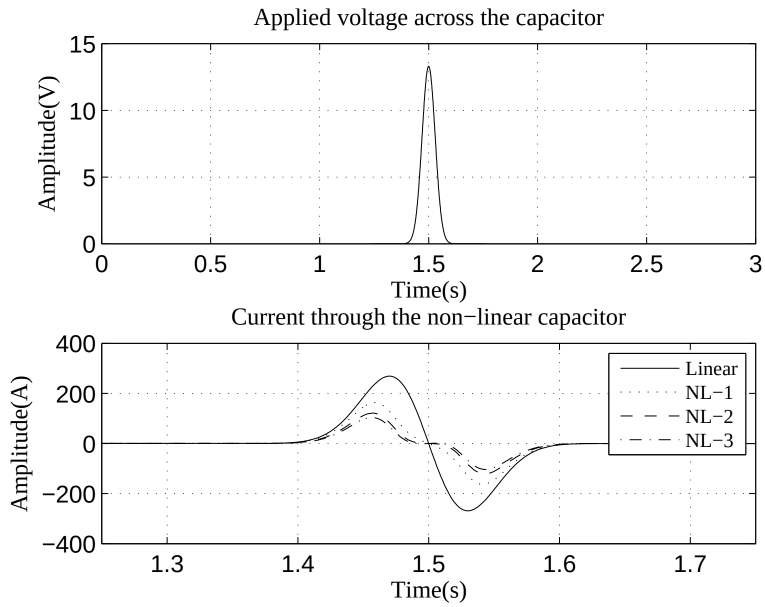

Figure 3.3.3: Time domain variations of voltage and current

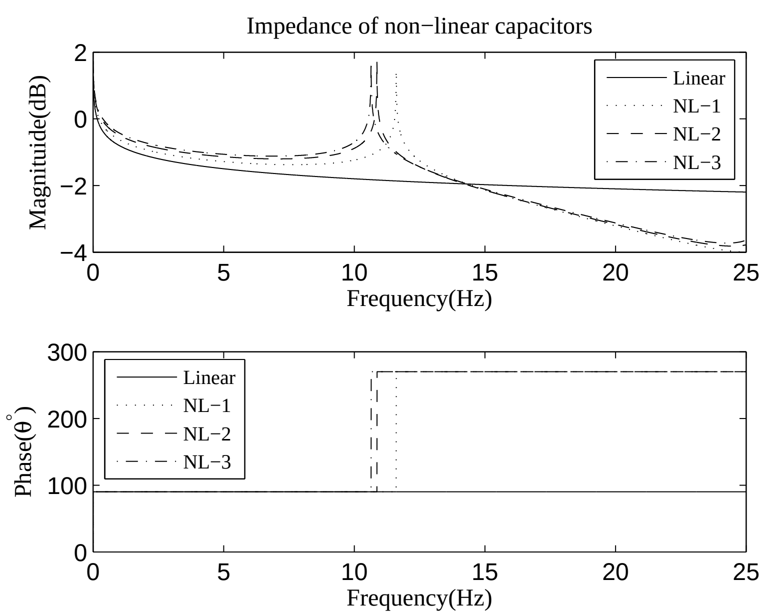

Figure 3.3.4: Impedance variations of non-linear capacitor

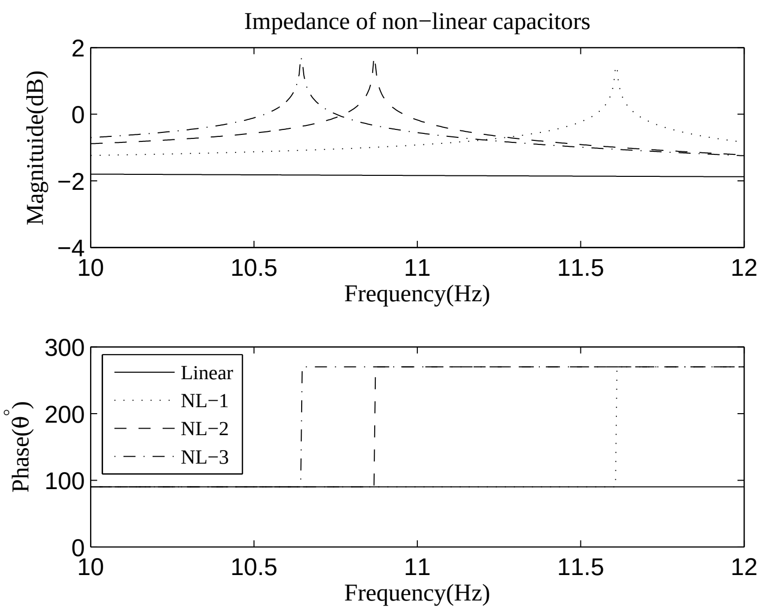

Figure 3.3.5: Changes in self resonating frequency with non -linearity

4 Analysis of conventional transmission line model with non-linear elements

4.1 Introduction

After the detailed analysis of non-linear inductor and non-linear capacitor is carried out, these elements are used in the transmission line equivalent circuit models to observe the effect of non-linearity on complex propagation constant. This analysis considers both the elements (inductor and capacitor) as non-linear. This chapter presents the analysis on low pass equivalent model with non-linear inductors and capacitors.

4.2 Methodology

The complex propagation constant and dispersion characteristics are obtained from the unit cell of the transmission line model [69, 70, 71]. To get the complex propagation constant, the scattering matrices [72] are obtained by exciting this unit with a voltage Gaussian pulse. Then the time domain voltages and currents are obtained by solving coupled ordinary differential equation [73]. These coupled differential equations are converted into state-space representation [74]. These equations are solved using the Runge-Kutta method [75]. All state-variables [76] such as voltage across the capacitor and currents through the inductors are obtained and then converted into frequency domain with the help of Fourier transform theory [5].

The scattering matrices are obtained from the impedance parameters. MATLAB is used in implementing the equations. These scattering parameters [77] are used to get the propagation constant using by first determining the effective dielectric constant and effective permeability as suggested in [78] . The same procedure is used for all models of transmission lines.

4.3 Different models considered in the analysis

In the analysis part, four different circuit models have been considered as listed below.

- Lowpass equivalent circuit model: In this, three different cases have been considered depending on which element is non-linear.
  - Shunt capacitor is non-linear.
  - Series inductors are non-linear.
  - Both series inductor and shunt inductors are non-linear.
- Highpass equivalent circuit model with shunt branch inductor as non-linear.
- Lowpass equivalent model with Miller loading.
- Highpass equivalent model with Miller loading.

In Miller loading two types of loads are considered. Firstly with the inductive loading and secondly with the capacitive loading.The analysis of Miller loading has been presented in next chapter.

4.4 Lowpass equivalent circuit model

The unit cell of the lowpass equivalent circuit model is as shown in Fig.4.4.1. This is also called as forward wave supporting structure. This circuit model is analyzed in three different cases. Initially with shunt capacitor as non-linear alone, secondly with the series inductors as non-linear and thirdly with both the series and shunt elements as non-linear.

