

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -3 EXAMINATION- 2023

B.Sc. (Mathematics and Computing) I Semester

COURSE CODE (CREDITS): 22BS1MA111 (04)

MAX. MARKS: 35

COURSE NAME: CALCULUS

COURSE INSTRUCTORS: Prof. K. Singh

MAX. TIME: 2 Hours

Note: (a) All questions are compulsory and they carry equal marks.

(b) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems

1. Suppose the derivative of the function $y = f(x)$ is $y' = (x - 1)^2(x - 2)(x - 4)$. At what points, if any, does f have a local minimum, local maximum, or point of inflection? [CO-1, CO-2]
2. Check the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+1)}{2n^2+n-1}$ for convergence/divergence. [CO-4]
3. Find the Taylor series of the function $f(x) = x^3 - 2x + 4$ with center at $a = 2$. [CO-4]
4. Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{2x}{(x^2+1)^2} dx$. [CO-3]
5. By considering different paths show that $\lim_{(x,y) \rightarrow (0,0)} \left(\frac{x^2}{x^2-y} \right)$ does not exist. [CO-4]
6. Check whether the function $f(x, y) = \frac{x}{y^2} + \frac{y}{x^2}$ is homogeneous or not? If yes, verify the Euler's theorem for homogeneous functions. [CO-4]
7. Use chain rule to evaluate $\frac{dw}{dt}$ at $t = 1$ if
 $w = f(x, y, z) = z - \sin(xy)$, $x = t$, $y = \ln t$, $z = e^{t-1}$. [CO-4]