JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- 2023

B.Tech-I Semester (B.Sc)

COURSE CODE (CREDITS):22BS1MA112 (04) MAX. MARKS: 35

COURSE NAME: LINEAR ALGEBRA

COURSE INSTRUCTORS: BKP*, RKB, MDS MAX. TIME: 2 Hours Minutes

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***Note:** (a)All questions are compulsory.*

(b)Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

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1. Express $v = (1, -2, 5)$ in R^3 as a linear combination of the vectors $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 3)$ and $u_3 = (2, -1, 1)$. (CO-3) [5]

2. Let $T: R^3 \to R^3$ be define as $T(x_1, x_2, x_3) = (3x_1 + x_3,\ -2x_1 + x_2,\ -x_1 + 2x_2 + 4x_3)$, then

   a) Show that T is a linear transformation.
   b) What is the matrix for T with respect to the standard basis $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ for R^3.
   c) Find the transition matrix P from the ordered basis $B' = \{(1,0,1),\ (-1,2,1), (2,1,1)\}$ to the ordered basis B.

   (CO-3)[2+2+3]

3. On $P_2(R)$ (set of polynomials up to degree 2 on set of real numbers), consider the inner product given by $< p,q >= \int_0^1 p(x)q(x)dx$. Apply Gram-Schmidt orthonormalization process for the basis $\{1, x, x^2\}$ to produce an orthonormal basis. (CO-4) [4]

4. (a) If α and β are orthogonal unit vectors then find the distance between them.

   (b) If α and β are orthogonal vectors in complex inner product space orthogonal unit vectors then show that $\|a\alpha + b\beta\|^2 = \|a\alpha\|^2 + \|b\beta\|^2$. (CO-4) [1+2]

5. (a) Decompose a vector $X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ into the sum of two vectors, one of which is parallel to $Y = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and one of which is perpendicular to Y. (CO-4) [2]

(b) Obtain a least square solution to the set of equations $x - 2y = 5;\ x + y = 3$ and $2x - y = 4$. [2]

6. Check whether the following matrices is linearly independent or linearly dependent:

$$\begin{bmatrix} 1 & 3 & 0 \\ -3 & 2 & -5 \\ 4 & 5 & 1 \end{bmatrix}$$

(CO-5) [4]

7. Find the eigenvalues and the corresponding eigenvectors of the following matrices:

$$\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$$

(CO-5) [4]

8. Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(CO-5) [4]

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