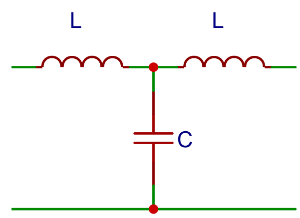

Figure 4.4.1: Unit cell of low-pass transmission line model

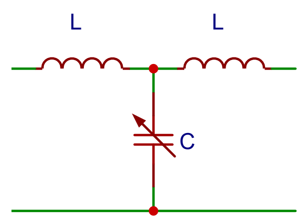

Figure 4.4.2: Unit cell of low-pass transmission line with non-linear capacitor

4.4.1 Non linear shunt capacitor in lowpass equivalent circuit model

The inductance value taken as $L = 0.01H$ and capacitor is a non-linear device whose relationship is given as:

$$\begin{aligned} q(t) \quad = \quad & -9.133\times10^{-16} + 0.08105v(t) + 5.273\times10^{-18}v^2(t) \\ & -1.574\times10^{-5}v^3(t) - 9.401\times10^{-22}v^4(t) + 1.456\times10^{-9}v^5(t) \end{aligned} \tag{4.4.1}$$

The variation of the capacitance as a function of applied voltages is represented in Fig.4.4.4.

Derivation of state-space equations for the lowpass equivalent circuit model with non-linear capacitor is obtained by simply using the KCL and KVL. In the state-space representation, current passing through the inductor and voltage drop across the capacitor are considered as state-variable. As shown in Fig.4.4.3, x_1, x_2 and x_3 are the state variables and after applying KCL and KVL, the following equations have been obtained.

Applying KVL at the input loop gives:

$$v_a = L_1\dot{x}_1 + x_2 \tag{4.4.2}$$

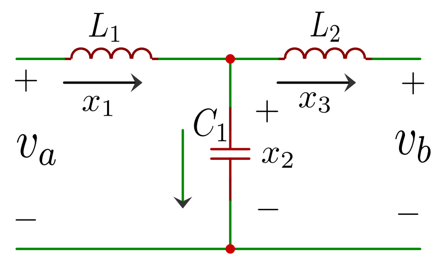

Figure 4.4.3: Lowpass equivalent circuit with state variables

Rearranging this, the following equation is obtained.

$$\dot{x}_1 = \frac{1}{L_1}\{v_a - x_2\} \tag{4.4.3}$$

Similarly, applying the KVL at the output loop gives:

$$x_2 = L_2\dot{x}_3 + v_b \tag{4.4.4}$$

Rearranging this, the following equation is obtained.

$$\dot{x}_3 = \frac{1}{L_2}\{x_2 - v_b\} \tag{4.4.5}$$

Finally, applying KCL at the capacitor junction, the following equation is obtained.

$$\dot{x}_2 = \frac{1}{C_1(x_2)}\{x_1 - x_3\} \tag{4.4.6}$$

Here capacitor has been considered as non-linear.

In the same manner, the general equations for all the elements as non-linear elements can be obtained as follows.

$$\dot{x}_1 = \frac{1}{L_1(x_1)}\{v_a - x_2\} \tag{4.4.7}$$

$$\dot{x}_3 = \frac{1}{L_2(x_3)}\{x_2 - v_b\} \tag{4.4.8}$$

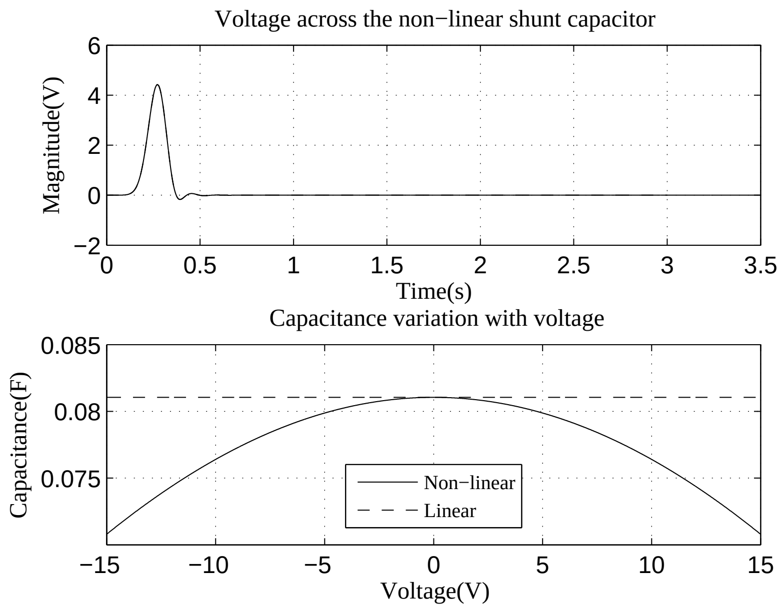

Figure 4.4.4: Variation of capacitance with applied voltage

$$\dot{x}_2 = \frac{1}{C_1(x_2)}\{x_1 - x_3\} \tag{4.4.9}$$

These equations are solved using the Runge-Kutta fourth order method.

This unit cell is excited with a Gaussian form voltage pulse of $\sigma = 0.05$ and $\mu = 0.25$. The time domain signals are converted into the frequency domain to observe the voltage gain of the unit cell in frequency domain. The input and output signals in time domain are represented in Fig.4.4.5. From this figure, it is observed that the gain of the unit cell is more than unity which indicates the amplification [79]. This is one of the important applications of non-linear circuit elements.

It is also observed that the phase variation is not monotonic which indicates that this phase response produces dispersion. Even though the distortion in the output pulse is not obvious from the Fig.4.4.5 in time domain, the variations are evident in the frequency domain with respect to gain and the phase variations.

The scattering matrices for the unit cell are obtained and shown in Fig.4.4.6. From this figure, it is obvious that around the frequency 20 Hz, both S_{11} and

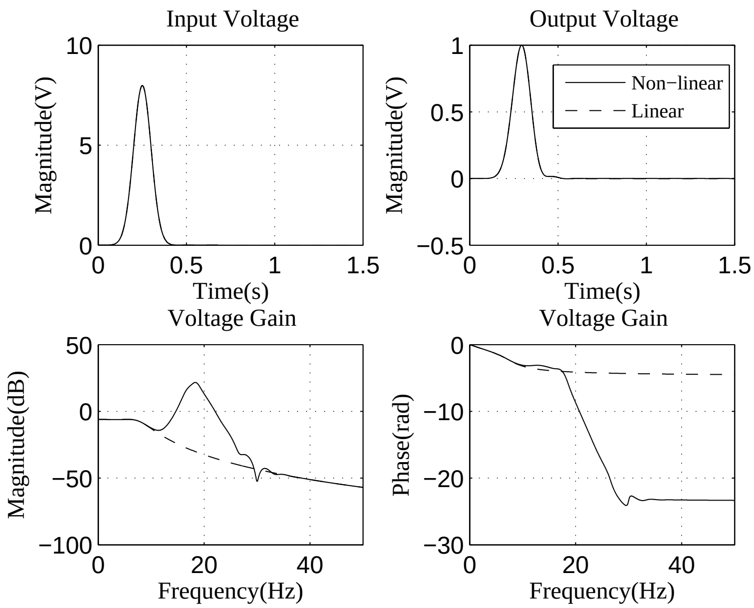

Figure 4.4.5: Gain of unit cell of lowpass transmission line model

S_{12} are beyond unity which is results in amplification of the signal due to the non-linearity. From the symmetry S_{12} is same as S_{21}. Usually, the phase of the scattering parameters is more important for the pulse propagation which gives the dispersion properties of the model. These Scattering parameters are used to obtain the complex propagation constant and a comparison is made between the non-linear and linear model unit cell.

