JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -1 EXAMINATION- 2025

Ph.D. (Mathematics)

COURSE CODE (CREDITS): 13P1WMA232 (3) MAX. MARKS: 15

COURSE NAME: Mathematical Analysis

COURSE INSTRUCTOR: P K Pandey MAX. TIME: 1 Hour

Note: *(a) All questions are compulsory.*

(b) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

Q.No.	Question	Marks
Q1	In the metric space $(\mathbb{R}^2, d_\infty)$ define and draw a sketch of the closed ball $B_r[0]$. Given that, for $x = (x_1, x_2)$ and $y = (y_1, y_2) \in \mathbb{R}^2$, $d_\infty(x, y) = Max\{\|x_1 - y_1\|, \|x_2 - y_2\|\}$.	2
Q2	Define a Cauchy sequence in a metric space, and show that every convergent sequence is a Cauchy sequence.	2.5
Q3	Find the lim sup and lim inf for the sequence $(x_n)_{n=1}^{\infty}$ where $x_n = \left(1 + (-1)^n + \frac{1}{2^n}\right)^{1/n}$.	2
Q4	Prove or disprove that $A \subset \mathbb{R}$, where by $A = \left\{\frac{1}{n} : n \in \mathbb{Z}^+\right\} \cup \{0\}$ is compact.	2
Q5	Check whether $\sum_{n=1}^{\infty} \frac{\sin(x^2 + n^2 x)}{n(n+1)}$ is uniformly convergent $\forall x \in \mathbb{R}$ or not?	2
Q6	Show that $\sin x$ is uniformly continuous on $[0, \infty)$.	2.5
Q7	Consider the metric space $(\mathbb{Q}, d)$ with usual metric d. Prove or disprove that it is a complete metric space.	2

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