JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST-1 EXAMINATIONS-2024

Ph.D.- II Semester (Mathematics)

COURSE CODE/CREDITS: 13P1WMA232/3 MAX. MARKS: 15
COURSE NAME: MATHEMATICAL ANALYSIS
COURSE INSTRUCTOR: SST MAX. TIME: 1 Hour

Note:(a) All questions are compulsory.

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make suitable numeric assumptions wherever required for solving problems.

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1. Let $\mathbb{R}_\infty$ be the extended set of real numbers i.e., the set of real numbers including $-\infty$ and ∞. The function d defined by $d(x,y) = |f(x) - f(y)|, \forall x, y \in \mathbb{R}_\infty$, where $f(x)$ is given by $f(x) = \begin{cases} \frac{x}{1+|x|}, -\infty < x < \infty \\ 1, x = \infty \\ -1, x = -\infty \end{cases}$
   Show that $(\mathbb{R}_\infty, d)$ is a bounded metric space. [4 Marks]
2. Prove that the space l_∞ is complete. [3 Marks]
3. Prove that every convergent sequence in a metric space is a Cauchy Sequence. Also for the metric space $((0,1], d)$, where $d(x,y) = |x - y|$, test whether the sequence $(\frac{1}{n})_{n\in\mathbb{N}}$ is complete or not. [2 Marks +1 Mark]
4. State and prove Bolzano-Weierstrass theorem for real sequences. [3 Marks]
5. (a) Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0,1]$.
   (b) Find the limit superior and limit inferior for the sequence $(\sqrt{n}sin\frac{n\pi}{2})_{n\in\mathbb{N}}$ and check whether the limit of the sequence exists or not. [1 Mark +1 Mark]