JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATIONS- 2024

Ph.D. - II Semester (Mathematics)

COURSE CODE (CREDITS): 13P1WMA232 (3) MAX. MARKS: 35

COURSE NAME: MATHEMATICAL ANALYSIS

COURSE INSTRUCTORS: SST MAX. TIME: 2 Hours

Note: *(a) All questions are compulsory.*

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

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1. Prove that the space $l^p, p \neq 2$, is not a Hilbert space. **(CO 3)[3]**

2. State and prove the Cauchy-Schwarz inequality for inner product space and also prove that $|\langle x, y\rangle| = \|x\|\|y\|$ if and only if the set $\{x, y\}$ is not linearly independent. **(CO 3)[4]**

3. Let X and Y be Banach spaces and $T: D(T) \to Y$ is a closed linear operator, where $D(T) \subset X$, then prove that if $D(T)$ is closed in X, the operator T is bounded. **(CO 3)[5]**

4. Let f be a bounded linear functional on a subspace Z of a normed space X. Prove that there exists a bounded linear functional $\tilde{f}$ on X, which is an extension of f to X and has the same norm, $\|\tilde{f}\|_X = \|f\|_Z$, where $\|\tilde{f}\|_X = \sup_{\substack{x \in X \\ \|x\|=1}} |\tilde{f}(x)|$, $\|f\|_Z = \sup_{\substack{x \in Z \\ \|x\|=1}} |f(x)|$, and $\|f\|_Z = 0$ in the trivial case $Z = \{0\}$. **(CO 3)[5]**

5. Let $T: D(T) \to Y$ be a linear operator, where $D(T) \subset X$ and X, Y are normed spaces, then prove that T is continuous if and only if T is bounded. **(CO 3)[4]**

6. Prove that the linear functional f, defined by, $f(x) = \int_a^b x(t)dt, x \in C[a, b]$ is bounded and has the norm $\|f\| = b - a$. **(CO 3)[4]**

7. If $f(z)$ is an analytic function for all finite values of z and is bounded for all values of z in $\mathbb{C}$, then prove that f is a constant function. **(CO 2)[5]**

8. Prove that the set $C[a, b]$ of all real-valued functions continuous on the interval $[a, b]$ with the function d, defined by, $d(f, g) = \left(\int_a^b (f(x) - g(x))^2 dx\right)^{\frac{1}{2}}$, is a metric space. **(CO 1)[5]**