The complex propagation constant for this model is obtained from the scattering matrices as shown in Fig.4.4.7. The main important observation made from this figure is that there is no difference in the pass band (up to 8 Hz approximately) of the linear and non-linear model. But there is a possibility for variation in phase constant from 10 Hz to 28 Hz approximately which indicates the propagation of the electromagnetic wave. The algorithm used is converging to constant values if there is no propagation of the wave through the model. From this figure it is observed that the non-linearity in the device can be used in generating the dual-band operating devices [80]. The original passband nature has no variations but there is another band of frequencies suitable for the wave propagation. In this case, the second pass band is around

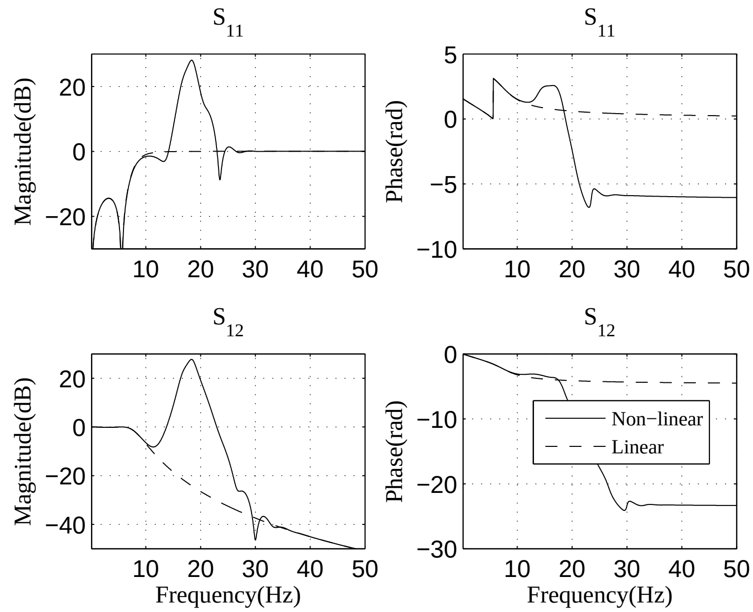

Figure 4.4.6: Scattering matrices of unit cell

20 Hz which has less attenuation and non-monotonic variation in the phase response which gives the opposite phase velocity and group velocity.

The reason for the variations in the phase constant has been observe by getting the equivalent shunt impedance of the non-linear capacitor. The negative resistance is not a new phenomenon in electronics but its presence in this model is an advantage as this can be used in controlling the gain of the unit cell. The shunt branch impedance is represented in Fig.4.4.8 . From this figure, it is observed that the negative resistance [81] is being produced which is oscillating over the band of frequencies.

4.4.2 Non-linear series inductors in lowpass equivalent circuit model

To see the effect of non-linearity in the series branches, the inductors are assumed to be non-linear as shown Fig.4.4.9. The values of the capacitor and inductirs are interchanged to compare this case with the previous case. The same approach is used to obtain the scattering parameters as shown

