Jaypee University of Information Technology, Waknaghat

TEST-2 Examination - October 2024

B.Tech - VII Semester (ALL)

Course Code/Credits: 22B1WMA731/3 — Max. Marks: 25
Course Title: Linear Algebra for Data Science & Machine Learning
Course Instructor: RAD — Max. Time: 90 mins

Note: (a) ALL questions are compulsory.

(b) The candidate is allowed to make suitable numeric assumptions wherever required.

Q.No	Question	CO	Marks
Q1	Which of the following is a *subspace* of $\mathbb{R}^3$? Justify your answer. (a) $\mathbf{W}_1 = \{(x_1, x_2, 1) \mid x_1, x_2 \in \mathbb{R}\}$ (b) $\mathbf{W}_2 = \{(x_1, x_1 + x_3, x_3) \mid x_1, x_3 \in \mathbb{R}\}$	CO-1	4
Q2	Let $\mathbf{T} : \mathbb{R}^2 \to \mathbb{R}^2$ be linear transformation: $$\mathbf{T}\left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \quad \mathbf{T}\left[\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right] = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$ (a) Compute $\mathbf{T}\left[\begin{pmatrix} 3 \\ 6 \end{pmatrix}\right]$. (b) Find a non-zero vector $\mathbf{v}$ such that $\mathbf{T}(\mathbf{v}) = 0$.	CO-1	4
Q3	Consider the vectors from $\mathbb{R}^3$: $$v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$ (a) Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent. (b) Find a linearly dependence relation among v_1, v_2, v_3.	CO-1	4
Q4	The linear transformation $\mathbf{T} : \mathbb{R}^5 \to \mathbb{R}^3$ is represented by matrix: $$\mathbf{A} = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$ (a) Find the `kernel` of the linear transformation $\mathbf{T}$. (b) Write down the basis for the `nullspace` of $\mathbf{A}$. (c) Give the values of $\texttt{rank}(\mathbf{T})$ and $\texttt{rank}(\mathbf{A})$.	CO-1	5

Q.No	Question	CO	Marks
Q5	Determine the *column space* of the following 3×4 matrix: $$\mathbf{B} = \begin{bmatrix} 1 & -10 & -24 & -42 \\ 1 & -8 & -18 & -32 \\ -2 & 20 & 51 & 87 \end{bmatrix}$$	CO-1	4
Q6	Let $\mathbf{W}$ be a subspace of $\mathbb{R}^n$ and $\mathbf{W}^\perp$ denote its *orthogonal complement.* Suppose that $\mathbf{W}_1$ is a subspace of $\mathbb{R}^n$ such that if $\mathbf{x} \in \mathbf{W}_1$, then $\mathbf{x}^T\mathbf{u} = 0$ for all $\mathbf{u} \in \mathbf{W}^\perp$: (a) $\mathtt{dim}(\mathbf{W}_1^\perp) \leq \mathtt{dim}(\mathbf{W}^\perp)$ (b) $\mathtt{dim}(\mathbf{W}_1^\perp) \leq \mathtt{dim}(\mathbf{W})$ (c) $\mathtt{dim}(\mathbf{W}_1^\perp) \geq \mathtt{dim}(\mathbf{W}^\perp)$ (d) $\mathtt{dim}(\mathbf{W}_1^\perp) \geq \mathtt{dim}(\mathbf{W})$ Which of the above statement is true? Justify your answer.	CO-2	2
Q7	In a manufacturing process, a robotic arm is designed to place objects onto a conveyor belt. The arm moves in 3D space, and the conveyor belt lies on the xy-plane that is represented by the subspace $\mathbf{W}$ spanned by the vectors $(1,0,0)^T$ and $(0,1,0)^T$. The current position of the robotic arm is given by the vector $\mathbf{v} = (3,4,5)^T$. (a) Find the *orthogonal projection* of the robotic arm's position vector $\mathbf{v}$ onto xy-plane. (b) What does $\mathtt{Proj}_{\mathbf{W}}(\mathbf{v})$ represent?	CO-2	2

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