COURSE CODE (CREDITS): 17P1WMA111 (3)   MAX. MARKS: 25

COURSE NAME: DIFFERENTIAL GEOMETRY

COURSE INSTRUCTORS: P K Pandey   MAX. TIME: 1 Hour 30 Minutes

Note: (a) All questions are compulsory.

(b)Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

1. Compute $\nabla_V W$ for a vector $V \in \mathbb{R}^3$ given by (α, β, γ) and a vector field W given by $W = (x_1^3 x_2 + x_3^2, \ 3x_2^2 + x_1, \ 2x_2^2 + x_1 x_3^2)$.   [3M] [CO2]

2. For what values of c the set $f = \{(x, y, z) : x^2 - y^2 + z^3 - z = c\}$ represents a smooth surface in $\mathbb{R}^3$.   [3M] [CO3]

3. Compute the first fundamental form of $f : \mathbb{R}^2 \to \mathbb{S}^2 - \{(0, 0, 1)\}$ given by
$f(x, y) = \left(\frac{2x}{1+x^2+y^2}, \ \frac{2y}{1+x^2+y^2}, \ \frac{x^2+y^2-1}{1+x^2+y^2} \right)$ [3M] [CO3]

4. If $\eta = F(x, y, z) \, dxdy + G(x, y, z) \, dydz + H(x, y, z) \, dzdx$, find $d\eta$.   [3M] [CO3]

5. Compute the second fundamental form for the surface given by: [5M] [CO4]
$r = (u \cos v, \ u \sin v, \ cv)$, where $c \neq 0$.

6. Explain the following with examples: [4M] [CO4]
   (i)   Surface
   (ii)   Regular surface

7. Explain differentiable manifold by means of a example and obtain a differentiable structure on it.   :   [4M] [CO4]

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