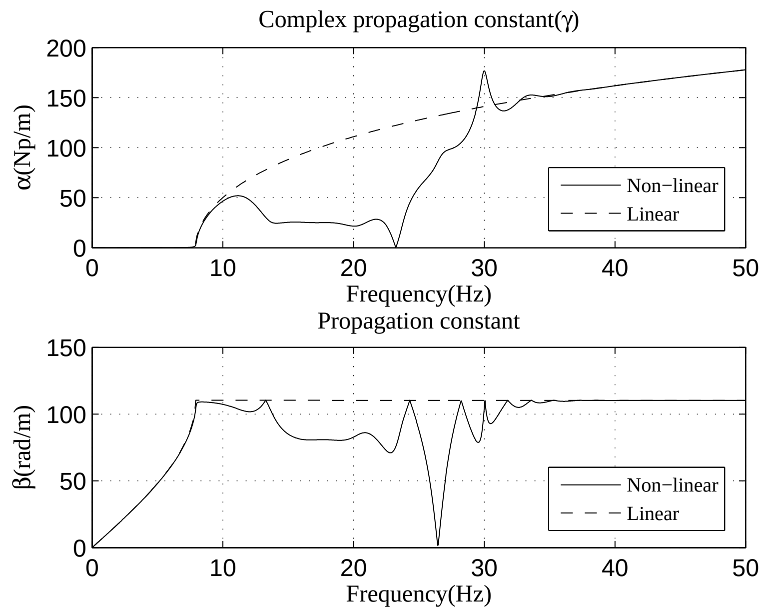

Figure 4.4.7: Complex propagation constant with non-linear capacitor

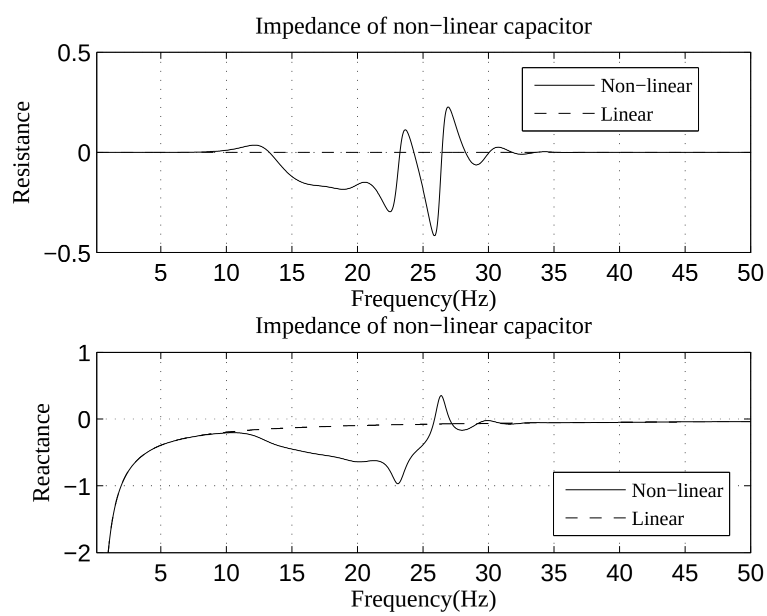

Figure 4.4.8: Impedance of non-linear capacitor

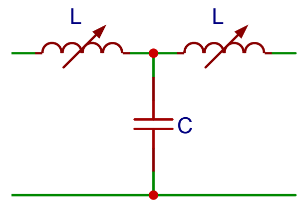

Figure 4.4.9: Unit cell of low-pass transmission line with non-linear inductors

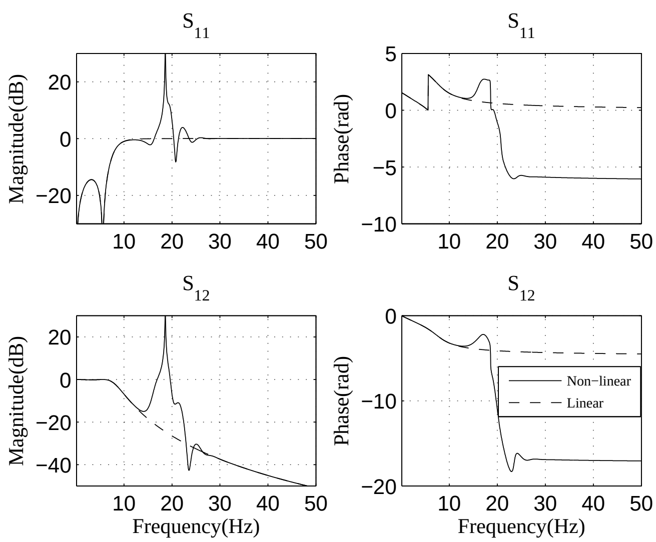

Figure 4.4.10: Scattering matrices of the unit cell with inductor as non-linear

in Fig.4.4.10 and there after the complex propagation constant as shown in Fig.4.4.11.

From Fig.4.4.10, it is again observed that there is no variation in the passband nature again but it is still affecting it in the stopband. Again there is a non-monotonic phase variation in the response of S_{21} which is same as the conclusion made in the previous section.

From the complex propagation constant for this model, it is observed that there is slightly smooth variation the propagation constant compared with the single non-linear capacitor. From this, it can be observed that more the non-linearity in the model, it can produce smooth variations in the complex propagation constant.

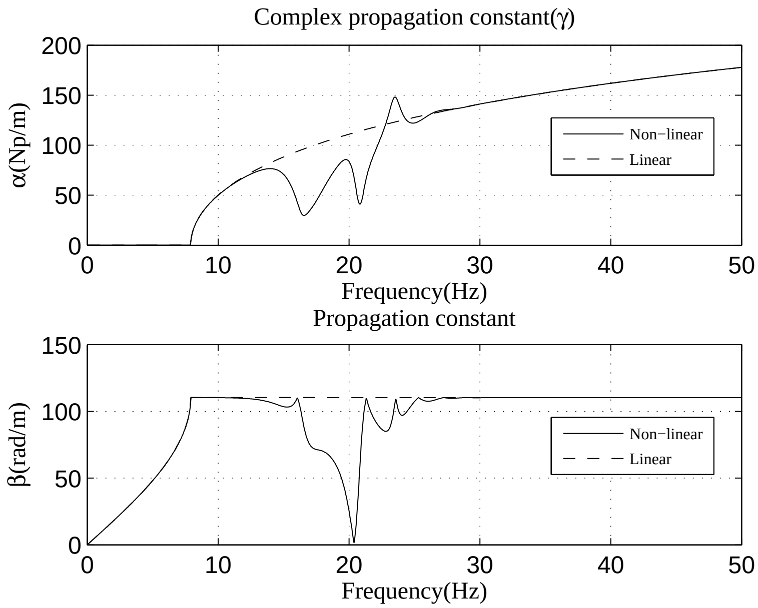

Figure 4.4.11: Complex propagation constant with series inductors as non-linear

4.4.3 Both non-linear elements in lowpass equivalent circuit model

Finally, the conventional transmission line model is analyzed by considering both the series element and the shunt element as non-linear as shown in Fig.4.4.12.The values of the capacitor and inductor are same. The complex propagation constant for this circuit model is represented in Fig.4.4.13. The same constitutive non-linear relationship is considered for both the cases as given as:

$$\begin{aligned} q(t) \;=\; & -9.133\times10^{-16}+0.08105v(t)+5.273\times10^{-18}v^2(t) \\ & -1.574\times10^{-5}v^3(t)-9.401\times10^{-22}v^4(t)+1.456\times10^{-9}v^5(t) \end{aligned} \tag{4.4.10}$$

for the capacitor and

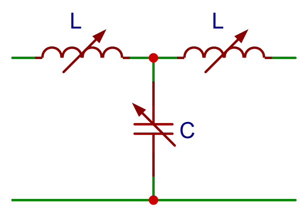

Figure 4.4.12: Unit cell of low-pass transmission line with non-linear elements

$$\begin{aligned}\phi(t) \quad = \quad & -9.133\times10^{-16}+0.08105i(t)+5.273\times10^{-18}i^2(t)\\ & -1.574\times10^{-5}i^3(t)-9.401\times10^{-22}i^4(t)+1.456\times10^{-9}i^5(t)\end{aligned} \tag{4.4.11}$$

for the inductor.

From the result it is observed that the propagation constant varies smoothly and it is in non-monotonic variation. And comparing these results with the previous case, this circuit model is producing variation in dispersion characteristic smoothly. Hence, these circuit models can be used for getting the opposite phase and group velocities.

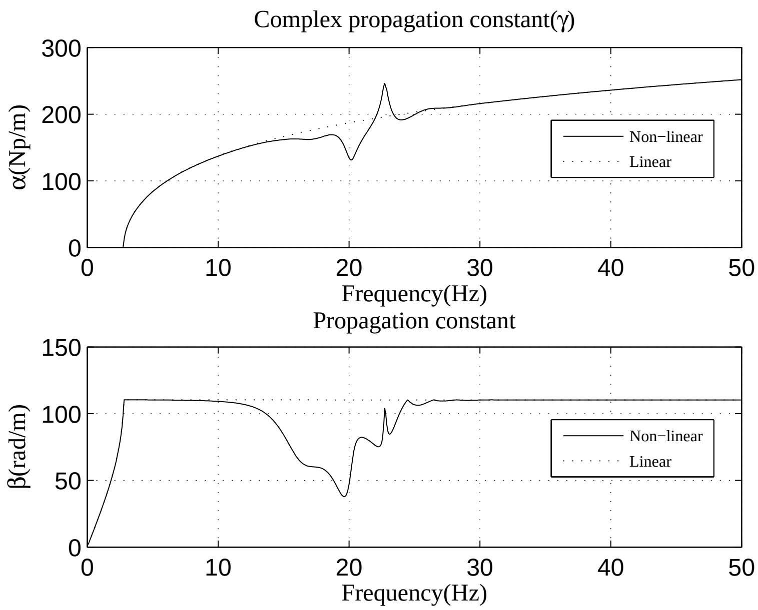

Figure 4.4.13: Complex propagation constant with both elements as non-linear

5 Analysis of highpass equivalent circuit model and Miller loading of transmission line models

5.1 Introduction

This chapter presents the analysis of highpass equivalent circuit model with non-linear inductor. This analysis is the dual of the previous models. In the highpass equivalent circuit model, only the shunt inductor has been taken as a non-linear element. From the circuit theory, using the principle of duality other models behavior can be obtained. Hence only one model has been analyzed and the results are presented for the single case.

Along with this, loading of conventional lowpass equivalent circuit model (which supports the forward waves) and high pass equivalent circuit model (which supports the backward waves) in Miller form [82] has been considered. These models are considered in order to come up with an equivalent circuit that produces opposite phase and group velocities with minimum number of components.

