JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- 2025

Ph.D. (Mathematics)

COURSE CODE (CREDITS): 13P1WMA232 (3) MAX. MARKS: 35

COURSE NAME: MATHEMATICAL ANALYSIS

COURSE INSTRUCTOR: Pradeep Kumar Pandey MAX. TIME: 2 Hours

Note: *(a) All questions are compulsory.*

(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

Q. No.	Question	Marks
Q1	(i) Consider the metric space $(\mathbb{R}, d)$ with standard metric given by $d(x,y) = \lvert x-y \rvert, \forall x,y \in \mathbb{R}$. Find the interior of $\mathbb{Q} \subset \mathbb{R}$. (ii) Consider the metric space $(\mathbb{Q}, d)$ with standard metric given by $d(x,y) = \lvert x-y \rvert, \forall x,y \in \mathbb{Q}$. Find the interior of $\mathbb{Q}$.	3
Q2	Prove or disprove that $f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is differentiable $\forall x \in \mathbb{R}$.	3
Q3	Obtain the singularities of $f(z) = \frac{1}{(z-1)^2} + \frac{\sin z}{z} + e^{1/(z-2)}$ and sub-classify them according to number of terms in principal part of the Laurent's series.	3
Q4	State and prove Cauchy's Inequality $\lvert f^{(n)}(z_0) \rvert \leq Mn!/r^n, n = 1,2,3,\cdots$	3
Q5	Define harmonic conjugate of a function $u(x,y)$. Given that $u(x,y) = x^3 - 3xy^2 + x$ find its harmonic conjugate using Milne-Thomson method.	3
Q6	Show that $f(z) = \begin{cases} \frac{z^5}{\lvert z \rvert^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not differentiable at the origin though Cauchy-Riemann equations are satisfied at the origin.	4
Q7	Define a bounded linear operator between normed linear spaces. Suppose X and Y are normed linear spaces, then show that the collection $L(X,Y)$ of all bounded linear operators from X to Y is a normed linear space.	3
Q8	(i) State the Hahn Banach theorem. (ii) Give an example of a sublinear functional.	3
Q9	Define a Hilbert space. Consider $\mathbb{C}^3$ with the inner product $\langle z, w \rangle = \sum_{i=1}^{3} z_i \overline{w_i}$, where $z = (z_1, z_2, z_3)$ and $w = (w_1, w_2, w_3) \in \mathbb{C}^3$. Is $(\mathbb{C}^3, \lVert \cdot \rVert)$ is a Hilbert space. Also compute $\lVert z \rVert$ for $z = (1+i, 2i, -1) \in \mathbb{C}^3$.	3
Q10	Using contour integral evaluate $\int_0^{2\pi} \frac{1}{7+6\cos\theta} d\theta$.	3
Q11	State and prove the Closed Graph Theorem.	4

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