JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

T-3 Examination- 2023

Ph.D. (Mathematics)- I Semester

COURSE CODE (CREDITS):17P1WMA231 (3) MAX. MARKS: 35

COURSE NAME: ADVANCED LINEAR ALGEBRA

COURSE INSTRUCTORS: Pradeep Kumar Pandey MAX. TIME: 2 Hours

Note: ***(a)*** *All questions are compulsory.*
(b)Marks are indicated against each question in square brackets.
(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems.

1. Suppose W is a vector subspace of $\mathbb{R}^4$ spanned by the vectors $v_1 = [1 \ \ -2 \ \ 5 \ \ -3]$, $v_2 = [0 \ \ 1 \ \ 1 \ \ 4]$, $v_3 = [1 \ \ 0 \ \ 1 \ \ 0]$. Find a basis for W and extend it to a basis for $\mathbb{R}^4$. [CO1] [6]

2. Suppose $B = \{[1 \ \ 0 \ \ 0], [0 \ \ 1 \ \ 0], [0 \ \ 0 \ \ 1]\}$, and $C = \{[1 \ \ 0 \ \ 1], [0 \ \ -1 \ \ 2], [2 \ \ 3 \ \ -5]\}$. Find a transition matrix from B to C and use it to find $[x]_C$ where $[x]_B = [1 \ \ 2 \ \ -1]^T$. [CO2] [6]

3. Using the inner product on $M_2(\mathbb{R})$ compute the angle between the vectors $X = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$, and $Y = \begin{bmatrix} 1 & -1 \\ 4 & 0 \end{bmatrix}$. [CO3] [6]

Hint: $\langle X , Y \rangle = tr\,(X^T Y)$.

4. Write the statement of *spectral theorem* for the real symmetric matrices. For the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Find an orthogonal matrix P and diagonal matrix D such that $P^T A\, P = D$. [CO4] [6]

5. Find the least squares solution of the following system: [CO4] [5]

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -3 & 1 & -5 \\ 1 & -1 & 2 & 0 \end{bmatrix}^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [1 \ \ 0 \ \ 1 \ \ 0]^T$$

6. Let A is a 3×3 matrix. If the characteristic and minimal polynomials of A are given by $f_A(x) = x^3 + x^2 - x - 1$ and $m_A(x) = x^2 - 1$. Then justify whether A is diagonalizable or not? [CO5] [3]

7. Does there exist a 3×3 diagonal matrix which is neither Hermitian, nor skew-Hermitian nor Unitary. Give an example in support of your answer. [CO5] [3]

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