After the analysis, it is observed that the forward wave supporting structure with capacitive loading is producing the opposite phase and group velocities in a similar manner as composite right hand left hand metamaterial (CRLH-MTM). Likewise, the backward wave supporting structure with inductive Miller loading is producing opposite phase and group velocities. Hence these circuits can be used in place of CRLH-MTM by selecting proper values of inductor and capacitors.

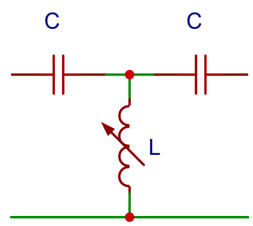

Figure 5.2.1: Unit cell of highpass transmission line model with non-linear inductor

5.2 Highpass equivalent circuit model with non-linear inductor

The dual of the previous model as shown in Fig.5.2.1 has been analyzed with the inductor as a non-linear element. The element values considered for this circuit are $C = 0.01F$ and the non-linear constitutive relationship for the inductor is given below:

$$\begin{aligned} i(t) = & -9.133\times10^{-16}+.08105\phi(t)+5.273\times10^{-18}\phi^2(t) \\ & -1.574\times10^{-5}\phi^3(t)-9.401\times10^{-22}\phi^4(t)+1.456\times10^{-9}\phi^5(t) \end{aligned} \tag{5.2.1}$$

The scattering parameters of this unit cell model with inductor as linear and non-linear element are represented in Fig.5.2.2. S_{21} is the gain of the unit cell and hence it is possible to use the phase response of S_{21} to get relative signs(positive or negative) of the phase velocity and group velocity. From this figure, it is observed that the phase of S_{21} is monotonic for the linear circuit where as the phase of $S_{21}$is non-monotonic due to the non-linearity. This results in non-monotonic variation in the dispersion relation for the unit cell.

From this figure, it is also observed that the gain of the unit cell is more than unity at which the phase changes its direction from decreasing to increasing. The other differentiating factor for this circuit models that, its gain has been changed in the original passband unlike the lowpass equivalent model.

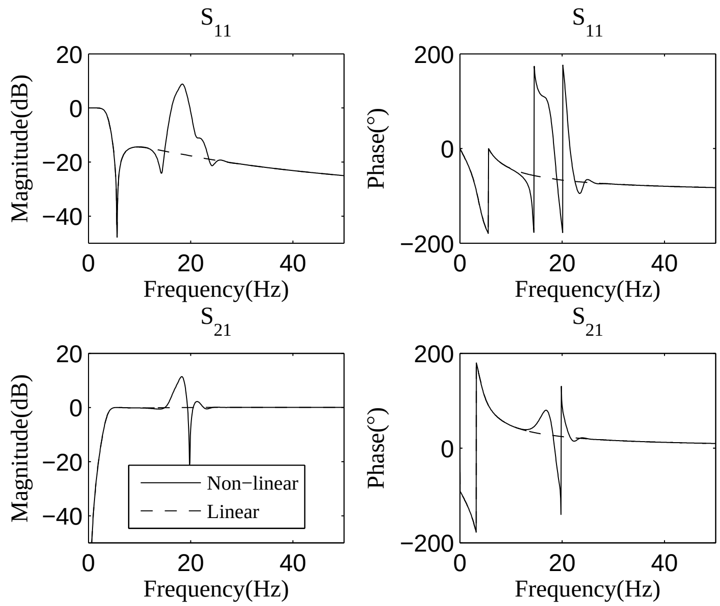

Figure 5.2.2: Scattering parameters for the unit cell of highpass equivalent circuit model with non-linear inductor

5.3 Loading of lowpass and highpass transmission line models in Miller form

As discussed in Chapter 2, the CRLH-MTM is used to get the opposite phase and group velocities which supports both forward waves and backward waves. To have the two modes of transmission (forward wave supporting structure and backward wave supporting structure) the transmission line model requires six components in total. But with the help of Miller loading, it is possible to get the same functionality as CRLH-MTM with respect to phase and group velocities with minimum number of components. This is the main advantage of the Miller loading with inductor and capacitor for the lowpass equivalent circuit model and highpass equivalent circuit model.

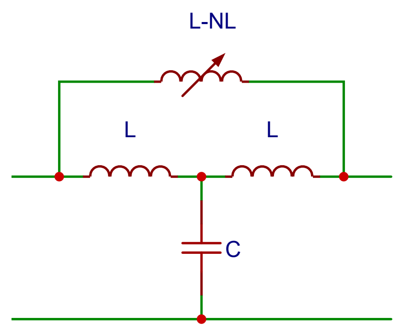

Figure 5.3.1: Unit cell of lowpass transmission line with Miller non-linearity inductor

5.3.1 Loading of lowpass equivalent circuit model with an inductor in Miller form

The conventional forward wave supporting structure has been loaded with an inductor as shown in Fig.5.3.1. A parametric study has been performed on this model for different values of Miller inductor. The gain of unit cell gain has been obtained and represented in Fig.5.3.2. The values taken for this circuit are $L = 1H$ and $C = 1F$. From this figure, it is observed that by increasing the values of the inductance, the gain is approaching towards the forward wave supporting structure without loading. But if an inductor has been connected, it is producing a resonant frequency at which the gain of the unit cell is zero.

At that resonant frequency the entire circuit model is acting like a short circuit [83] . This resonant frequency is small if the Miller inductor is small and tends to infinity if the value inductance is infinity. From the same figure, it is also observed that the variation in the phase is still monotonic no matter what may be the value of inductance. Hence this circuit model cannot produce opposite phase and group velocities.

To see actually whether this structure is supporting the wave propagation or not, its Bloch impedance [84, 85, 23] has been obtained and shown in Fig.5.3.3. The Bloch impedance for the T-network can be obtained by using the formula

$$Z_{Bloch} = \frac{\sqrt{(ZY)^2 + 2ZY}}{Y} \tag{5.3.1}$$

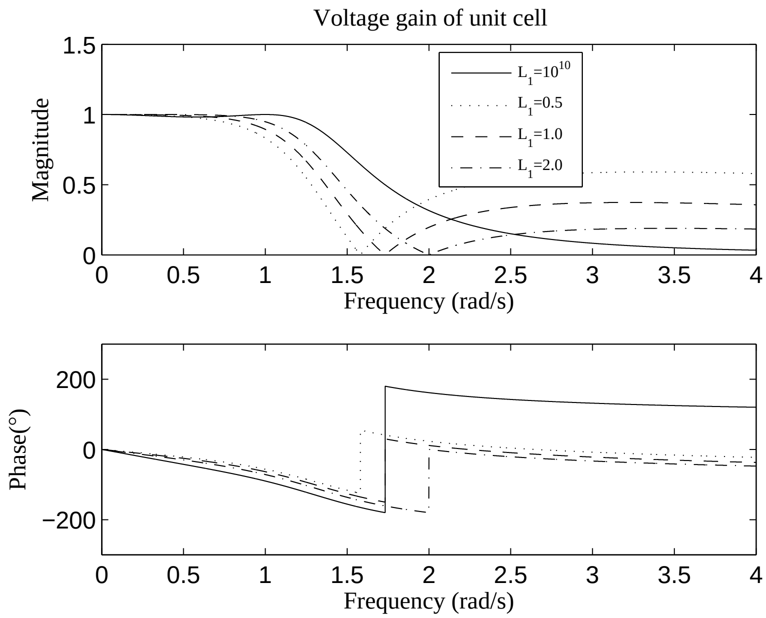

Figure 5.3.2: Voltage gain of unit cell with inductive loading

where Z is the series impedance of the T-network while Y is the shunt admittance of the symmetric T-network.

