JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2023

Ph.D.-I Semester (Mathematics)

COURSE CODE (CREDITS): 17P1WMA231 (3) MAX. MARKS: 25

COURSE NAME: ADVANCED LINEAR ALGEBRA

COURSE INSTRUCTOR: Pradeep Kumar Pandey MAX. TIME: 1 Hour 30 Minutes

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Note: *(a) All questions are compulsory.*

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems.

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1. For any $u, v \in \mathbb{R}^+$ and $\alpha \in \mathbb{R}$, consider the operations $\oplus$ and $\odot$ defined as: $u \oplus v = uv$ and $\alpha \odot u = u^{\alpha}$. Prove or disprove that the algebraic structure $(\mathbb{R}^+, \oplus, \odot)$ is a vector space over the Field $\mathbb{R}$. [CO-1] [5M]

2. State the Cayley-Hamilton theorem, and using it reduce the degree of polynomial $A^3 - 5A^2 + 2I$ for the matrix $A = \begin{bmatrix} 3 & 7 \\ 0 & 1 \end{bmatrix}$. [CO-1] [5M]

3. Suppose $B = \{(1,0,1), (1,-1,0), (2,0,-1)\}$ is an ordered basis of $\mathbb{R}^3$. Find the dual basis of B. [CO-2] [5M]

4. Check consistency of the following linear system, and solve it by method of least squares

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}^T \begin{bmatrix} x \\ y \end{bmatrix} = [0 \quad 3 \quad 4 \quad 4]^T$$

[CO-3] [5M]

5. Using Gram-Schmidt method on the columns of matrix A find its QR decomposition

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

[CO-3] [5M]

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