To get the Bloch impedance for the Miller circuit, it has been converted in to T-network using the impedance transformations [86]. The magnitudes of the Bloch impedance have been shown in Fig.5.3.3. If the Bloch impedance is real, it supports the wave propagation, if the Bloch impedance is imaginary, then it cannot support the wave propagation [87]. From this figure it is observed that by increasing the value of inductance, the Bloch impedance is approaching the characteristic impedance.

For this model, the complex propagation constant has been obtained and shown in Fig.5.3.4. This complex propagation constant is obtained by assuming that this medium is of infinite line length. For this $\gamma = \sqrt{ZY}$ has been used. From this figure, it is observed that this model is having a cutoff frequency above which it is not allowing the propagation of waves. The cutoff frequency of the model is increasing with increasing the values of inductance. Since the variations of β are monotonic for the entire band of frequencies, it is not possible to get the opposite phase and group velocities. Hence the

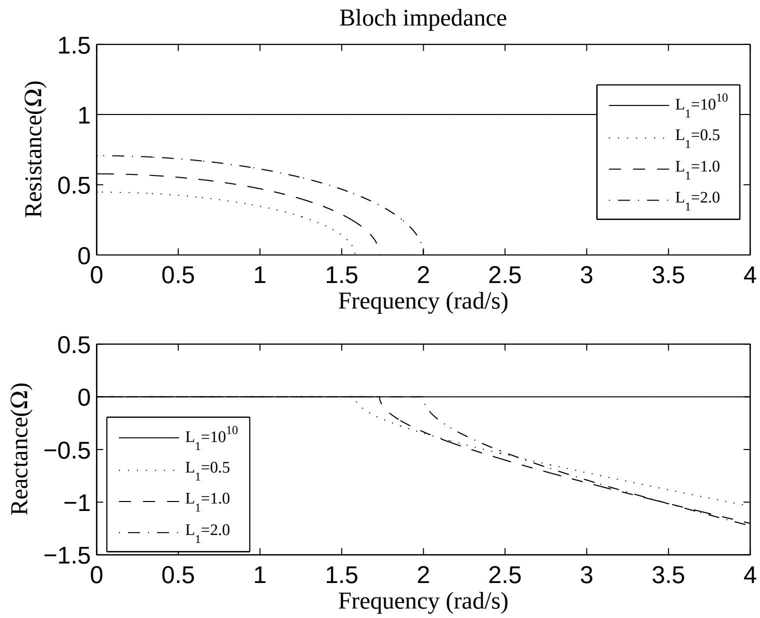

Figure 5.3.3: Bloch impedance of the lowpass equivalent circuit with Miller inductive loading

loading has been changed from inductor to capacitor.

5.3.2 Loading of lowpass equivalent circuit model with a capacitor in Miller form

The lowpass structure has been loaded with capacitive loading as shown in Fig.5.3.5. The voltage gain of the unit cell with different values of capacitance is represented in Fig.5.3.6. From this it is observed that there is a transition happening in the nature of the circuit from lowpass to bandstop for different values of capacitance. The effect of this variation has been analyzed further on its complex propagation constant. The Bloch impedance is obtained for this circuit model and shown in Fig.5.3.7. From this figure, it is observed that this model is supporting both the forward wave supporting structure and the backward supporting structure. The main advantage with this circuit is the minimum number of elements in it for the realization in microstrip version.

The complex propagation constant for different values of Miller capacitance as a linear element is shown in Fig.5.3.8.The values of the series branch

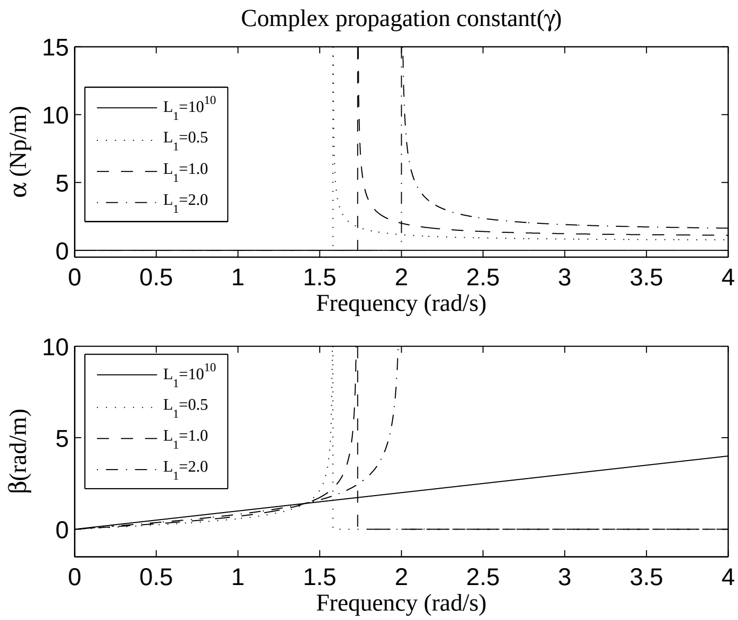

Figure 5.3.4: Complex propagation constant for lowpass with Miller inductive loading

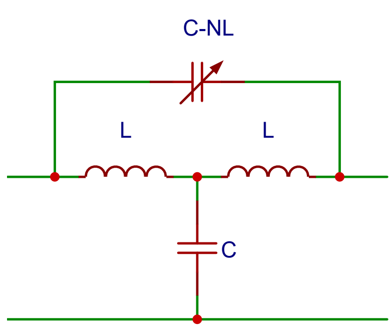

Figure 5.3.5: Unit cell of lowpass transmission line with Miller capacitive loading

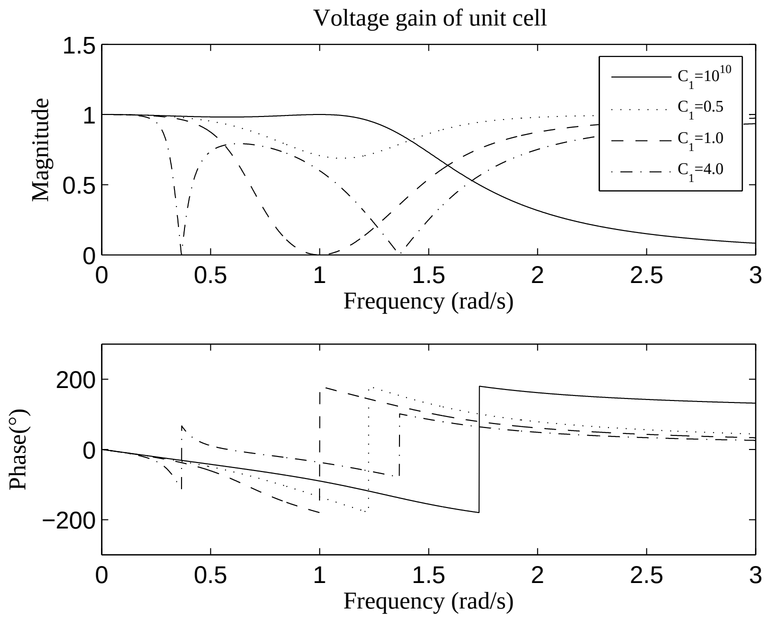

Figure 5.3.6: Voltage gain of unit cell with Miller capacitive loading

inductor has been taken as 1H and shunt capacitance has been taken as 1F. Different values of capacitance has been used to observe the complex propagation constant and the Bloch impedance variations. Bloch impedance variations are represented in Fig.5.3.7

The same circuit model is analyses with the non-linear capacitor to see the effect of non-linearity on the scattering matrices. The values considered for this circuit are $L = 0.01H$, $C = 0.01F$ with the non-linear capacitor constitutions relationship as

$$\begin{aligned} q = \quad & -1.996 \times 10^{-19} + 1.724 \times 10^{-4} v - 2.118 \times 10^{-21} v^2 \\ & -4.637 \times 10^{-7} v^3 + 2.764 \times 10^{-23} v^4 + 6.418 \times 10^{-10} v^5 \end{aligned} \tag{5.3.2}$$

From Fig.5.3.9 it is observed that there is a phase change in non-monotonic variation which represents the opposite phase and group velocities.

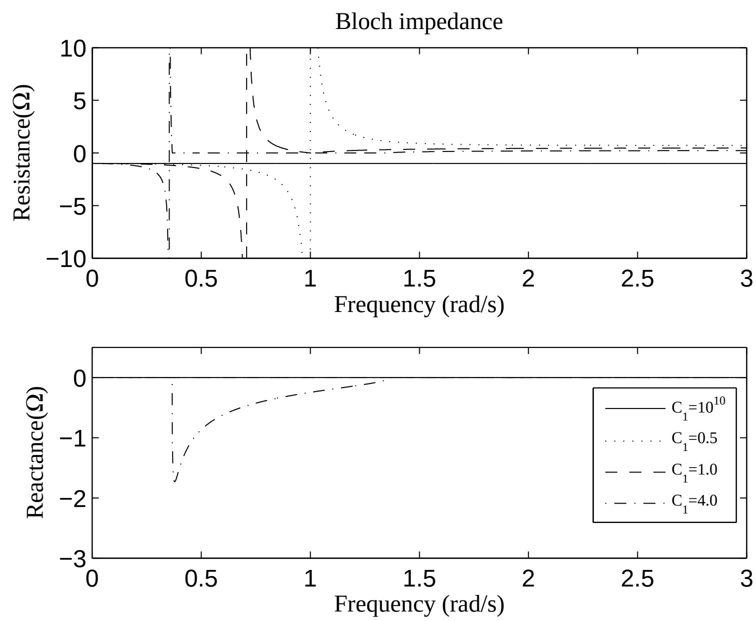

Figure 5.3.7: Bloch impedance of unit cell of Miller capacitive loading

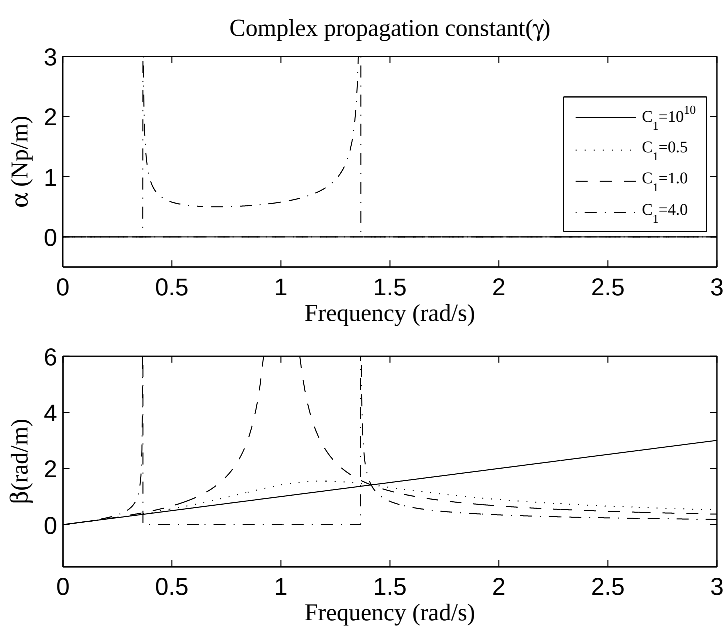

Figure 5.3.8: Complex propagation constant for lowpass Miller capacitive loading

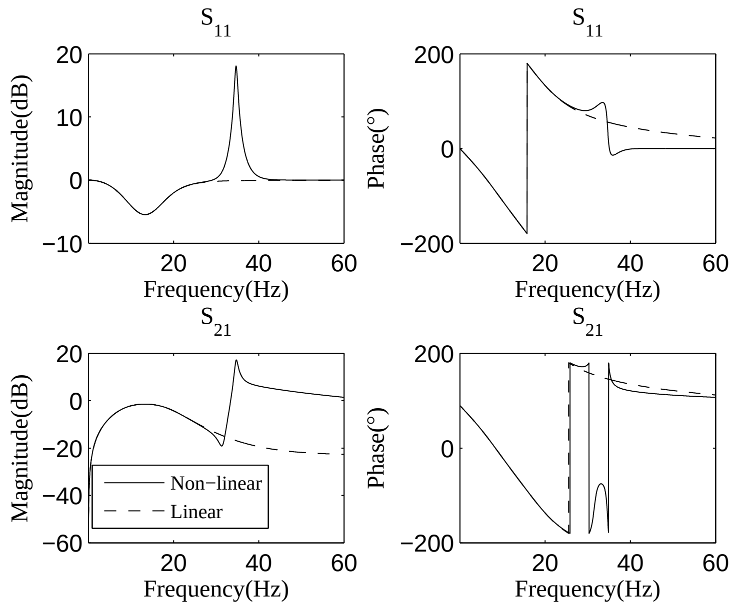

Figure 5.3.9: Scattering matrices for the lowpass Miller non-linear capacitive loading

5.3.3 Loading of highpass equivalent circuit model with an inductor in Miller form

This section presents the analysis of the highpass equivalent circuit model with the inductive Miller loading. This is the dual of the above case and hence only S-parameters are presented. The highpass equivalent circuit model with inductive loading as shown in Fig.5.3.10 is considered. The values taken for this circuit are $L = 0.01H$, $C = 0.01F$ and the non-linear relationship for the inductor has been taken as

$$\begin{aligned} i = \quad & -1.996\times10^{-19} + 1.724\times10^{-4}\phi - 2.118\times10^{-21}\phi^2 \\ & -4.637\times10^{-7}\phi^3 + 2.764\times10^{-23}\phi^4 + 6.418\times10^{-10}\phi^5 \end{aligned} \tag{5.3.3}$$

.

The scattering matrices of this model are as shown in Fig.5.3.11. From this it is observed again that the phase of S_{11} and $S_{21}$is varying smoothly in non-monotonic manner while the magnitude of S_{21}above-20dB.For this

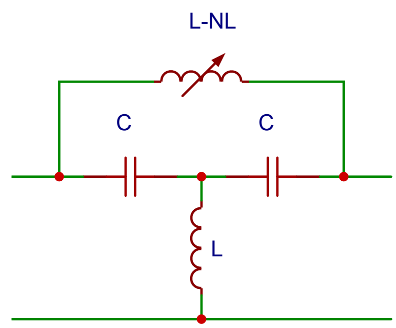

Figure 5.3.10: Highpass equivalent circuit with non-linear inductor Miller loading

circuit model with the selected values of inductance and capacitance, even though gain is less but the change in the gain is smooth which can be compensated by an amplifier. Hence this model is better choice compared with the lowpass and high pass circuit models with non-linear elements.From the above discussion, it is possible to say that the highpass circuit with inductive Miller loading is producing opposite phase and group velocities

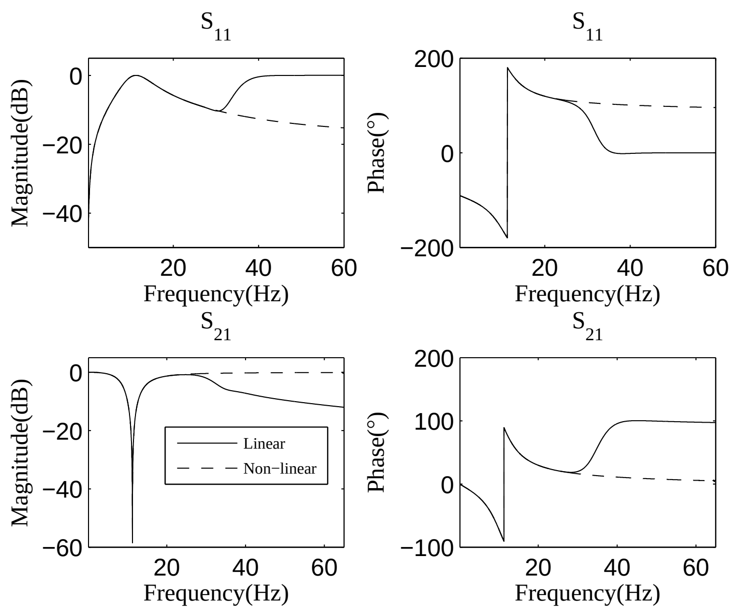

Figure 5.3.11: Scattering matrices for highpass equivalent circuit with non-linear inductor Miller loading

6 Conclusions, future scope and limitations

6.1 Conclusions

In this thesis, a detailed analysis of the non-linear inductor and capacitor has been performed and their frequency response is compared with the linear inductor and the linear capacitor. From this analysis, it is observed that the non-linearity in the device's constitutive relationship is producing the self resonance (changing the initial reactance nature from capacitive to inductive and vice-versa) which is characterized by its self resonating frequency. Variations in the self resonating frequency are observed separately for the non-linear inductor and capacitor.

Three different non-linear capacitors are analyzed with the same excitation and compared their self resonating frequencies with each other. From this it is observed that the self resonating frequency is occurring at lower frequencies if there is more non-linearity in the device.

After the analysis of individual non-linear element is performed, the effect of these non-linear elements on the complex propagation constant of the conventional transmission line models is observed. For all the models that have been analyzed with the non-linearity in the capacitor or inductor, it is observed that the considered non-linearity is not changing the frequency response of the unit cell in the original passband but it is effecting in the stopband.

The non-linearity results in allowing the signal propagation in the original stopband. The main conclusion from this analysis is that, with the small non-linearity, it is possible to have the transmission line models with dual bands. Along with this, the non-linearity is producing opposite phase and group velocities in the new passband. Hence the transmission line models with non-linear elements find applications where dual-bands are required.

This thesis also proposes a simpler circuit to produce opposite phase and group velocities by loading the conventional forward wave supporting structure (lowpass equivalent circuit model) and backward wave supporting structure (highpass equivalent circuit model) with suitable linear and non-linear elements. From the analysis, it is observed that Miller loading of backward wave supporting structure with inductor and Miller loading of forward wave supporting structure with capacitor are potential transmission line models to get the opposite phase and group velocities.

6.2 Future scope

In this analysis, the non-linear constitutive relationships considered for the inductor and the capacitor are of odd order polynomials. And only first three odd order terms (first, third and fifth degree) are considered in the constitutive relationships. Even though it is a reasonably good choice to approximate the practical non-linearity with first few odd order terms in the polynomial, it is also possible to have the devices with even ordered non-linearity as well.

In the future this analysis will be extended to even order non-linearity along with the combinations of even and odd non-linearity. In this thesis, hysteresis of the inductor is neglected, but in future this will also be incorporated.

6.3 Limitations

Even though all practical devices exhibit some sort of non-linearity, it is difficult to design the practical device with the desired non-linear constitutive relationship. The other main limitation of non-linear elements is about the stability. Most of the time the non-linear devices produce saturated outputs due to the supply conditions.

The other limitation of this work is that, even though non-linear capacitors and inductors are available as of now, these non-linear devices models are more accurate at low frequencies with high magnitudes of currents and voltages but not at high frequencies. But this work presents the general conclusions with non-linear elements in the transmission line models.

List of Publications

1. Salman Raju Talluri, Sunil Vidya Bhooshan, “Effect of a relatively small non-linear capacitor in the unit cell of transmission line model on scattering parameters and propagation constant”, International Journal of Applied Engineering Research ,Vol.11, no.16, pp. 3795-3798 , March , 2016. [Scopus]

2. Salman Raju Talluri, “Delay properties of a transmission line model with non linear Miller loading”, International Journal of Applied Engineering Research ,Vol.10, no.14, pp. 34985-34988, July 2015. [Scopus]

3. Salman Raju Talluri, “Analysis on phase properties of a transmission line model with non-linear elements”, Ain Shams Engineering Journal ,Vol.7 ,no.1, pp. 185-189,March 2016. doi:10.1016/j.asej.2015.11.007 [Scopus]

4. Salman Raju Talluri, Sunil Vidhya Bhooshan, “Fundamental study of a non-linear capacitor to use it in non-linear transmission line models”, International Journal of Advanced Research in Computer and Communication Engineering, Vol.3,no.7, pp.7594-7497, July 2014.

5. Salman Raju Talluri, Sunil Vidhya Bhooshan,“Analysis on some other models of transmission lines for CRLH meta-materials” International Journal of Advanced Research in Computer and Communication Engineering, Vol.3,no.7,pp.7395-7398, July 2014.

6. Salman Raju Talluri, Sunil Vidhya Bhooshan, “Effect of non-linearity on delays and velocities in a CRLH-MTM transmission line model” International Journal of Advanced Research in Computer and Communication Engineering, Vol.3,no.10,pp.8155-8159,October 2014.doi: 10.17148/IJARCCE.2014.31017